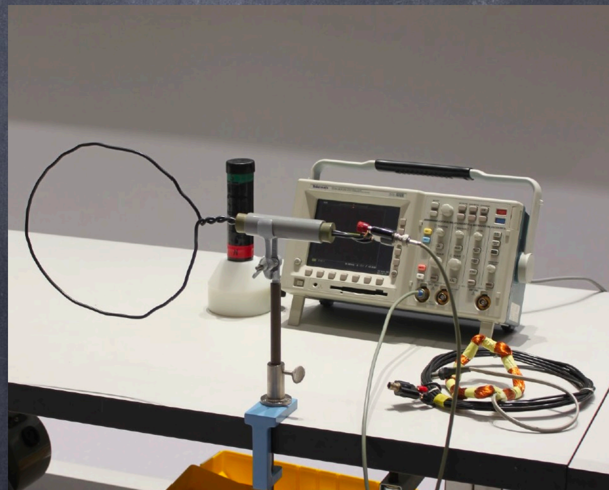
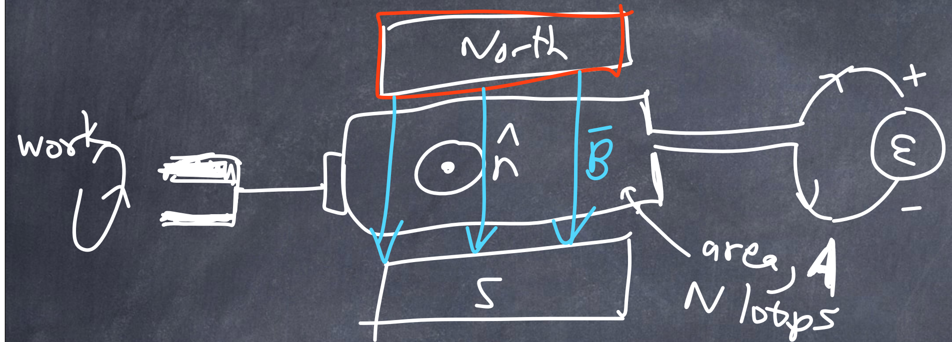
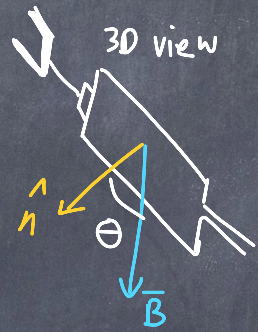
PHY117 HS2023

Week 11, Lecture 1
Nov. 28th, 2023
Prof. Ben Kilminster



Most electrical energy used today produced by AC (alternating current) electric generators. mechanical work → electrical energy



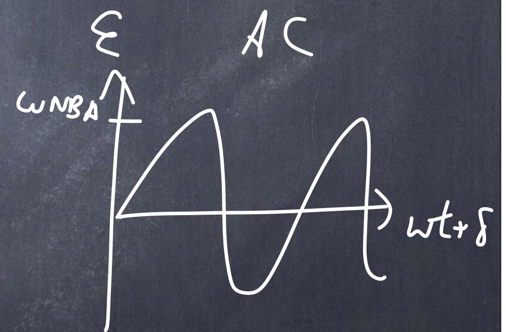


when $\bar B \perp \hat n$, $\theta = 90^\circ$, $\cos\theta = 0$ (no flux)

magnetic flux through loop: $\phi_m = NBA\cos\theta$

$\theta = \omega t + \delta$ ← starting phase

↑ angular velocity

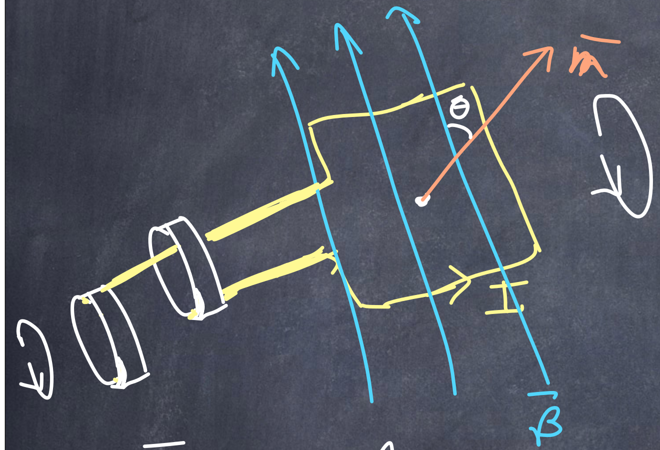


$$\phi_m = NBA\cos(\omega t+\delta)$$

$$\varepsilon = -\frac{d\phi_m}{dt} = -NBA\frac{d}{dt}\cos(\omega t+\delta) = +\underbrace{NBA\omega}_{\text{Amplitude, max Voltage}}\sin(\omega t+\delta)$$

A **motor** is a generator run in reverse; electrical energy → mechanical work

An AC current in the loop creates an alternating magnetic moment, $\bar{m}$.

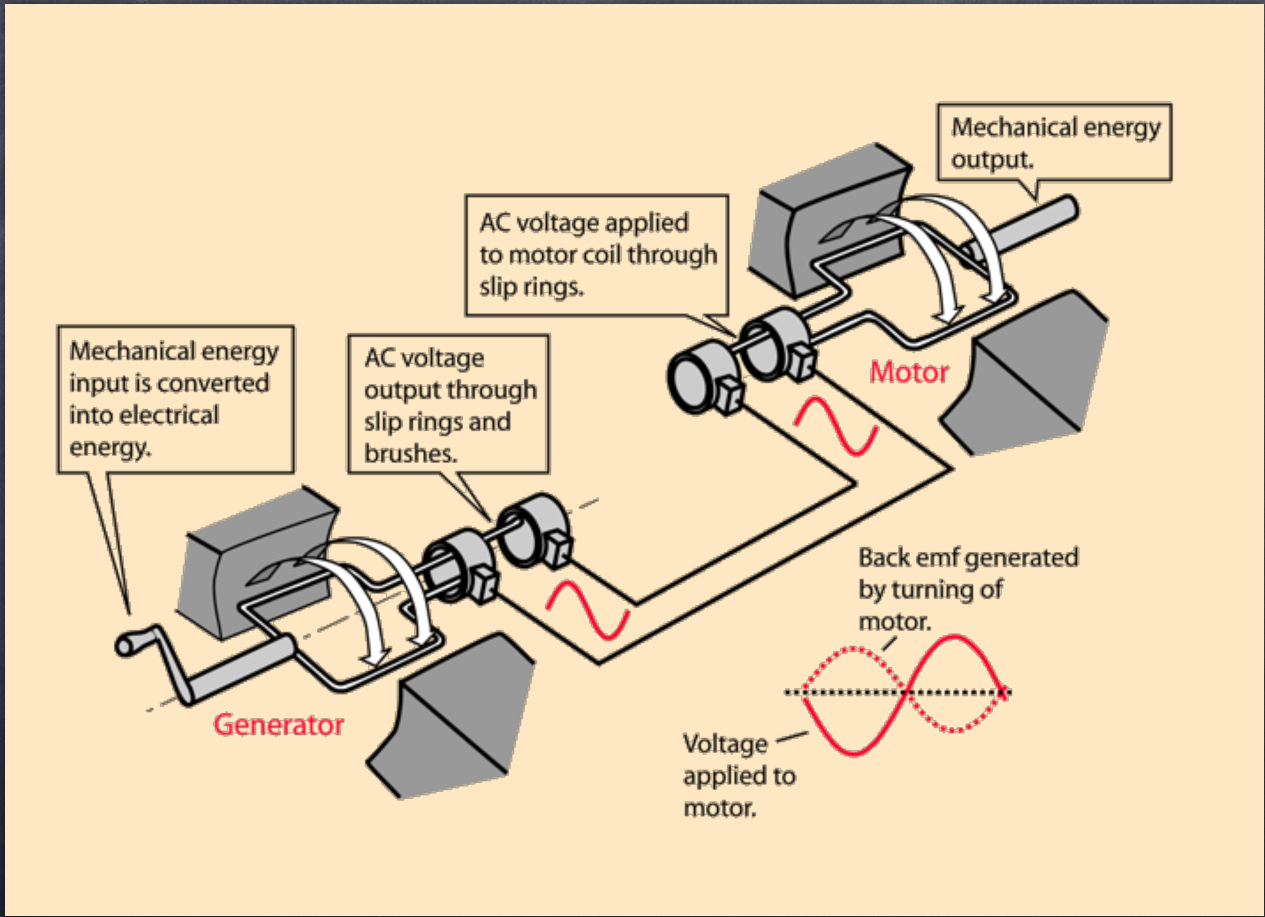


$\bar{m} = IA\,\hat{n}$

Torque $\bar{\tau} = \bar{m} \times \bar{B}$

By putting an alternating current through the loop, torque on the $\bar{m}$ from the B-field makes the loop spin.

Alternating current generator and motor run in series.



Magnetization - atomic level

Atoms have magnetic moments.

consider charged particle orbiting with mass m_q, charge q.

$A = \pi r^2$

Angular momentum

$$L = m_q v r$$

mass, velocity

I am using a funny "m" for magnetic moment.

The magnetic moment is in general, $m = IA = I\pi r^2$

The current $I = \frac{q}{T}$ charge per time to go in a circle.

$v = \frac{2\pi r}{T} \Rightarrow T = \frac{2\pi r}{v}$

So we get $I = \frac{qv}{2\pi r}$ and then $m = \frac{qvr}{2}$

now substitute in $L = m_q v r \Rightarrow m = \frac{qL}{2m_q}$ magnetic moment of spinning charged particle

For a positive charge, $\bar{m} = \frac{q\bar{L}}{2m_q}$

$\bar{m}$ + $\bar{L}$ are in the same direction.

For a negative charge, $\bar{m} = -\frac{q\bar{L}}{2m_q}$

$\bar{m}$ + $\bar{L}$ opposite direction

Classical relations

(assumes electron is continuously moving in orbit around atom)

Holds also for quantum theory, but...

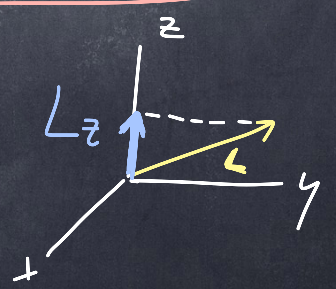
in quantum theory, orbital angular momentum is quantized. Typically, we talk about L_z (z-component of the angular momentum).

Why? Because often we have a B-field that is by convention in z-direction

What is L_z?

Assume $\bar{L}$ is our angular momentum.

L_z is the projection onto the z-axis.



L_z is quantized, $L_z = nh$ where $n = 0, \pm 1, \pm 2, \ldots$

$h = 6.63 \mathrm{E}{-34}$ J.s

we sometimes use $\hbar = \frac{h}{2\pi} = 1.05 \mathrm{E}{-34}$ J.s

The magnetic moment is then $m_z = \frac{-q}{2m_q} L_z$ for a negative charge

or $m_z = -m_B \frac{L_z}{\hbar}$ where m_B = Bohr magneton $= \frac{eh}{2m_e}$ for an electron

$= 9.27 \mathrm{E}{-24} \frac{J}{T}$

The magnetic moment of any atom is roughly $\sim |m_B|$ but depends on # of electrons and pairings

In a material, if magnetic moments align,

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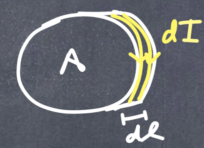
(a) Each atom has its own small current loops, and their own magnetic moment.

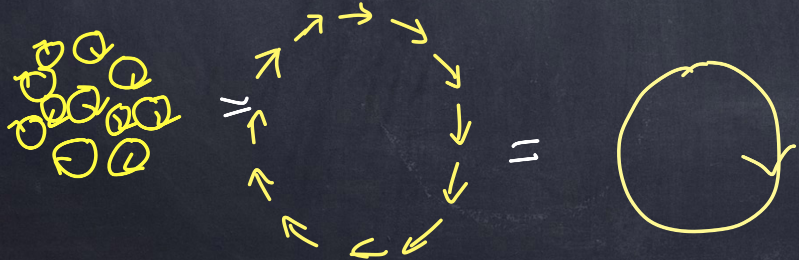
(b) One can think of the microscopic currents adding up to one big one.



The currents cancel out

the magnetization $\bar{M}$ is

$$\bar{M} = \frac{\text{sum of magnetic moments}}{\text{volume}} = \frac{(\text{sum of currents})\cancel{(\text{area})}}{\cancel{(\text{area})}\ \text{length}}$$



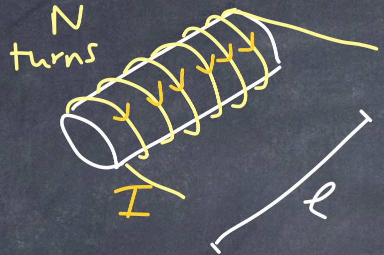
$$\bar{M} = \frac{\text{current}}{\text{length}}$$

$$\left(\text{or} \quad \bar{M} = \frac{dm}{dV} = \frac{dI}{d\ell}\right)$$

Magnetic moment of hollow solenoid:

$\bar{M}$ = magnetic moment
= magnetization

$$M = \frac{NI}{\ell}$$



previously, we calculated $B = \mu_0 \frac{N}{\ell} I$ for a solenoid,

So we see how magnetic field $\bar{B}$ relates to magnetization, $\bar{M}$ in this case:

$$\bar{B} = \mu_0 \bar{M}$$

In general,
The magnetization depends on the material and an external B-field.

$$\boxed{\bar{M} = \chi_m \left(\frac{\bar{B}_{ext}}{\mu_0} \right)} \quad (1)$$

χ_m: magnetic susceptibility $\qquad \chi_m = \frac{\mu}{\mu_0} - 1$

material	χ_m
Al	2.3 E -5
Gold	-3.6 E -5
Bismuth	-1.66 E -5
nickel	600
iron pure	200,000
copper	-9.6 E -6
water	-9 E -6
graphite	(1E-5, 1E-3)

(depends on orientation)

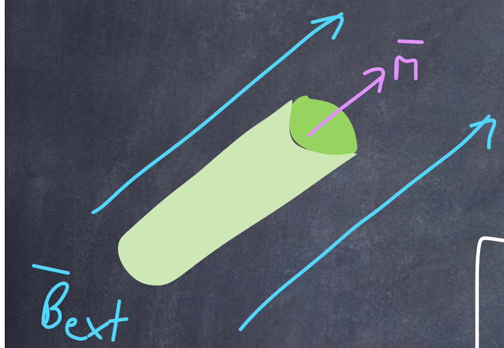
3 types of magnetic materials

paramagnetic materials have small positive χ_m

diamagnetic materials have small negative χ_m

feromagnetic materials have large positive χ_m

If we have an external magnetic field, $\bar{B}_{ext}$, in our material, the total magnetic field is a combination of the magnetization $\bar{M}$ + $\bar{B}_{ext}$



$$\bar{B} = \bar{B}_{ext} + \mu_0 \bar{M}$$

substitute in $\bar{M} = \chi_m \left(\frac{\bar{B}_{ext}}{\mu_0} \right)$

$$\boxed{\bar{B} = \bar{B}_{ext} (1 + \chi_m)} \quad (2)$$

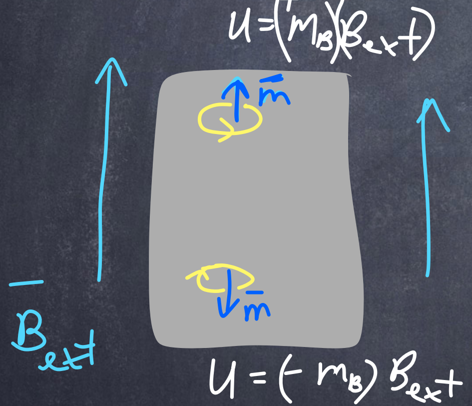
inside the material where field is uniform.

Paramagnetism

- Materials with a small, positive χ_m. $\bar{m}$ are randomly aligned, but $\bar{m}$ tend to align in an external $\bar{B}$-field. But thermal motion counteracts this tendency. The two effects compete.

Assume $B = 1T$

Assume $m = m_B$



Magnetic potential energy $= U = -\bar{m} \cdot \bar{B}$

$\Delta U =$ Energy to flip $\bar{m} = 2 m_B B = 2\left(9.27E{-}24 \frac{J}{T}\right)(1T)$

$$\Delta U = 2E{-}23\ J$$

Thermal energy at room temperature:

energy of atoms due to thermal energy $= kT = \left(1.38E{-}23 \frac{J}{K}\right)(300K) = 4E{-}21\ J$

k ↑ Boltzmann's constant

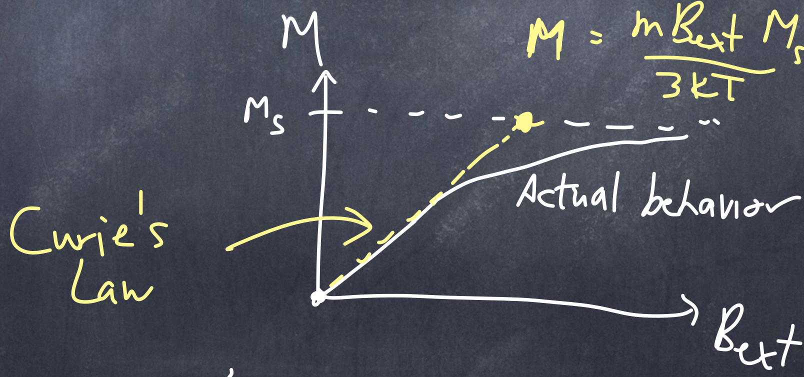
We see that typically (300 K) the thermal energy is much more than the magnetic potential energy. The magnetization of a material is much stronger at lower temperatures.

$$\boxed{M = \frac{m B_{ext}}{3kT} M_s}$$

Curie's Law

↑ $3kT$ (to do with 3 dimensions)

M_s is the saturation value (the maximum value of magnetization when all the magnetic moment are aligned.)



Curie's Law is a good approximation

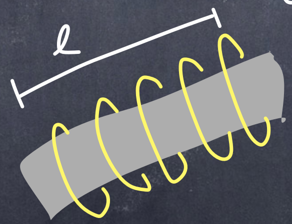
Ferromagnetism – materials with large, positive values of χ_m.
(iron, cobalt, nickel)

Atoms exert strong force on neighbors, causing alignment in groups called domains.

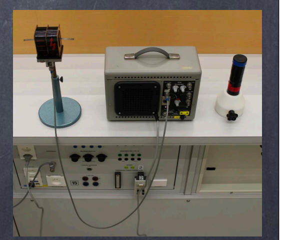
By increasing $\bar{B}_{ext}$, we can get domains to align.

Barkhausen effect – flipping of domains (sound!)

Put iron into a solenoid.

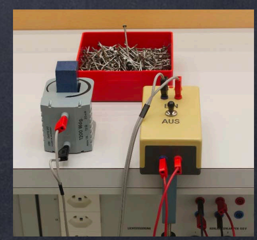


$n = \frac{N\,loops}{\ell}$



Hollow: $B_{ext} = \mu_0 n I$

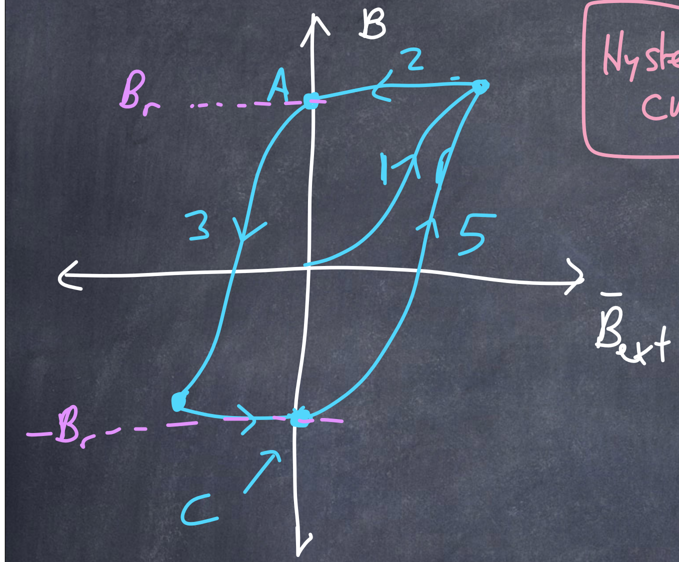
From ① $\bar{M} = \chi_m \left(\frac{\bar{B}_{ext}}{\mu_0}\right)$



$$\bar{B} = \bar{B}_{ext} + \mu_0 \bar{M} = \mu_0 n I (1+\chi_m) = \mu n I$$

total $\bar{B}$-field

Start with a piece of unmagnetized iron, we can increase the $\bar{B}_{ext}$ and measure $\bar{B}$ (total $\bar{B}$-field)



Hysteresis curve

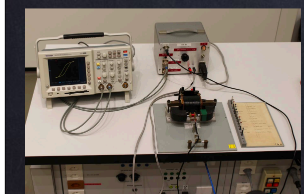
1) we increase $\bar{B}_{ext}$ and $\bar{B}$ increases.
2) we decrease $\bar{B}_{ext}$ to zero, there is still a $\bar{B}$ in the material since we have aligned domains.
3) we switch the direction of $\bar{B}_{ext}$, $\bar{B}$ becomes negative.
4) Increase $\bar{B}_{ext}$ to zero, $\bar{B}$ stays negative.
5) we increase $\bar{B}_{ext}$, $\bar{B}$ becomes positive

B_r : remnant magnetic field

$\bar{B}_r = \cancel{\bar{B}_{ext}} + \mu_0 \bar{M} = \mu_0 \bar{M}$

This is known as a hysteresis curve.

The magnetization of a material depends on its history of $\bar{B}_{ext}$.



Points A + C are when ferromagnet becomes permanent.

Diamagnetism – Materials with a small, negative value of χ_m. These materials have $\bar{m}$ that do not align in $\bar{B}_{ext}$.

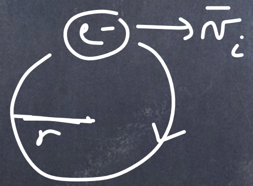
Discovered in 1846, when Faraday found that bismuth is repelled (slightly) by either side of a magnet.

Why? Atomic version of Lenz's Law!

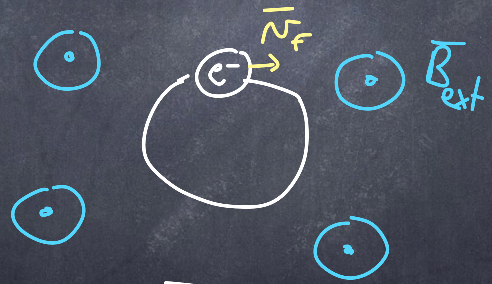
In a magnetic field, the electrons speed up or slow down, creating an opposing magnetic field.

from earlier:

Before:



After:



$$\bar{m} = \frac{e v_i r}{2} \odot$$

left-hand rule

$$\longrightarrow \bar{v}_i$$

$$\overrightarrow{v_f} \leftarrow \Delta v$$

$$\bar{m} = \frac{e v_f r}{2} \odot$$

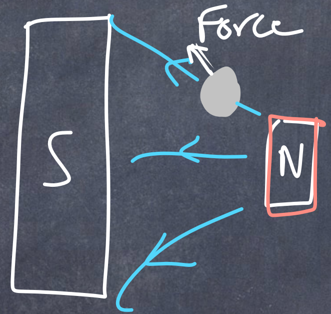
smaller

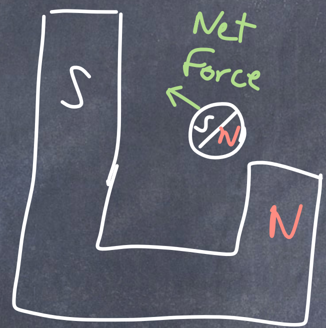
change in magnetic moment

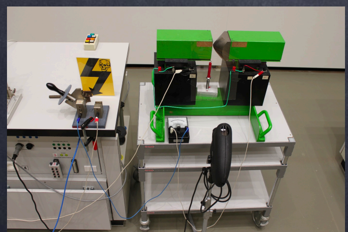
$$\Delta \bar{m} = \frac{e \Delta v r}{2} \otimes$$

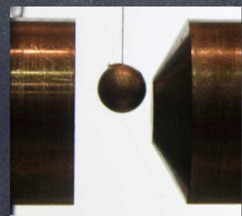
A diamagnet will have an induced magnetization opposing the direction of an external B-field.

For example, in a static $\bar{B}$-field, that is diverging, the $\bar{B}$-field is stronger on one side.

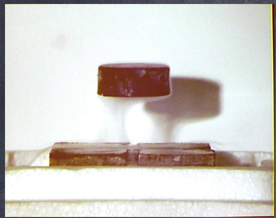








A superconductor is a perfect diamagnet.
It creates a magnetic field that cancels out $\bar{B}_{ext}$



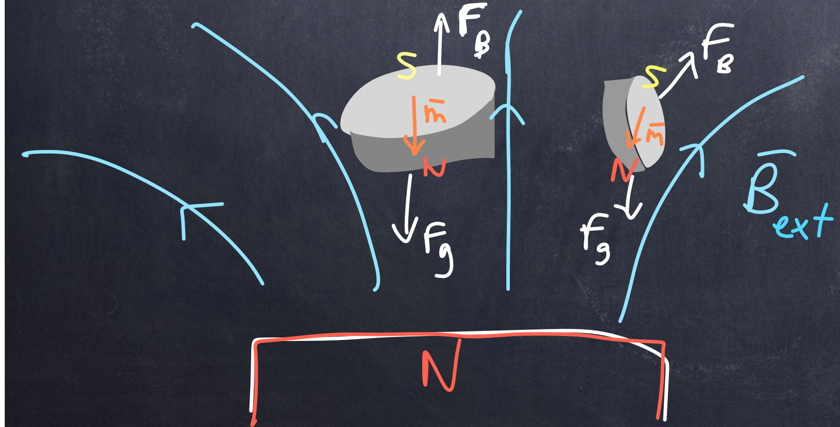
from ② $\bar{B} = \bar{B}_{ext}(1+\chi_m) = \emptyset$

$\underbrace{\chi_m = -1}_{\text{superconductor}}$

The magnet falls when temperature increases.

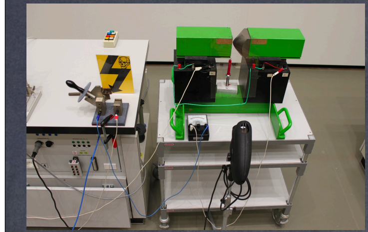
Alex Müller: UZH

1986 Nobel Prize for a high-temperature superconductor   35 K



$\bar{B}_{ext}$ is decreasing as we set higher.

At some height, $\bar{F}_B$ is equal & opposite to F_g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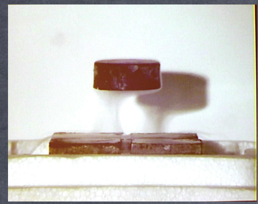
ED29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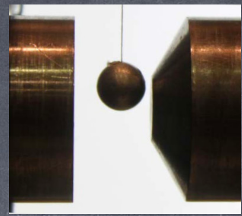
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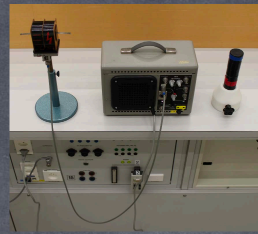
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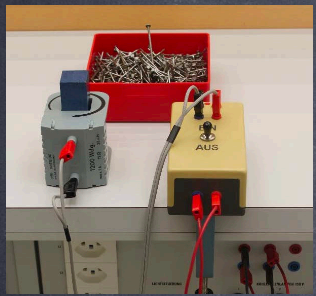

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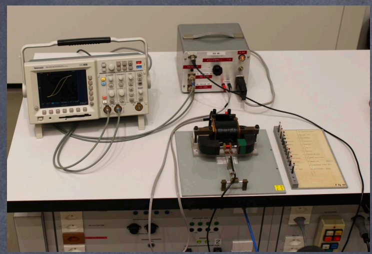
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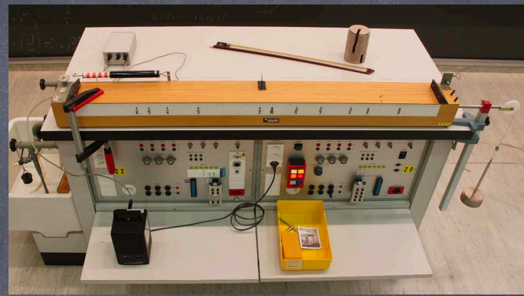
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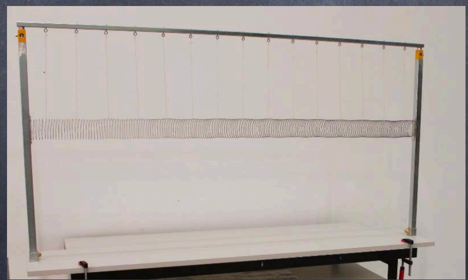
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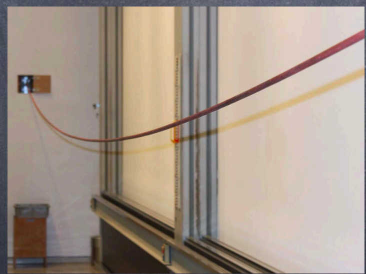
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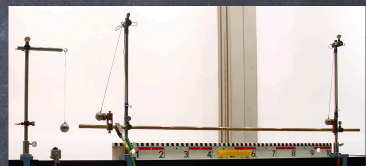
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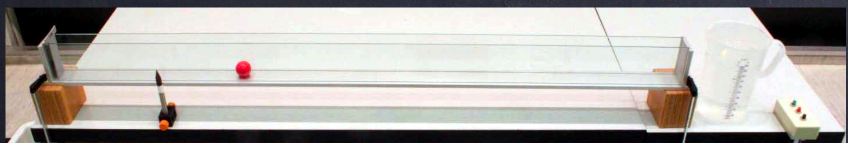
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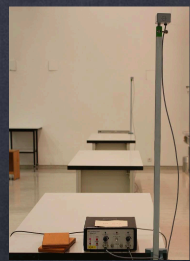
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