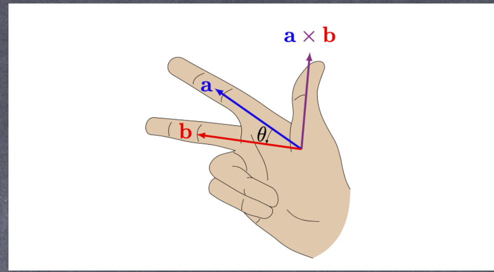
starting next week, the lecture
will only be in this lecture hall.

PHY 117 HS2023

Week 4, Lecture 1
Oct. 10th, 2023
Prof. Ben Kilminster

vector product or the cross product



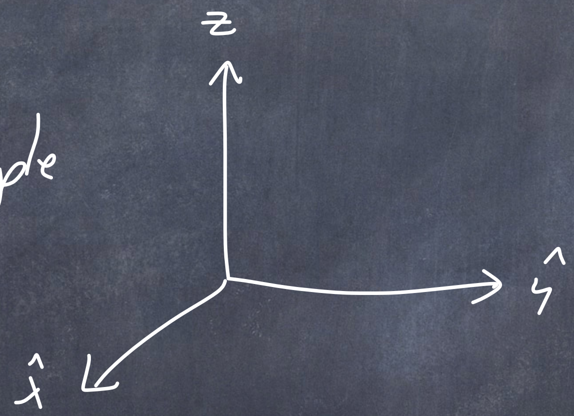
$$\bar{c} = \bar{a} \times \bar{b}$$

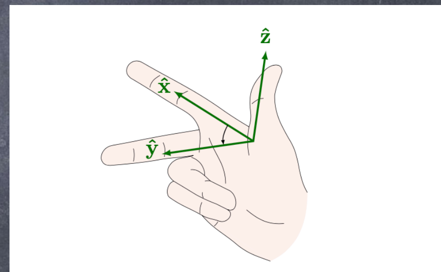
$\bar{c}$ is $\perp$ to both $\bar{a}$ and $\bar{b}$

$$\bar{c} = \bar{a} \times \bar{b} = |\bar{a}||\bar{b}|\sin\theta$$

θ is angle from $\bar{a}$ to $\bar{b}$

Example





what is $\hat{x} \times \hat{y}$? $\Rightarrow \hat{z}$

what is $\hat{y} \times \hat{z}$? $\Rightarrow \hat{x}$

what is $\hat{y} \times \hat{x}$? $\Rightarrow -\hat{z}$

Torque: • a force applied at a radius of rotation

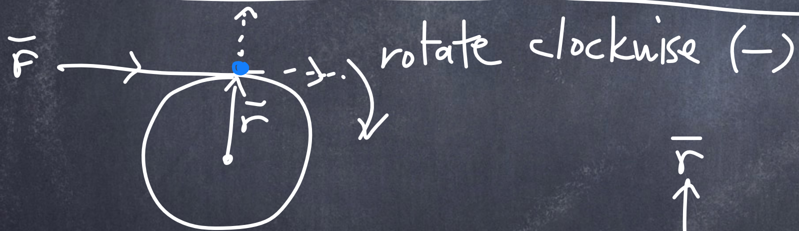
• tends to cause the object to rotate.

• symbol, $\bar{\tau}$ vector

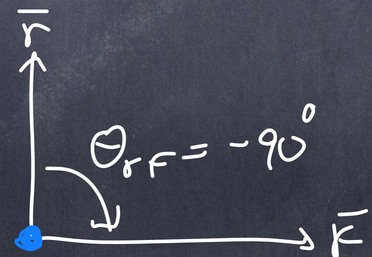
• units $F \cdot x = [N \cdot m]$

$$\boxed{\begin{aligned} \text{torque} = \bar{\tau} &= \bar{r} \times \bar{F} = |\bar{r}||\bar{F}| \sin\theta_{rF} \\ &= \underbrace{rF}_{\text{magnitudes}} \; \underbrace{\sin\theta_{rF}}_{\text{angle from } \bar{r} \text{ to } \bar{F}} \end{aligned}}$$

angle can be (−) or (+)



Draw the $\bar{r}$ and $\bar{F}$ vectors so that they start at the same point.



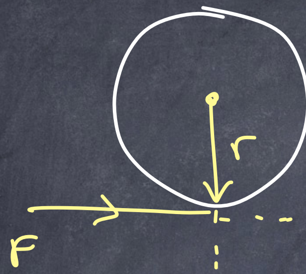
θ_{rF} is the angle that rotates $\bar{r}$ into $\bar{F}$

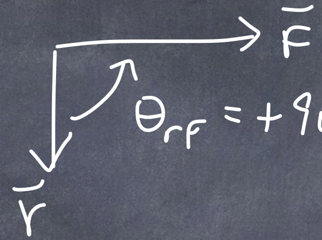
$$\tau = rF\sin(-90^\circ)$$

$$\tau = -rF$$

(−) clockwise

calculate the torque for each case:

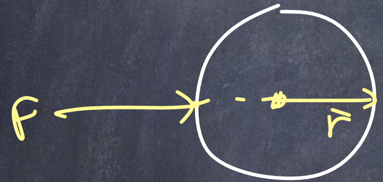


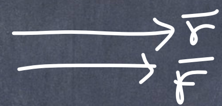


$\theta_{rF} = +90^\circ$

$\tau = rF \sin 90^\circ = rF$

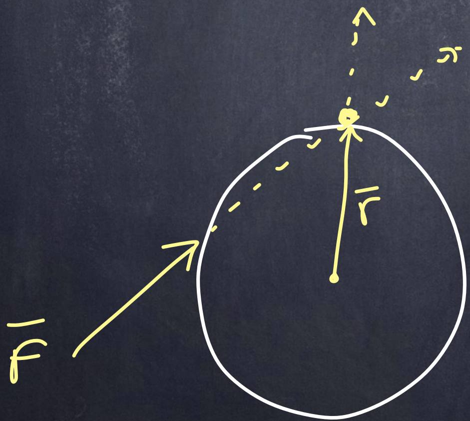
(+) direction or counterclockwise (CCW)

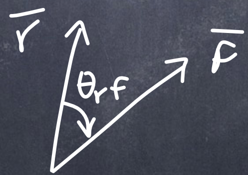




$\theta_{rF} = 0^\circ$

$\tau = rF(\sin 0^\circ) = 0$





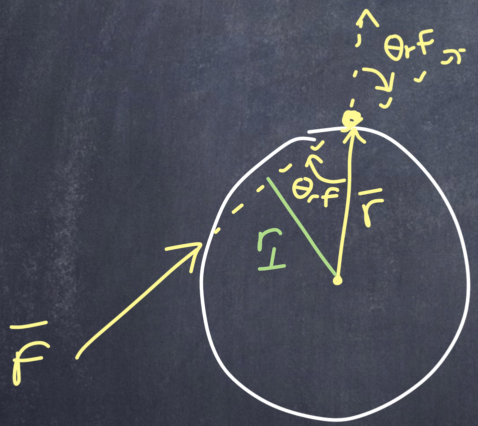
θ_{rF} is (−)

$$\bar{\tau} = \bar{r} \times \bar{F} = rF \sin \underbrace{\theta_{rF}}_{(-)}$$

(−) direction clockwise (CW)

Torque can be also thought of as the product of the force with the "lever arm" $= r_\perp =$ the part or component of $\bar{r}$ that is perpendicular to the force.

$$\tau = r_\perp F$$



$r_\perp$: component of $\bar{r}$ that is $\perp$ to $\bar{F}$

$$\tau = r_\perp F = (r \sin\theta_{rf}) F$$

add direction $\left[\tau = (r \sin\theta_{rf}) F \right.$ in clockwise (−) direction.

same as previous answer.

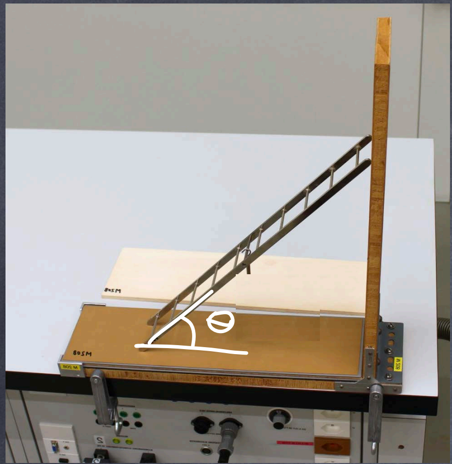
$\bar{\tau}$ vector points into page $\otimes$

static equilibrium:

2 conditions: $\sum \bar{F} = 0$ (so no acceleration)

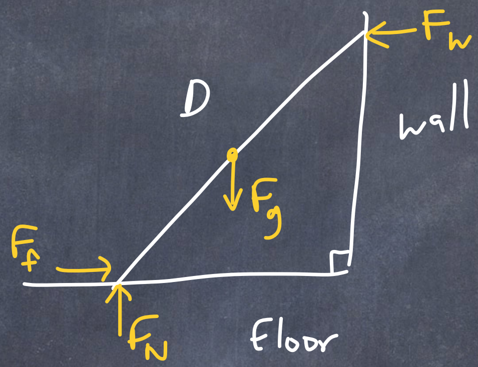
$\sum \bar{\tau} = 0$ $(\tau_{cw} = \tau_{ccw})$

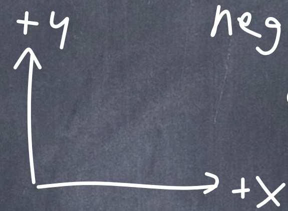
Trick: for calculating, the point of rotation matters (the right point makes the problems easier to solve)



What is the smallest angle, θ_{min}, such that the ladder does <u>not</u> fall?

Ladder has mass, M
length, D
angle, θ





For now, we neglect friction of the wall.

<u>First, consider forces:</u>

In equilibrium, $\Sigma F = 0$

<u>y-direction:</u>

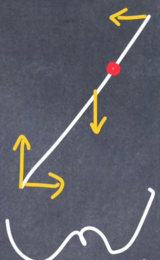
$$\Sigma F = 0$$
$$F_g = Mg$$
$$\Sigma F_y = F_N - F_g = 0 \Rightarrow F_N = F_g = Mg$$

<u>x-direction</u>

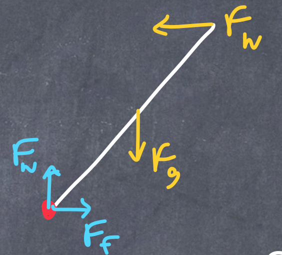
$$F_f = \mu_s F_N = \mu_s Mg$$
$$\Sigma F_x = F_f - F_W = 0$$
$$F_W = F_f = \mu_s Mg$$

Next, consider what should be the rotation point

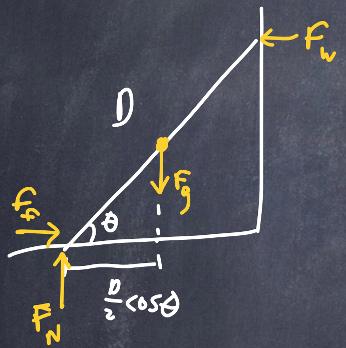
$$\sum \tau = 0 \qquad \tau_{cw} = \tau_{ccw}$$

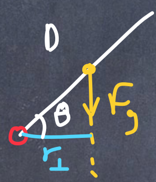


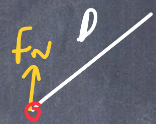
bad choice
all 4 forces put τ on •



No torque for $F_f + F_N$ since they point through the axis of rotation.







$$\sum \tau_{\bullet} = \tau_g + \tau_w + \tau_N + \tau_f$$

$$0 = -Mg\frac{D}{2}\cos\theta + F_w D\sin\theta + 0 + 0$$

$$\frac{\sin\theta}{\cos\theta} = \frac{Mg}{2F_w} \Rightarrow \tan\theta = \frac{Mg}{2F_w}$$

insert ① $\rightarrow$ $\tan\theta = \frac{\cancel{Mg}}{2\mu_s \cancel{Mg}} = \frac{1}{2\mu_s}$

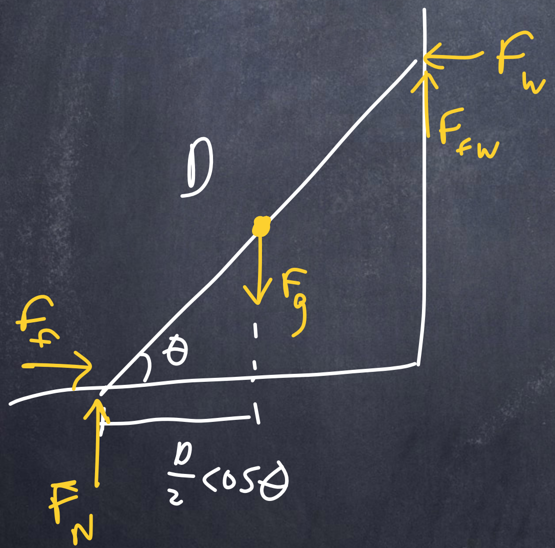
$$\theta = \tan^{-1}\left(\frac{1}{2\mu_s}\right)$$

we want $\theta_{min} \Rightarrow \theta_{min} = \tan^{-1}\left(\frac{1}{2\mu_s}\right)$

when θ is at minimum, so is $\tan\theta$

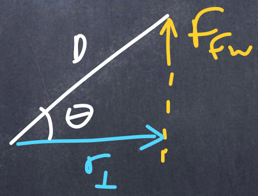
If $\mu_s = 0.3 \Rightarrow$ then $\theta_{min} = 59°$

If $\theta < 59°$, then the ladder falls.

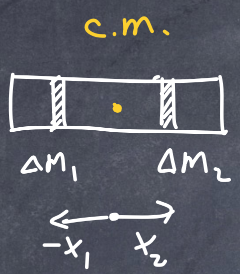


What if there is friction on the wall?

$$\tau_{ccw} = r_\perp F_{fw} = (D\cos\theta)\, F_{fw}$$



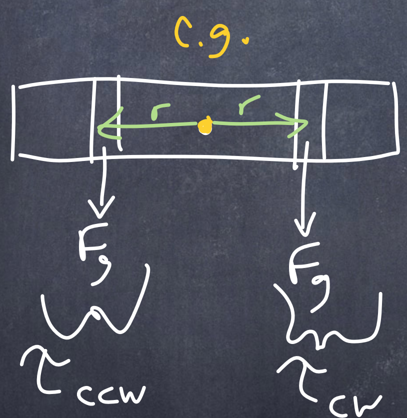
Center of mass: the point at which there is an equal amount of mass on all sides.



$$(\Delta m_1)(-x_1) + (\Delta m_2)(x_2) = 0$$

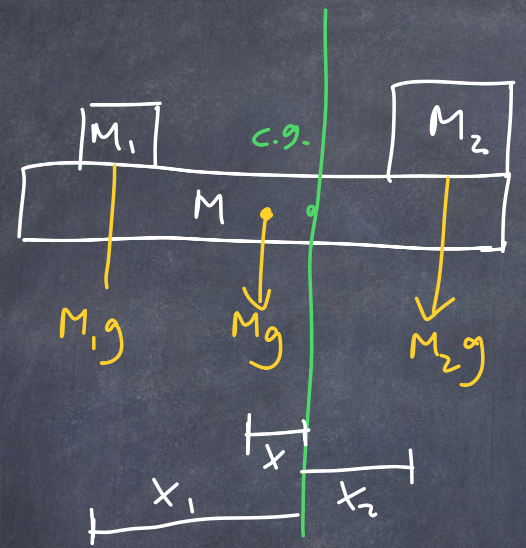
$$(x_1 = x_2)$$

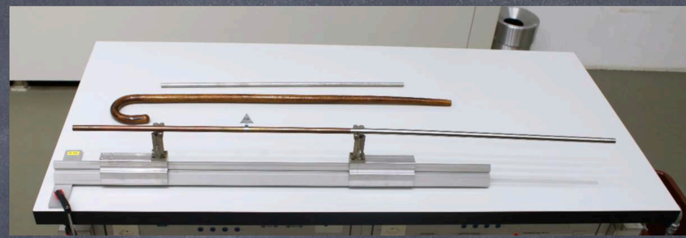
center of gravity: point at which all torques due to the force of gravity cancel out.



center of mass is equivalent to the center of gravity on earth.

we can choose our origin of rotation at the center of gravity





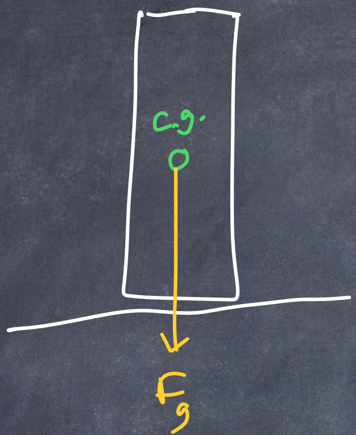
when origin is the center of gravity, then $\sum \tau_{cw} = \sum \tau_{ccw}$

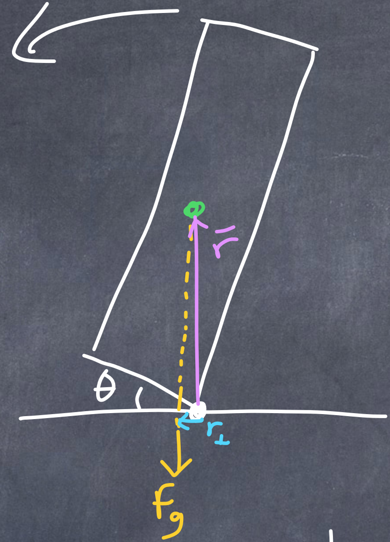
and so

$$x_2 M_2 g = x_1 M_1 g + x M g$$

object is balanced.

Stability



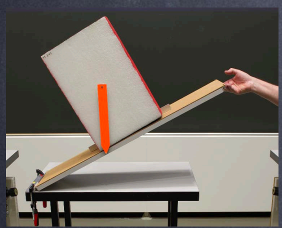


τ is CCW

$\odot$

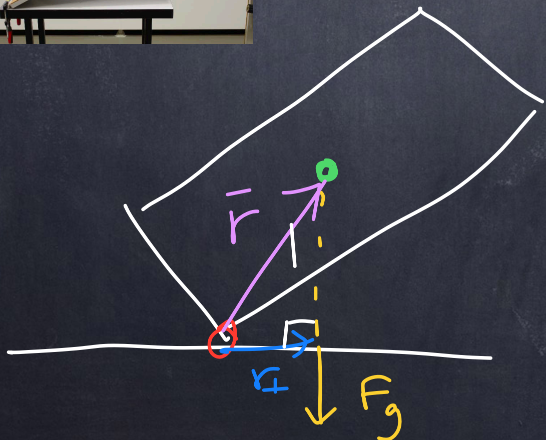
$\bar{\tau} = \bar{r} \times \bar{F}$

$\tau = r_\perp F$ (+) direction



An Object is stable when the torque due to gravity tends to restore the object to equilibrium.

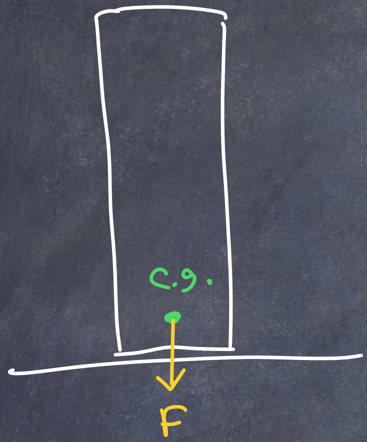
This depends on the direction of the torque with respect to the pivot point.

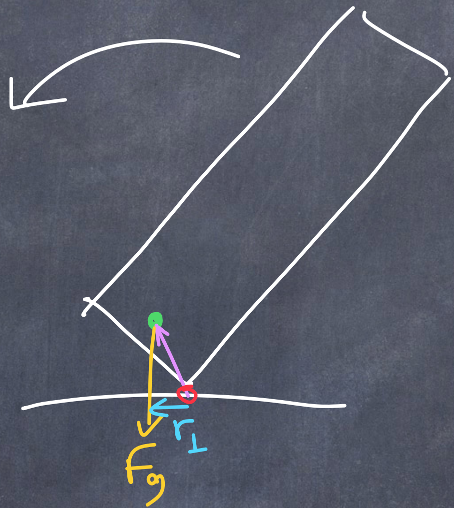


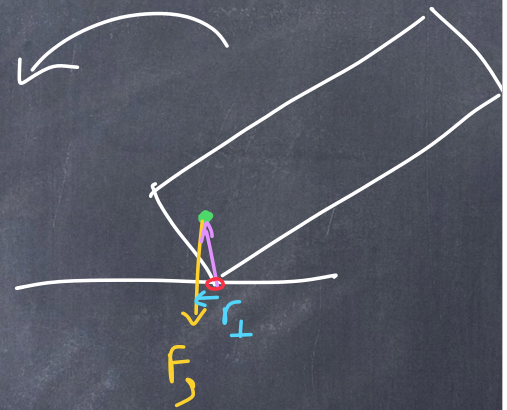
τ is CW, $\otimes$

This is not stable

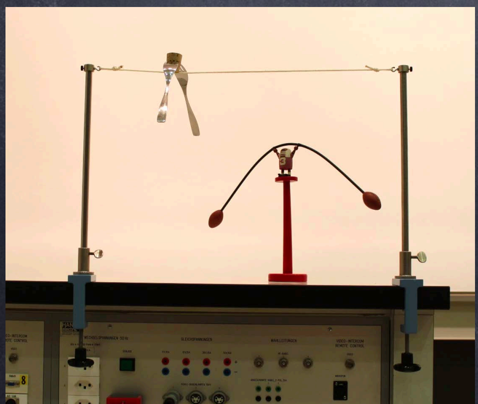
To Improve stability, lower the center of gravity (heavier at the bottom).





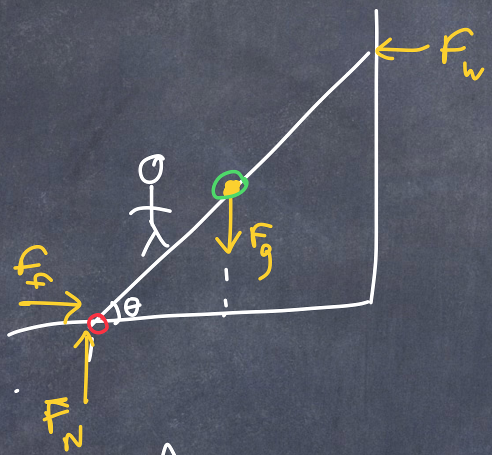


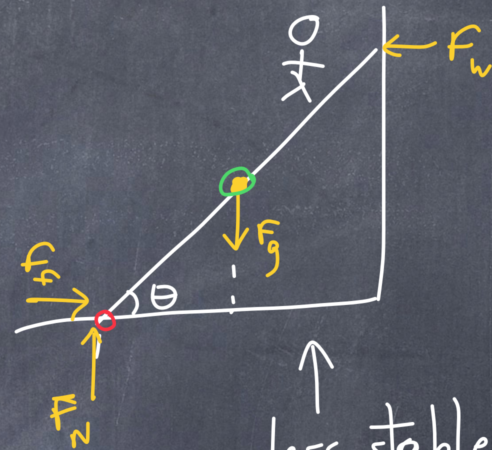
both stable:
τ rotates it back to initial position.



what if we add a person?

More or less stable?

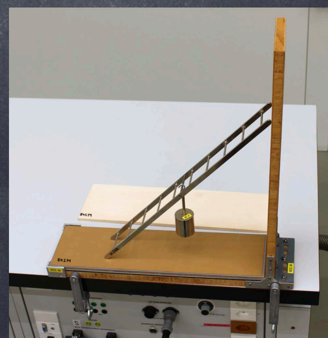




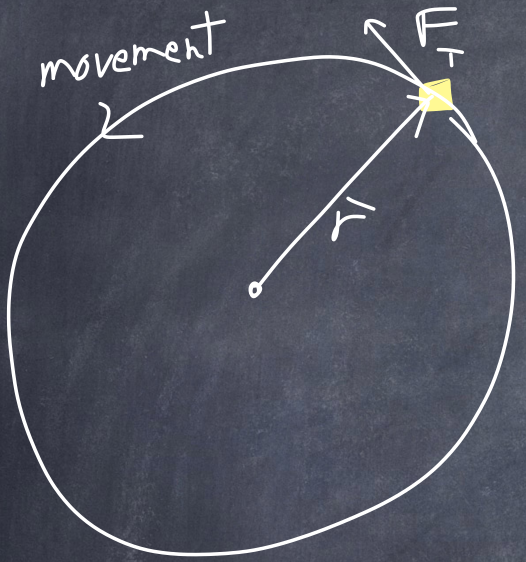
less stable

More stable.

θ_{min} can be smaller and ladder still stable.



Consider a force on an object constrained to move in a circle.



The force is tangential, F_T

The force is unbalanced, so

$$F_T = ma = m(r\alpha)$$

we multiply both sides by r

$$rF_T = mr^2\alpha$$

$$\underbrace{\underbrace{r_\perp F}_{}}_{\tau} = I\alpha$$

$$\boxed{\tau \equiv I\alpha}$$

$I \equiv mr^2$: "moment of inertia" : this is for a particle of mass m at a radius r from the center of rotation linear motion

$$F = ma$$

rotational motion

$$\tau = I\alpha$$

I is kind of like mass, m

$(I = mr^2)$

Newton's second law of rotation $\boxed{\Sigma\tau = I\alpha}$