Today's lecture PDF is online if you want to download and follow.

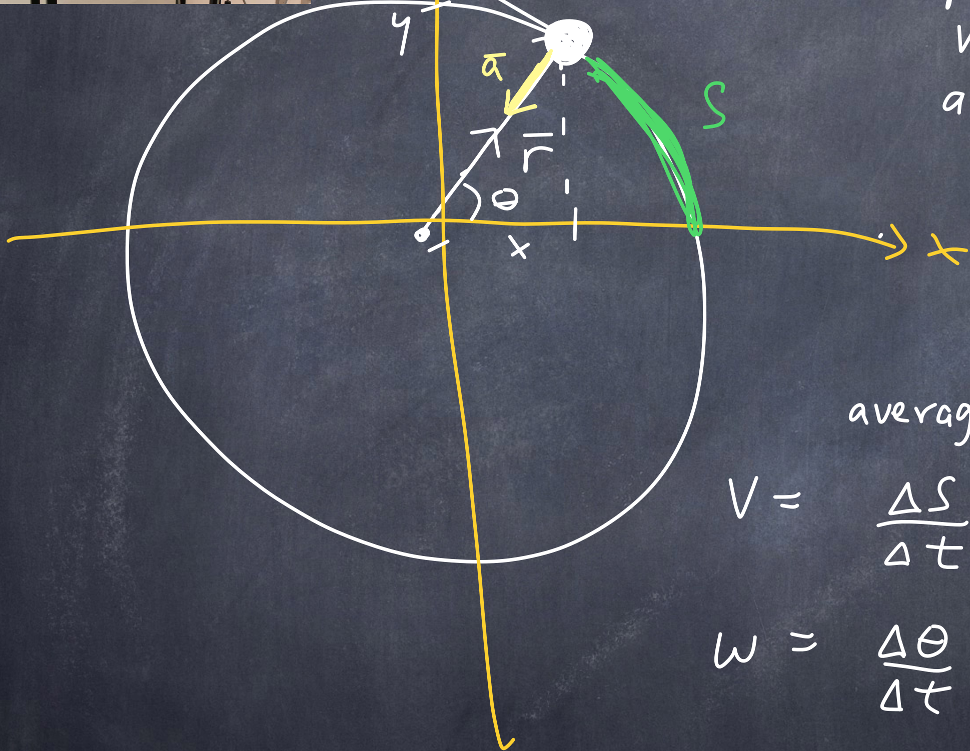
PHY 117 HS2023

Week 2, Lecture 1
Sept. 26, 2023
Prof. Ben Kilminster

Quiz: The acceleration vector for an object moving in a circle is toward the center of the circle.

(Incorrect online)

Circular motion (in 2D) at constant speed.



quantity	in terms of distance	in terms of angles	relation
position	s	θ	$s = r\theta$
velocity	v	ω	$v = r\omega$
acceleration	a	α	$a = r\alpha$

$$a = \frac{v^2}{r} = \frac{(r\omega)^2}{r} = \omega^2 r$$

$$\alpha = \frac{a}{r} = \omega^2$$

average $\rightarrow$ instantaneous

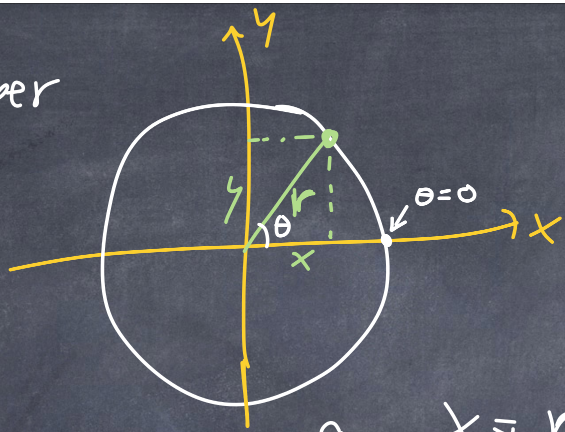
$$v = \frac{\Delta s}{\Delta t} \rightarrow \frac{ds}{dt}$$

$$\omega = \frac{\Delta \theta}{\Delta t} \rightarrow \frac{d\theta}{dt}$$

$$\alpha = \frac{\Delta \omega}{\Delta t} \rightarrow \frac{d\omega}{dt}$$

Equation of motion for constant speed: $\theta = \omega t + \underset{\text{initial angle}}{\theta_0}$

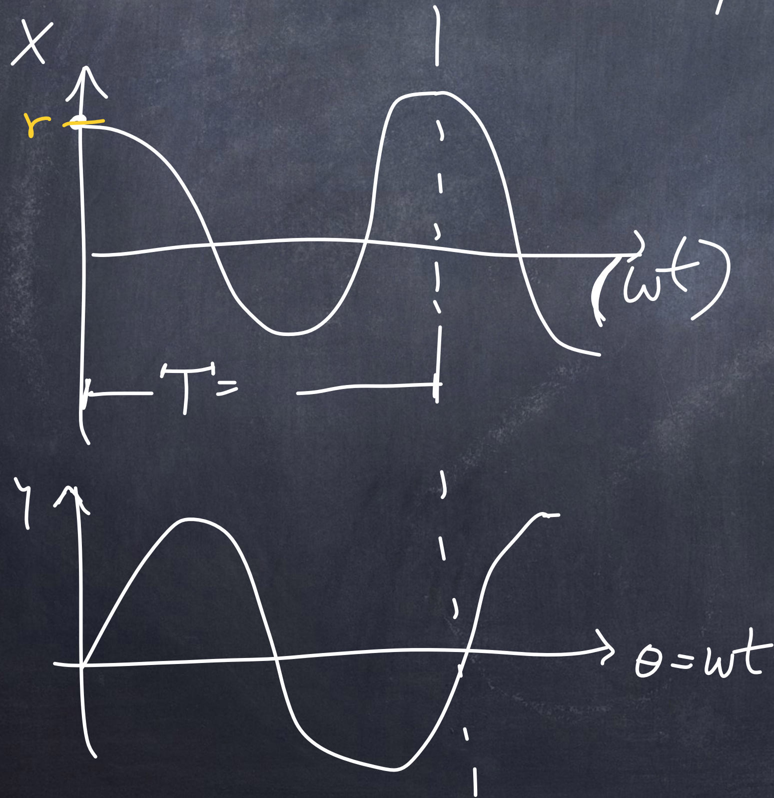
Remember



$$x = r\cos\theta$$
$$y = r\sin\theta$$

If speed is constant,
then $\theta = \omega t$

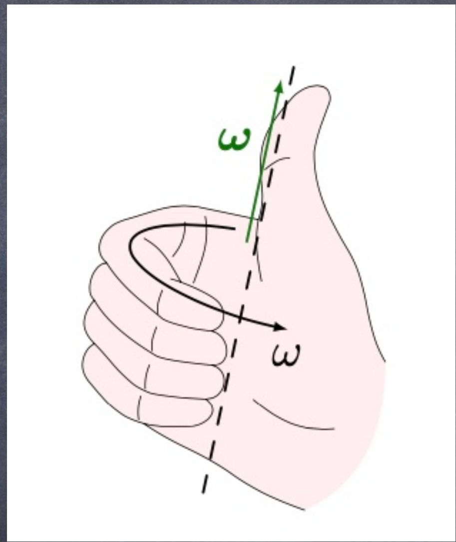
So
$$x = r\cos(\omega t)$$
$$y = r\sin(\omega t)$$



$$T = \frac{2\pi}{\omega} = \frac{2\pi r}{v}$$

Notice: circular motion means x+y components are simple harmonic oscillators.

Direction of $\bar{w}$ vector



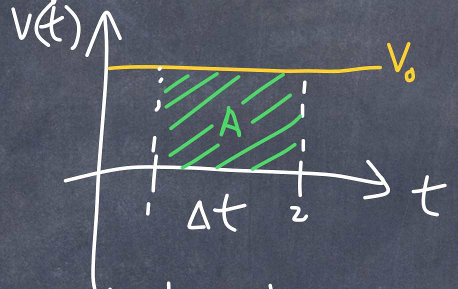
picture: $\bar{w}$ is out of the page

$\odot$ arrow coming towards you

If $\bar{w}$ is into the page, we use $\otimes$ (arrow moving away) we have seen that $\frac{dv}{dt} = a$: the slope of v vs. t is the acceleration

$\frac{dx}{dt} = v$: the slope of x vs. t is the velocity

Consider a particle moving at constant velocity:



$v(t) = V_0 = \text{constant}$

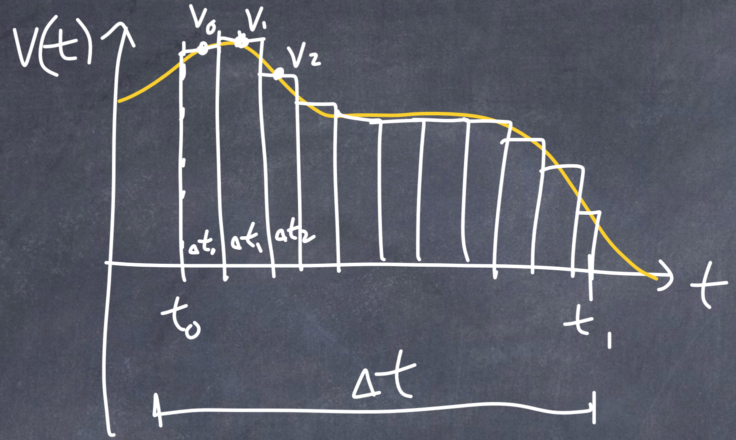
$t_2 - t_1$

we know that $V_0 = \frac{\Delta x}{\Delta t} \Rightarrow \Delta x = V_0 \, \Delta t$

from the figure we see that $V_0 \Delta t$ is the area of the rectangle $A = V_0 \Delta t$

So the change in position Δx is the area under the v vs. t curve.

For a more complicated v vs. t curve:



we can approximate Δx by summing up many small rectangles

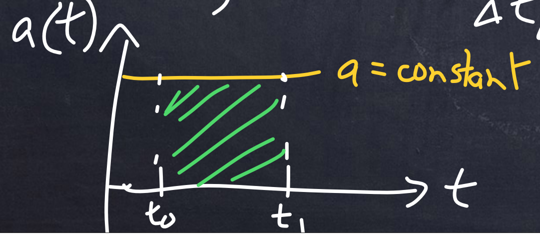
As Δt_i gets smaller, we get more precise

$$x_1 - x_0 = \Delta x = \sum_i v_i \Delta t_i$$

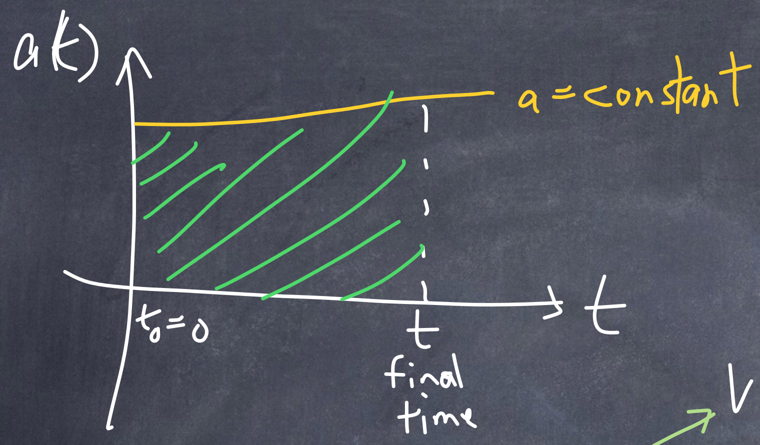
$$\Delta x = \lim_{\Delta t_i \to 0} \sum_i v_i \Delta t_i = \int_{t_0}^{t_1} v\, dt$$

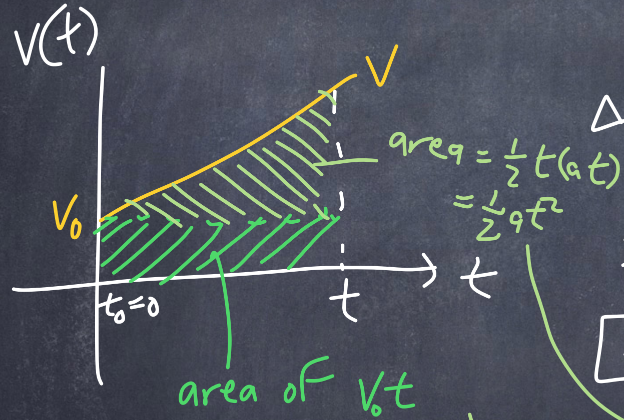
So Δx = the integral of the v vs. t curve from t_0 to t_1

Likewise, $\Delta V = \lim_{\Delta t_i \to 0} \sum a_i \Delta t_i = \int_{t_0}^{t_1} a\, dt$



ΔV = area of curve under a vs. t





$$\Delta V = V - V_0 = \int_{t_0=0}^{t} a\,dt = at\Big]_0^t =$$
$$= at - 0 = at$$

$$V - V_0 = at \rightarrow \boxed{V = V_0 + at}$$

formula from last week

$$\Delta x = x - x_0 = \int_{t_0=0}^{t} V\,dt = \int_{t_0=0}^{t} (V_0 + at)\,dt = V_0 t + \frac{1}{2}at^2\Big]_0^t$$

$$x - x_0 = V_0 t + \frac{1}{2}at^2$$

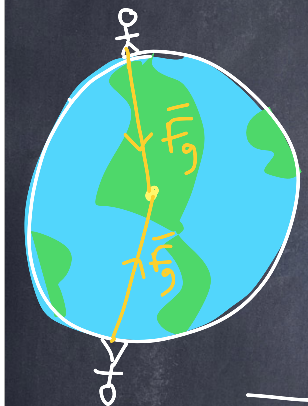
$$\boxed{x = x_0 + V_0 t + \frac{1}{2}at^2}$$

our formula from last week

So starting with a = constant, then we integrated twice, we get our formulas for V + x

Forces: A force is something that pushes or pulls an object.

One of the most common forces is "weight", which comes from gravity.



$$\text{weight} = \bar{F}_g = \underset{\text{gravity}}{\text{Force of}} = \underset{\text{mass}}{m}\,\bar{g}$$

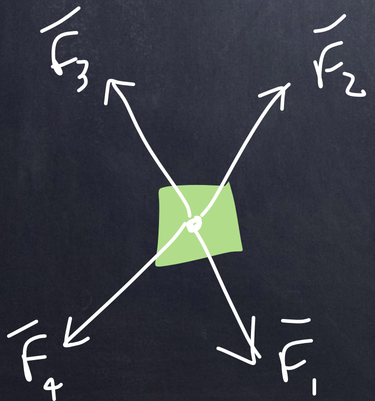
on earth

$g = 9.81 \ m/s^2$

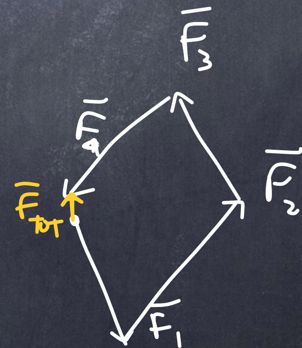
$\bar{g}$ points to the center of the earth

Forces are vectors, and can be added.

$$\bar{F}_{TOTAL} = \bar{F}_{TOT} = \bar{F}_{NET} = \sum_i \bar{F}_i$$



the "tail to tip" method.

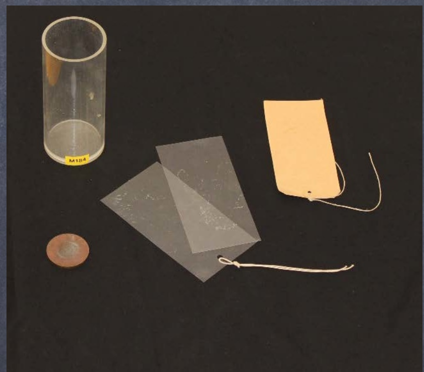


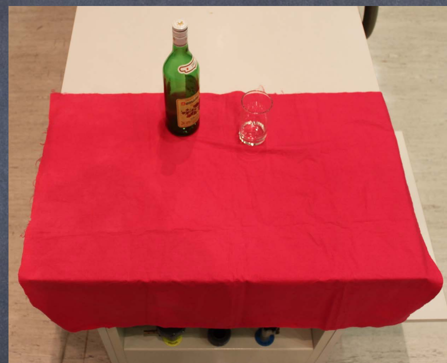
If $\bar{F}_{TOT} = 0$,
there is no "net" force.
we have equilibrium.

Newton's three laws:

<u>Law of inertia</u>: 1) An object will remain at rest, or continue to move in a straight line unless acted upon by a "net" force

↑ non-zero.

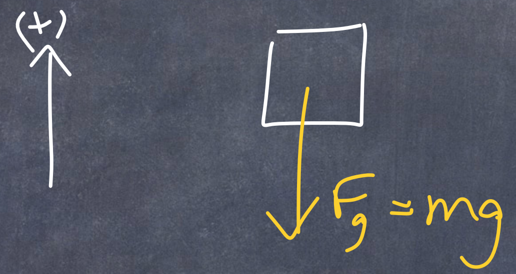




Newton's three laws:

2) A net force will cause an object to accelerate according to $\sum \bar{F} = m\bar{a}$

A common example is a falling object



$$\sum \bar{F} = -\bar{F}_g = -\cancel{m}\bar{g} = \cancel{m}\bar{a}$$

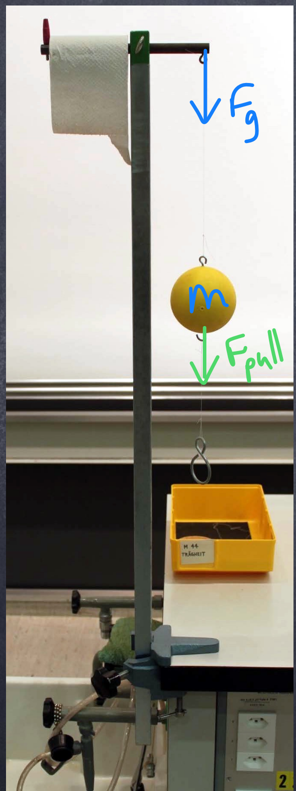
so $\bar{a} = -\bar{g}$

$$\sum F = ma$$

$$\cancel{m}g = \cancel{m}a$$

$$a = g$$

Tests of Newton's first and second laws:



pull quickly: breaks on bottom string, (law of inertia)

pull slowly: breaks on top because there is more force

top: $\bar{F}_g + \bar{F}_{pull}$

bottom: $\bar{F}_{pull}$

toilet paper

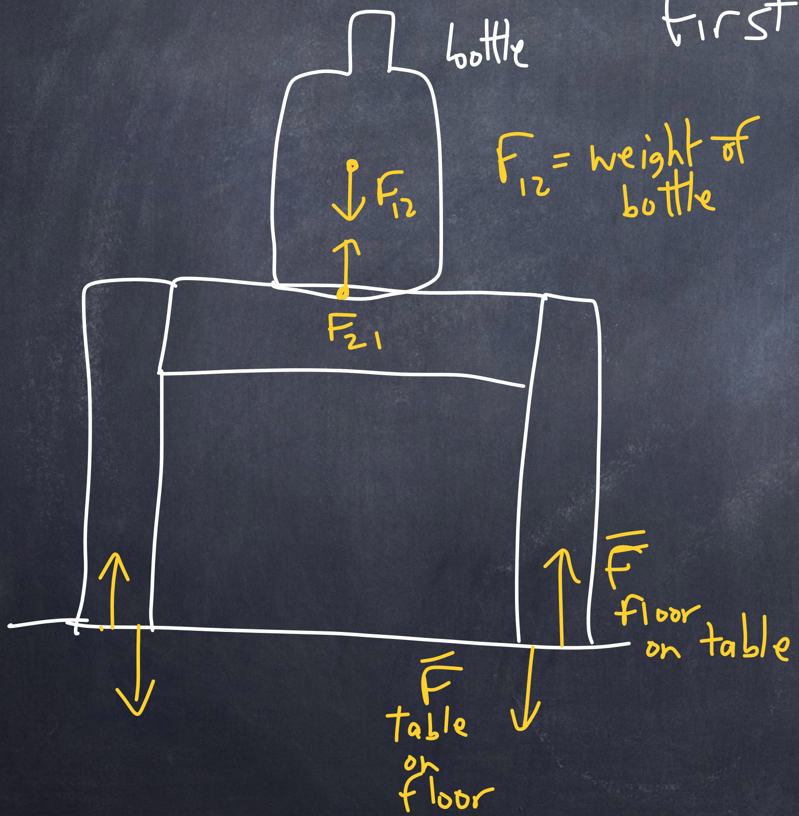
pull it fast : first law

pull it slow : second law

Newton's third law:

$\bar{F}_{12} = -\bar{F}_{21}$

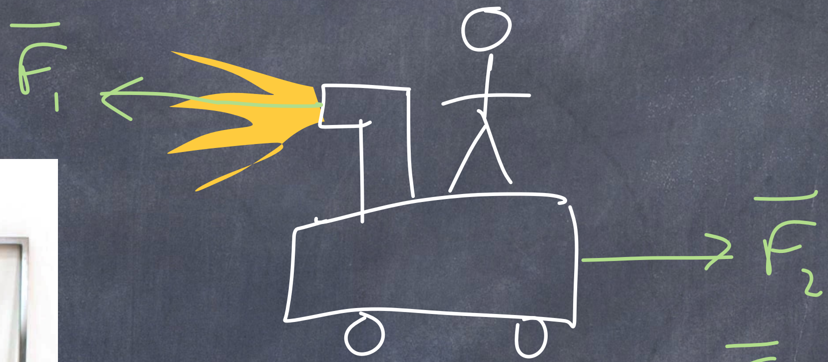
3) when one object exerts a force on a second object, the second object exerts a force equal in magnitude, but opposite in direction, on the first object.



$\bar{F}_{12}$: force exerted by object 1 on object 2.

$\bar{F}_{21}$: force exerted by object 2 on object 1.

third law: action - reaction law



$$\bar{F}_1 = -\bar{F}_2$$

Summary: Newton's three laws:

Law of inertia — 1) An object will remain at rest, or continue to move in a straight line unless acted upon by a "net" force

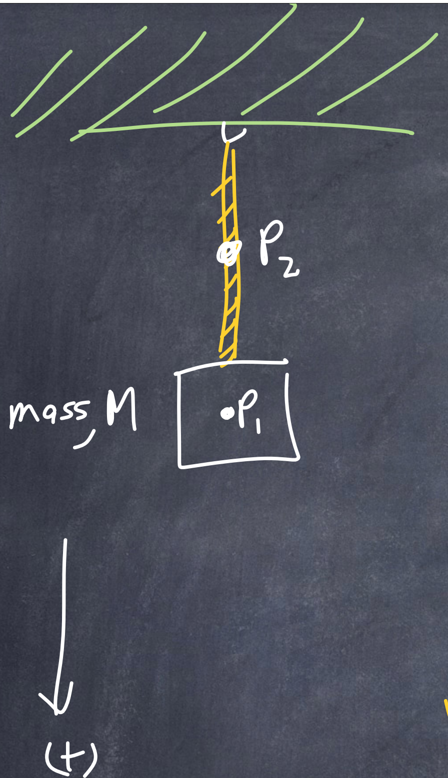
non-zero total force

$\Sigma \bar{F} = ma$ — 2) A net force will cause an object to accelerate according to $\Sigma \bar{F} = m\bar{a}$

sum

$\bar{F}_{12} = -\bar{F}_{12}$ — 3) When one object exerts a force on a second object, the second object simultaneously exerts a force equal in magnitude but opposite in direction on the first object.

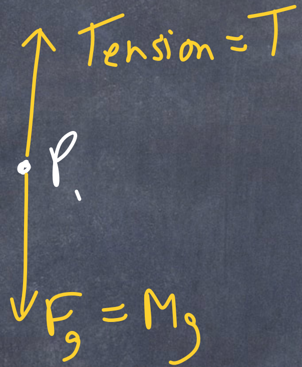
$$\bar{F}_{12} = -\bar{F}_{21}$$



A mass M hangs from a string to the ceiling.

Draw the forces acting on P_1.

Or . . . also on P_2?



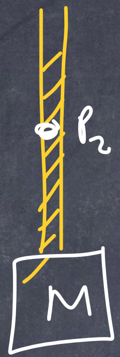
$$\Sigma F = \underbrace{F_g - T = 0}_{T = F_g} = ma$$

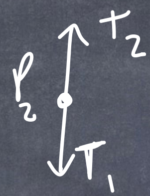
But we must specify the <u>direction</u> + magnitude

$F_g = Mg$ in + direction

$T = Mg$ in − direction

What about P_2 ?





$$\Sigma F = T_1 - T_2 = 0$$

$$T_1 = T_2$$

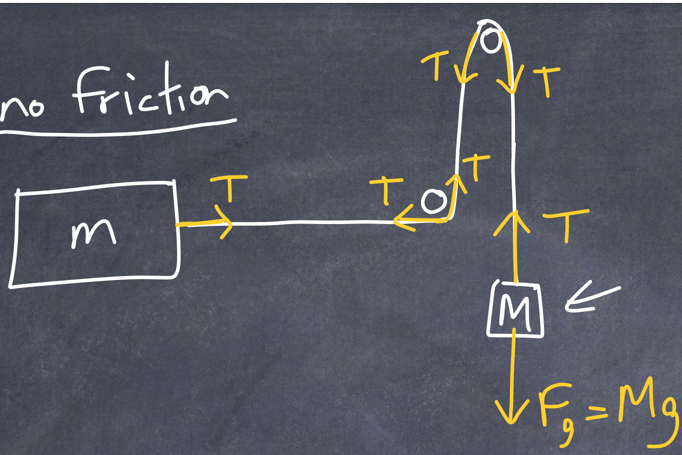
From previous page, we know that $T = Mg$

so here $T_1 = Mg$ in (+) direction

$T_2 = Mg$ in (−) direction.

Tension has the same magnitude everywhere in the string.

assume no friction



$M = 7\,g$

$m = 227\,g$

$\Sigma F = (\text{total mass})\, a$

$Mg = (M+m)\, a$

$$a = \frac{M(g)}{(M+m)} = \frac{(7)(9.81\ \frac{m}{s^2})}{(227+7)} = \boxed{0.293\ \frac{m}{s^2}}$$ predicted

measure $x = \cancel{x_0}^{\,0} + \cancel{v_0}^{\,0} t + \frac{1}{2} a t^2 = \frac{1}{2} a t^2 \rightarrow a = \frac{2x}{t^2}$

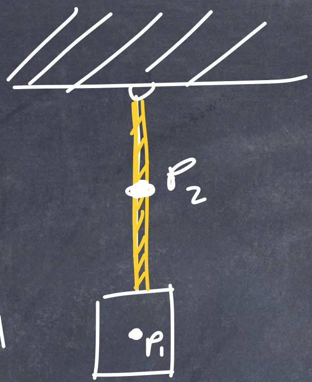
$x = 1.04\ m$

$t = ? = 2.8\ s$

$$a = \frac{2(1.04\,m)}{(2.8\ s)^2} = \boxed{0{,}265\ \frac{m}{s^2}}$$

measured

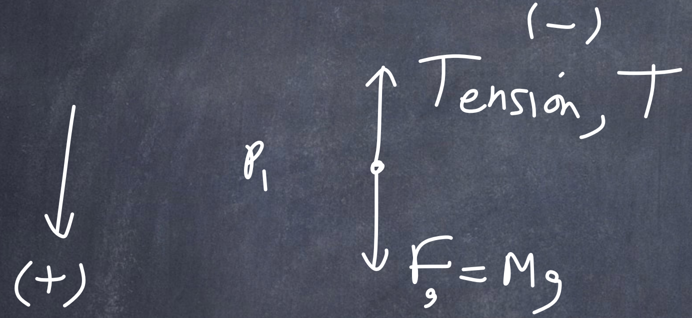
Aside:



Exercise:

A mass M hangs from a string to the ceiling.

Draw the forces acting at P_1.
what about P_2?



If we use vectors for $\bar{F}_g$ and $\bar{T}$, then we don't need to explicitly put negative signs in our sum, ΣF.

$$\Sigma \bar{F} = \bar{F}_g + \bar{T} = 0$$

then $\bar{T} = -\bar{F}_g$

so $\bar{F}_g = M\bar{g}$

$\bar{T} = -M\bar{g}$

If we use T and F_g as scalars, then we need to keep track of negative signs.

We state T is in (−) direction

$$\Sigma F = F_g - T = 0 = ma$$

and $T = F_g$

But we must specify the direction

$F_g = Mg$ in (+) direction

$T = Mg$ in (−) direction

In both cases F_g points down
+ T points up.

Aside: Sometimes people write $\frac{df(x)}{dx}$ as $f'(x)$.

These two are the same.

Since $\frac{df(x)}{dx} = f'(x) \Rightarrow df(x) = f'(x)\,dx$

And if you take the integral of both sides:

$$\int d\,f(x) = \int f'(x)\,dx$$

This becomes:

$$f(x) = \int f'(x)\,dx$$

which is the definition of an integral

Also, $\frac{d^2 f(x)}{dx^2} = f''(x)$