PHY127 FS2024

Prof. Ben Kilminster

Lecture 9

May3rd, 2024

Penetration of x-rays

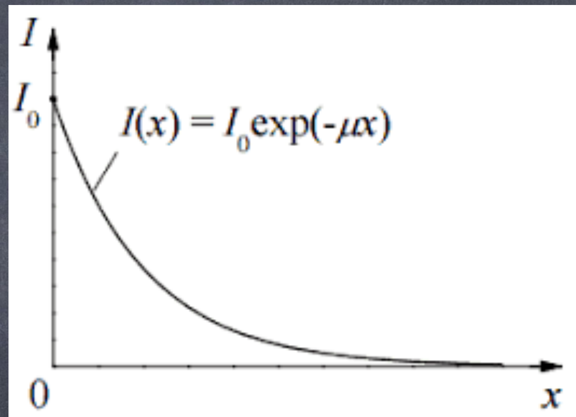
combined effect of Thomson scattering, photoelectric effect + compton scattering generate attenuation of the X-ray beam.

$$I(x) = I_0 e^{-\mu \cdot x}$$

I_0: initial beam intensity

$I(x)$: intensity at a depth, x

μ: attenuation coefficient with units $[m^{-1}]$



The mass attenuation coefficient, μ/ρ, is the attenuation per unit density of the material being penetrated.

Units: $\left[\frac{cm^2}{g}\right]$

photon energy	μ/ρ_{water}	$\mu/\rho_{dry\ air}$	μ/ρ_{bone}	μ/ρ_{muscle}	$\mu/\rho_{breast\ tissue}$
100 keV	0.17	0.15	0.18	0.16	0.16
10 keV	5.3	5.1	28	5.3	4.3
5 keV	43	40	190	42	34

Observations: The higher the X-ray energy, the farther the X-rays penetrate.
(If μ is large, attenuation is more & the distance traveled is less)

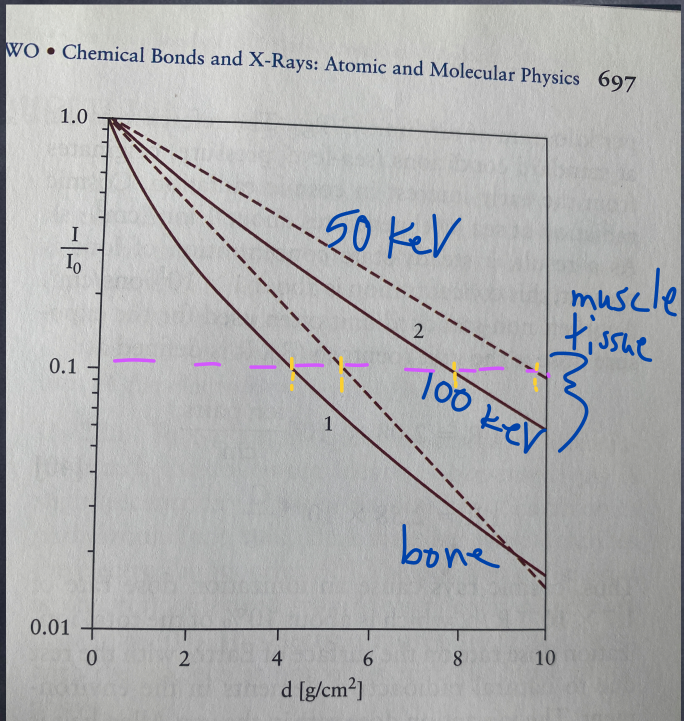


FIGURE 22.25
Intensity attenuation of X-ray beams of 100 keV (solid lines) and 50 keV (dashed lines) in human bones (curves 1) and muscle tissues (curves 2). The depth is given in unit g/cm^2, which results from the product of density and the path length in the tissue.

The x-axis is given as the product of density ρ and the path length, x. $d = \rho \cdot x$
(Because the combination is more meaningful)

For instance, X-ray intensity is reduced to 10% ($I/I_0 = 0.1$)...

for muscle tissue, at $d = 8\text{-}10\ g/cm^2$

$$x = \frac{d}{\rho_{mus}} \cong \frac{9\ g/cm^2}{1\ g/cm^3} = 9\ cm$$

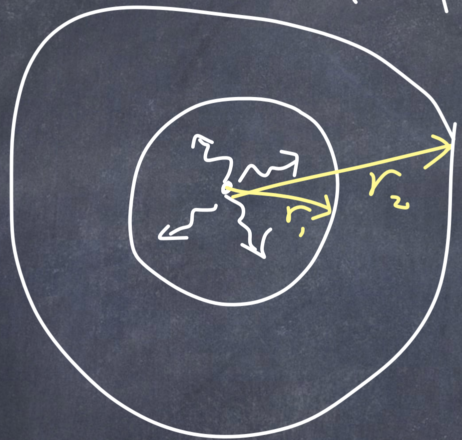
for bone tissue, $d = 4.5\text{-}5.5\ g/cm^2$

$$x = \frac{d}{\rho_{bone}} \cong \frac{5\ g/cm^2}{1.2\ g/cm^3} \cong 4\ cm$$

From [1]: “Physics of the Life Sciences” by Martin Zinke-Allmang

Reminder:

$$\text{Intensity} = \frac{\text{Power}}{\text{area}}$$



Intensity will decrease like $\frac{1}{r^2}$

Surface Area of a sphere is $A = 4\pi r^2$

If you have $r_2 = 2r_1$,

the intensity is 4 times less at r_2 than r_1

Where does the γ-ray intensity go?

γ-rays are either scattered or absorbed by bone or tissue.

Absorbed radiation has an adverse biological effect.

Measured radiation is reported in 2 ways:

1) amount of ionization ocurring in the material due to the radiation → <u>exposure dose</u>

2) energy deposited by radiation in the material → <u>absorbed dose</u>

Dose = total amount of ionization or energy deposited in a given amount of material.

Dose rate = dose per unit time.

There are different measures:

exposure dose: total charge generated by ionization per kg of air (std. atmosphere)

units: $\left[\frac{C}{kg}\right]$

Cosmic radiation at sea level generates 1-10 ions/cm³/s. This equilibrium value is maintained despite recombination of ions.

Another unit is roentgen (R)

$$1\ R = 2.08 \times 10^9\ \frac{\text{ion pairs}}{cm^3}$$

$$= 2.58 \times 10^{-4}\ \frac{C}{kg}$$

So cosmic rays cause ionization dose rate of 1.7 E-6 R/h. This is about 10% of dose rate at earth's surface. The other 90% is natural radioactivity (Radon, ... ^{41}K (bananas))

More commonly used is the energy dose, the energy deposited per kg of air in units of $\left[\frac{J}{kg}\right]$

Units are called "gray"

$$1\,\text{gray} = 1\,Gy = \frac{1\,J}{kg}$$

Sometimes an older unit is the "rad"

$$1\,Gy = 100\,\text{rad}$$

$$1\,R \cong 1\,\text{rad} = 0.01\,Gy$$

Biological effect of radiation defined as equivalent dose, $D_{equivalent}$, with units of sievert (Sv)

Defined so that the same value of D_{equiv} has the same impact on living tissue, for any type of radiation.

$$D_{equiv} = w_R \cdot w_T \cdot D_{absorbed}$$

radiation factor, expresses the physiological damage relative to γ-ray radiation:

$w_R = 1$ for γ-rays, electrons, positrons

$w_R = 5\text{-}10$ for neutrons

$w_R = 10$ alpha particles (He nucleus)

w_T: tissue weighting factor. For whole body, this sums to 1. w_T is the physiological damage with respect to a whole-body exposure.

$D_{absorbed}$: radiation exposure in Gray.

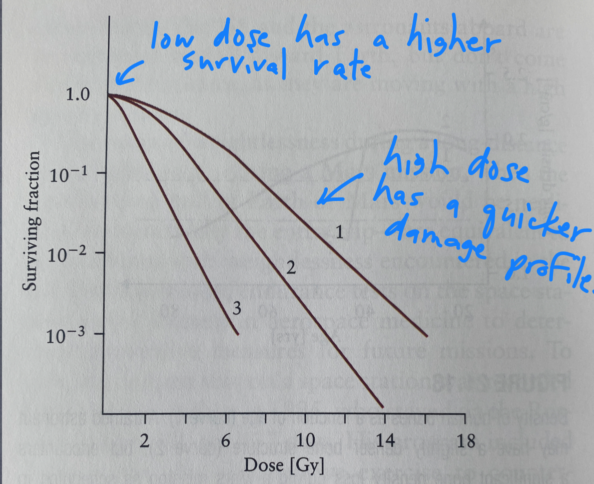


FIGURE 21.15

The surviving fraction of three types of human cells as a function of energy dose in unit Gy. The energy dose is the energy deposited by the radiation per kilogram of tissue. Note the lower steepness at doses below 1 Gy, which is due to self-repair mechanisms in living cells. Various cells respond with different sensitivity to radiation: (1) thyroid cells, (2) mammary cells, and (3) bone marrow.

[1]

Effect of dose

Equivalent dose (Sv)	pathological diagnosis
1 – 5	serious temporary alterations of blood count
4 - 5	50% death rate in 30 days.
10 - 50	vomiting + nausea (die sooner)
50 - 100	brain + nerve damage (death in ~1 week)

↑ for acute dose (all at once)

Tissue weighting factors (w_T) from ICRP

	Female	Male
Testes	0	0.08
Ovaries	0.08	0
Bone surface	0.01	0.01
Bladder	0.04	0.04
Bone marrow, red	0.12	0.12
Brain	0.01	0.01
Breast	0.12	0.12
Colon	0.12	0.12
Liver	0.04	0.04
Lungs	0.12	0.12
Oesophagus	0.04	0.04
Salivary glands	0.01	0.01
Skin	0.01	0.01
Stomach	0.12	0.12
Thyroid	0.04	0.04
Remainder[a]	0.12	0.12

[a]Component organs for remainder in ICRP 103: adrenals, extrathoracic airways, gallbladder, heart, kidneys, lymphatic nodes, muscle, oral mucosa, pancreas, prostate, small intestine, spleen, thymus and uterus/cervix.

↑ sum should add up to 1.

x-ray tricks for medical use

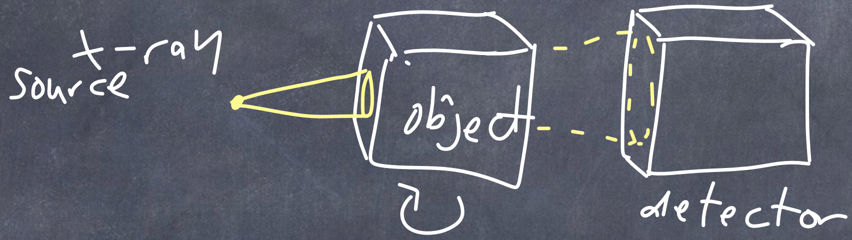
1) gastrointestinal tract can be imaged by x-rays if filled with dense Barium ($\rho = 3.5 \frac{g}{cm^3}$) solution for increased contrast

2) similarly, iodine ($\rho = 4.93 \frac{g}{cm^3}$) + water, make a soluble organic compound, used for cardiovascular system, urinary tract, + the brain

3) mammography (lower energy x-rays, softer)

4) Improved images with computed tomography (CT) to obtain 3-D images from a collection of 2-D images (x-ray images)

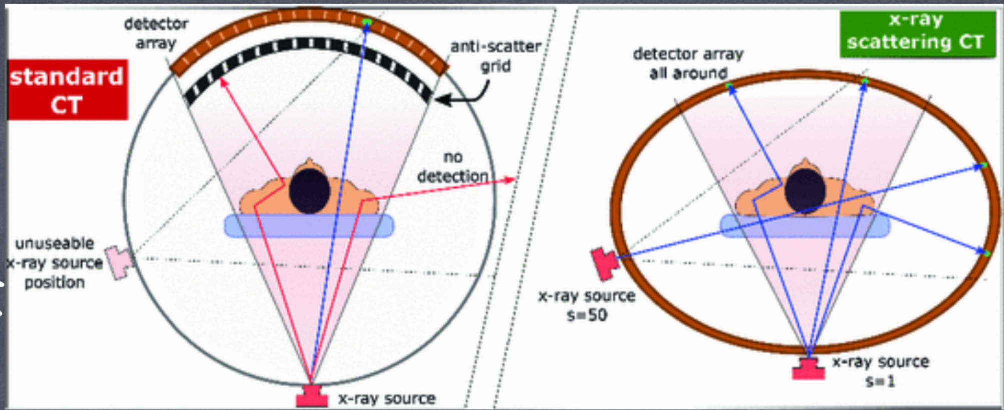
Originally, CT scan would have a single x-ray source, and a detector opposite.



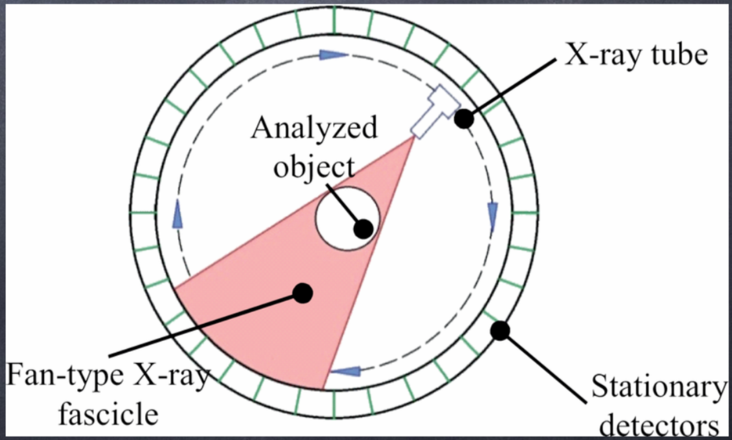
Rotate by 1°, take another x-ray.

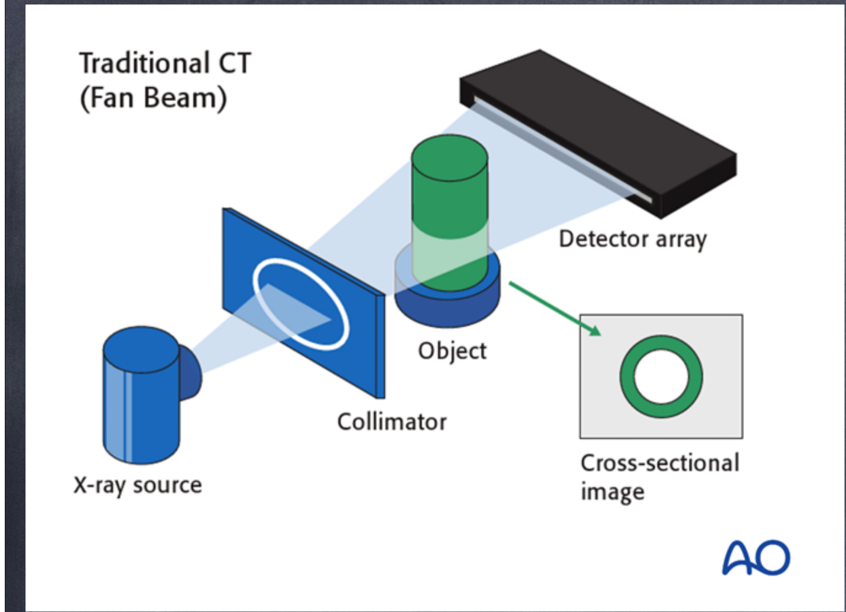
~ minutes to do x-ray.

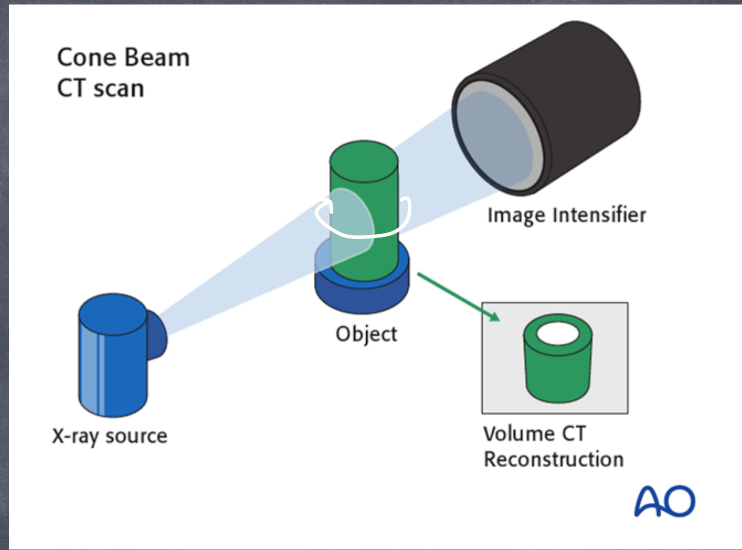
Today, wide fan-like beam, hundreds or thousands of detectors,
⟹ decreases time down to a few seconds



Newest: stationary detectors, and the beam sweeps around the patient.
Typically 50 ms per angle.

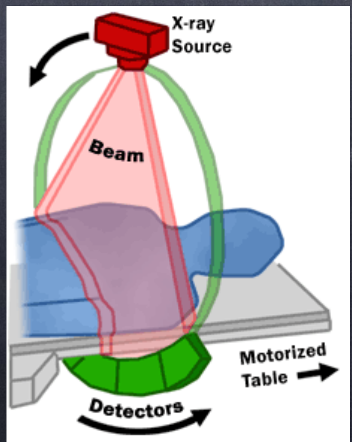






rotating source

fan beam



detectors opposite source

Gray scale can be converted into color to represent brightness level, set according to absorption coefficient of tissue, μ, compared to water, μ_w:

Hounsfield unit

$$HU = 1000 \frac{\mu - \mu_{water}}{\mu_{water} - \mu_{air}}$$

OR

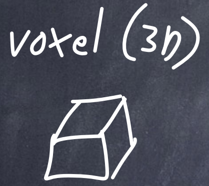
$$\text{CT number} = 1000 \frac{\mu - \mu_w}{\mu_w}$$

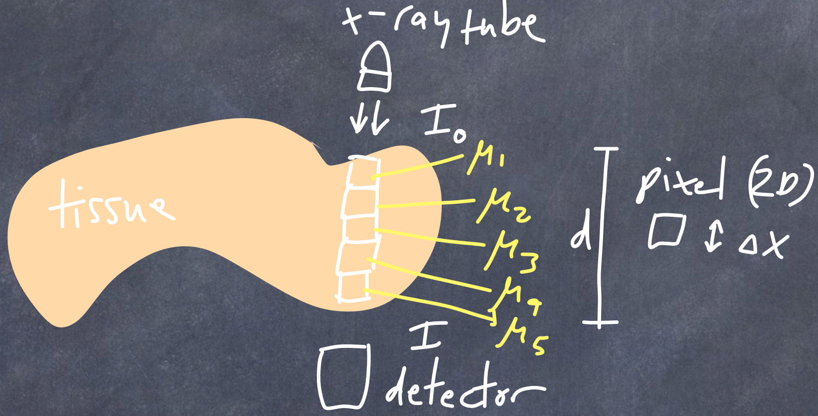
Note: Since ρ_{air} is 800 times smaller than ρ_{water}, CT number ~ HU

material	CT number for 60 KeV x-rays
water	0
air	-1000
bone	808
muscle	-48
fat	-142

How to do and reconstruct a CT scan

x-ray beam goes through patient, different types of tissue are encountered, with different μ.





we have 5 unknowns: $\mu_1, \mu_2, \mu_3, \mu_4, \mu_5$

$$I = I_0 e^{-\sum_{i=1}^{5} \mu_i \Delta x}$$

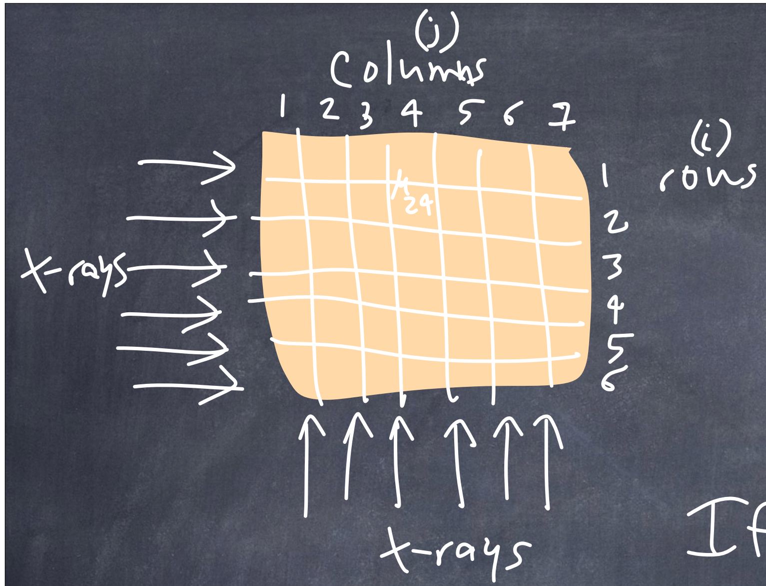
I: we measure this

I_0: we know this

$$\log \frac{I_0}{I} = \sum_{i=1}^{5} \mu_i \Delta x$$

If we keep making Δx smaller ($\Delta x \to 0$), then

$$I = I_0 e^{-\int_0^d \mu(x)\, dx}$$



we can measure I along different rows + columns

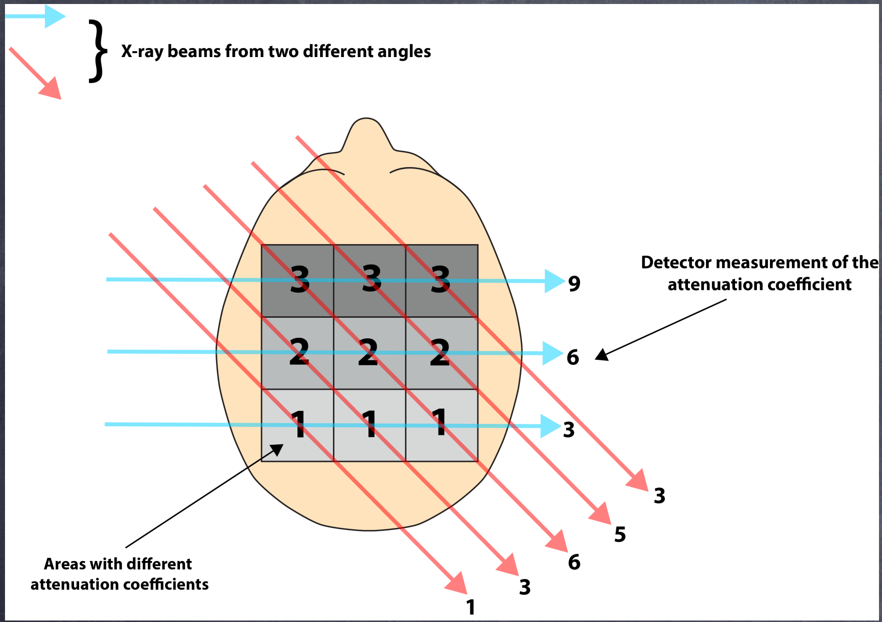
For the i^{th} row:

$$\mu_i(\Delta x) = \sum_{j=1}^{7} \mu_{ij} \Delta x$$

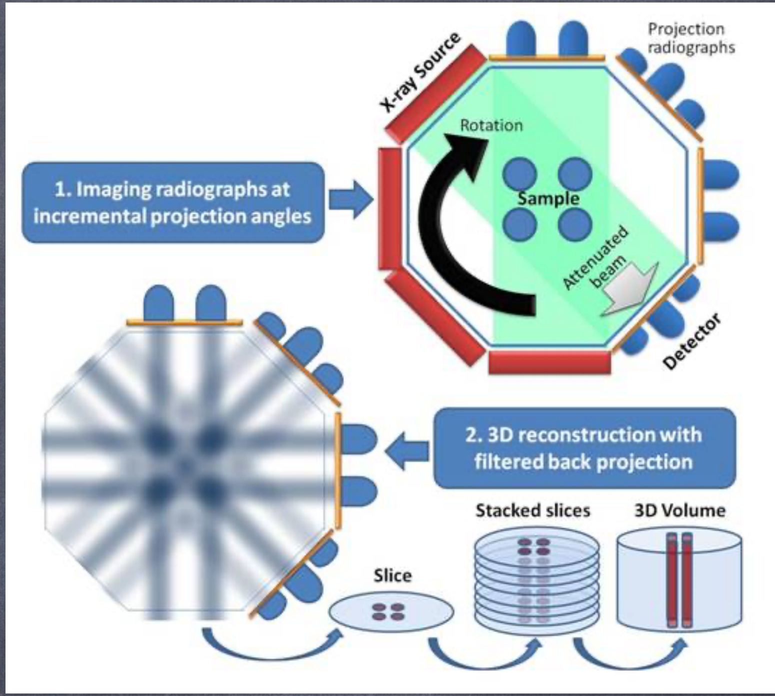
If we have N^2 pixels ($N \times N$ grid, here 7×7)

then you have N^2 unknowns. Then you need N^2 equations to solve for the unknowns.

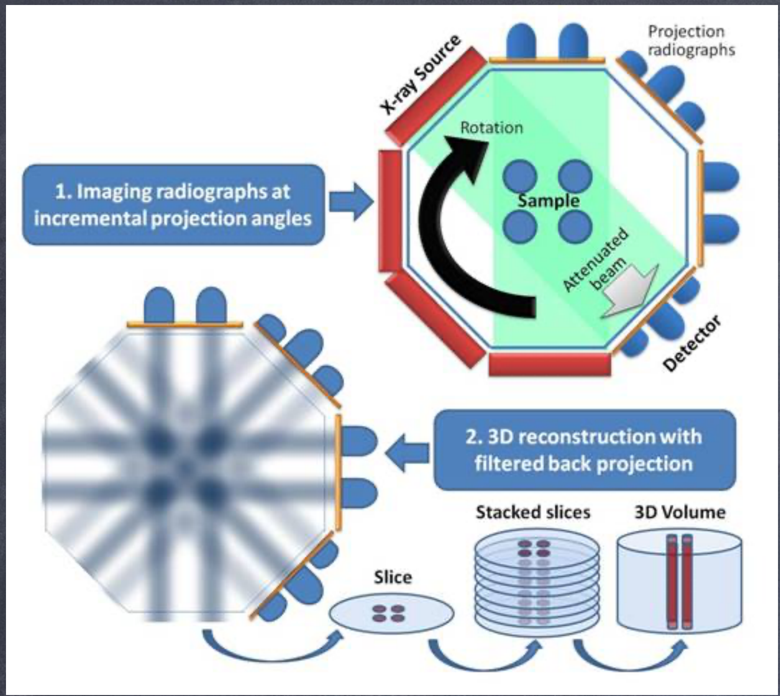
2D → 3D

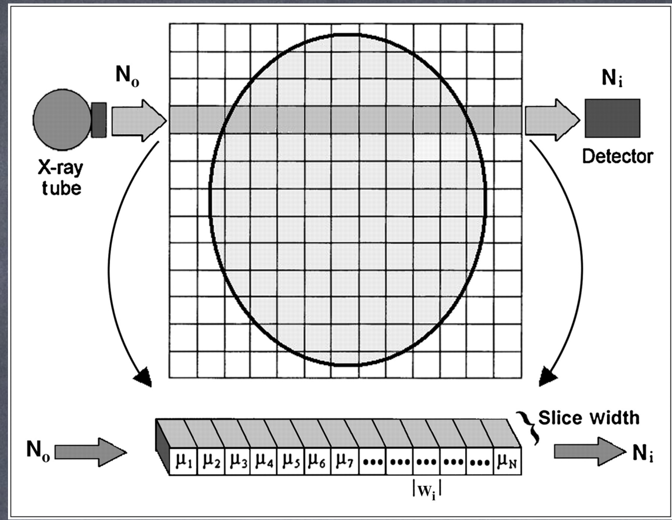


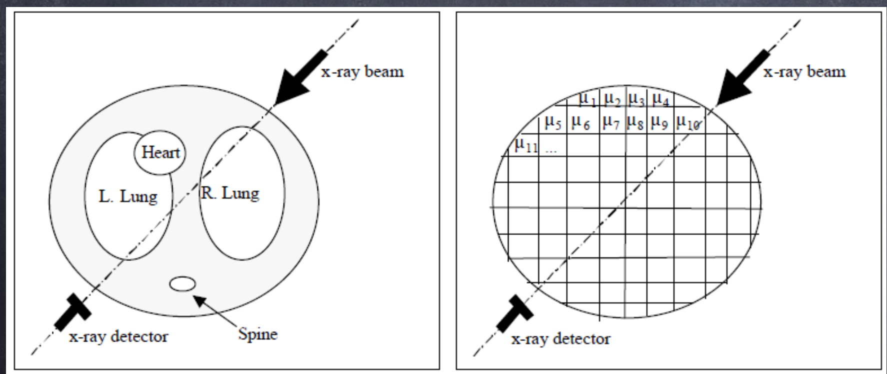
X-ray beams from two different angles
3 3 3
2 2 2
1 1 1
9
6
3
3
5
6
3
1
Detector measurement of the attenuation coefficient
Areas with different attenuation coefficients

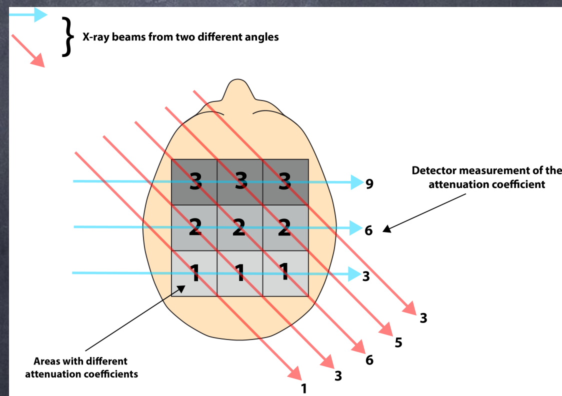


X-ray Source
Projection radiographs
Rotation
Sample
Attenuated beam
Detector
1. Imaging radiographs at incremental projection angles
2. 3D reconstruction with filtered back projection
Stacked slices
3D Volume
Slice


1. Imaging radiographs at incremental projection angles
X-ray Source
Rotation
Sample
Attenuated beam
Projection radiographs
Detector
2. 3D reconstruction with filtered back projection
Slice
Stacked slices
3D Volume


N_o
X-ray tube
N_i
Detector
Slice width
μ_1 μ_2 μ_3 μ_4 μ_5 μ_6 μ_7 ... μ_N
$|w_i|$

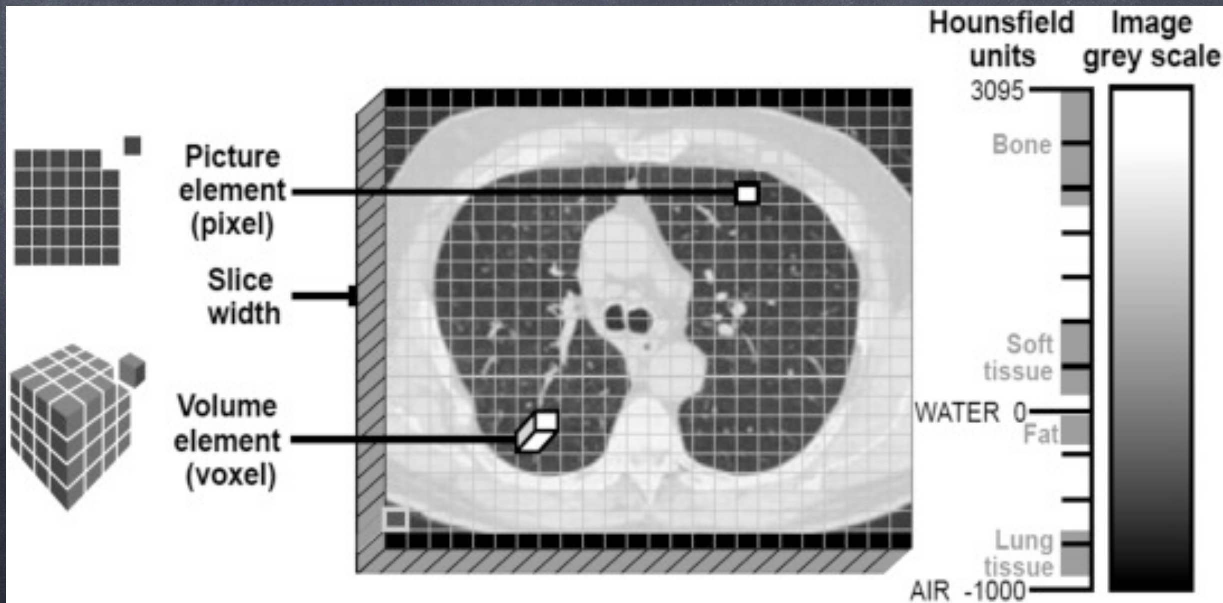

x-ray beam
Heart
L. Lung
R. Lung
Spine
x-ray detector
x-ray beam
μ_1 μ_2 μ_3 μ_4
μ_5 μ_6 μ_7 μ_8 μ_9 μ_{10}
μ_{11} ...
x-ray detector


X-ray beams from two different angles
Detector measurement of the attenuation coefficient
Areas with different attenuation coefficients

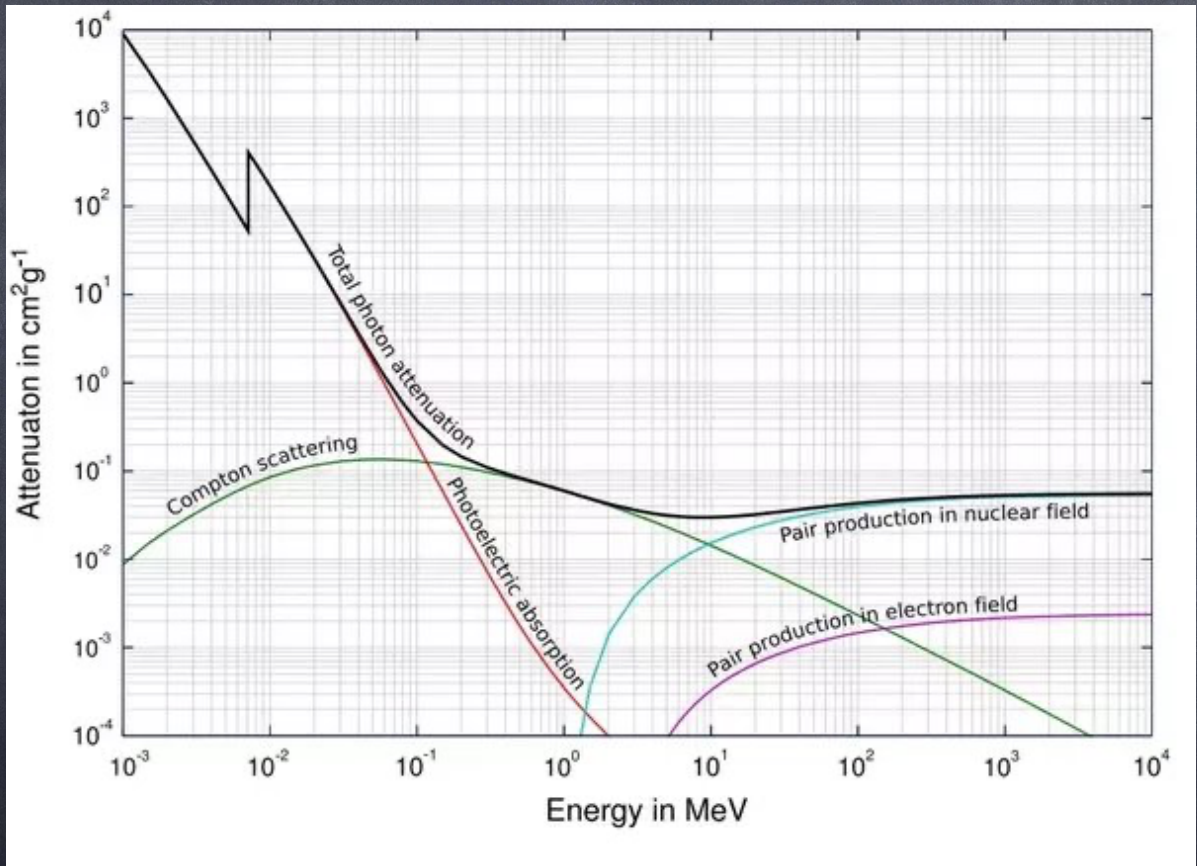
Typically, N : 256 ~ 1024

N^2 (# pixels) ~ 1 million unknowns.

Typically solved by computers



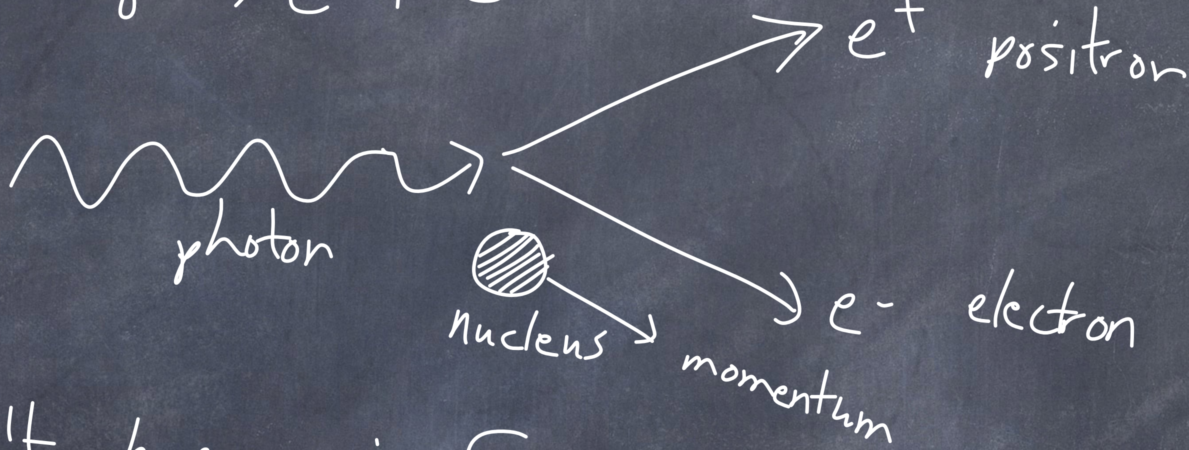
Reminder of attenuation of x-rays + gamma rays



x-rays ↑ gamma rays

If a photon has enough energy, it can convert its energy entirely to charged particles. The lightest charged particle is the electron.

$$\gamma \to e^+ + e^-$$



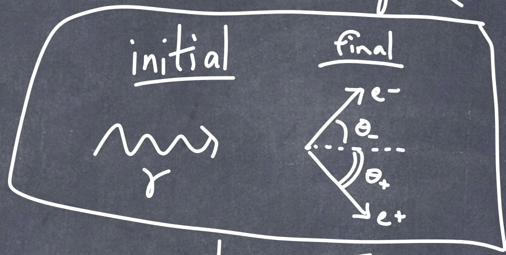
This can't happen in free space, only near a massive object, such as a nucleus, such that the nucleus can supply momentum so that the momentum is conserved.

How much photon energy is enough?

$$E = h\nu \geq 2m_e c^2 \qquad m_e = 0.511\ \mathrm{MeV}/c^2$$

$$E > 1.022\ \mathrm{MeV}$$

start S3: supplementary proof that $\gamma \to e^+ + e^-$ can't happen in free space



we assume no nucleus:

initial: photon has energy : $h\nu$

final : no photon anymore,
electron has energy E_- and momentum $\bar{p}_-$
positron has energy E_+ and momentum $\bar{p}_+$

$m = m_e$: mass of electron

From conservation laws:

	initial	final	
Energy :	$h\nu =$	$E_+ + E_-$	①
Momentum x :	$\frac{h\nu}{c} =$	$p_- \cos\theta_- + p_+ \cos\theta_+$	②
momentum y :	$0 =$	$p_- \sin\theta_- + p_- \sin\theta$	③

Rewrite ② we get : $h\nu = cp_- \cos\theta_- + cp \cos\theta_-$ ④

Insert formula for relativistic energy $\left(E^2 = (cp)^2 + (mc^2)^2\right)$ into ①:

$$h\nu = \sqrt{(cp_+)^2 + (mc^2)} + \sqrt{((cp_-)^2 + (mc^2)^2} \quad ⑤$$

The maximum value of $h\nu$ in ④ is when
$\cos\theta_- = \cos\theta_+ = 1$.

Then ④ becomes $$h\nu = cp_- + cp_+$$ ⑥

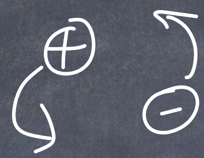
But if we look at eq. ⑤, we see that
$(h\nu)^2$ must be greater than $(cp_-)^2 + (cp_+)^2$
because of the electron + positron masses.

Therefore, since we have 2 equations, ⑤ and ⑥, which can't be both true at the same time, this reaction is not valid, because energy & momentum can't be conserved simultaneously.

end

(53) Finished

what happens to positrons ?



They orbit each other
(positron exists for about 10^{-10} s)

Then they annhilate

$$e^- + e^+ \rightarrow \gamma + \gamma$$

(Two photons instead of one is because of momentum & energy conservation)

This is the principle of Positron emission tomography (PET). We start with a positron.

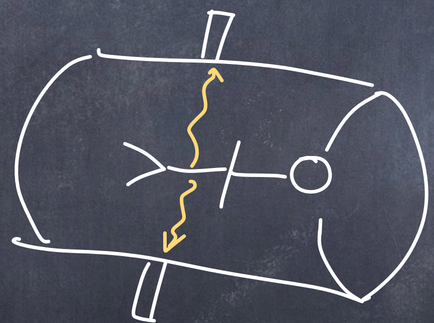
1) A positron-emitting radioactive element containing ^{15}O, ^{11}C, ^{13}N, ^{18}F, ^{68}Ga is attached to a pharmaceutical + (ingested or injected)

Radioative elements are usually prepared at an accelerator.

This is the principle of Positron emission tomography (PET).

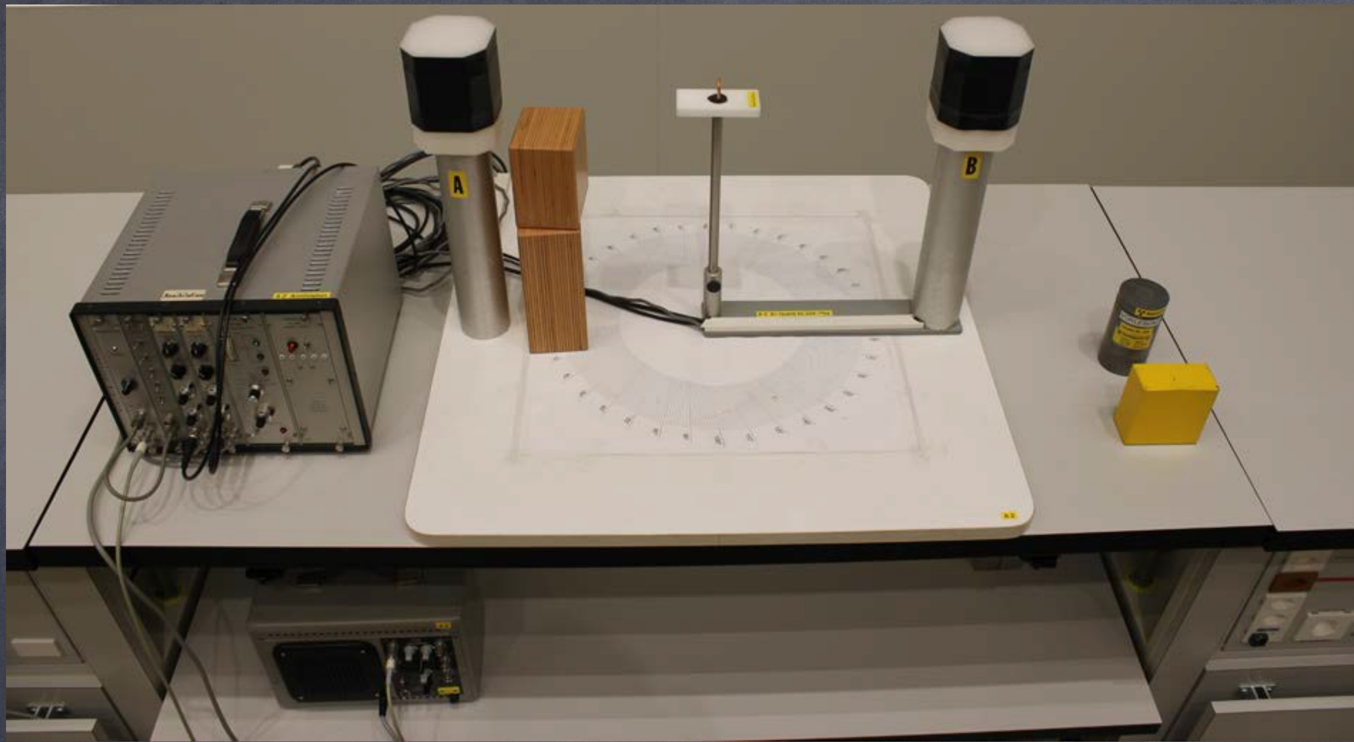
1) A positron-emitting radioactive element containing ^{15}O, ^{11}C, ^{13}N, ^{18}F, ^{68}Ga is attached to a pharmaceutical, + (ingested or injected)

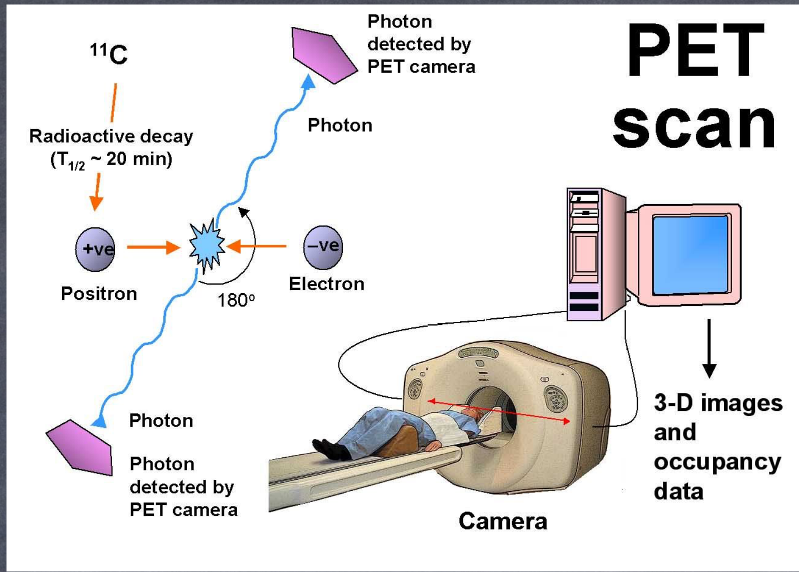
Radioative elements are usually prepared at an accelerator.

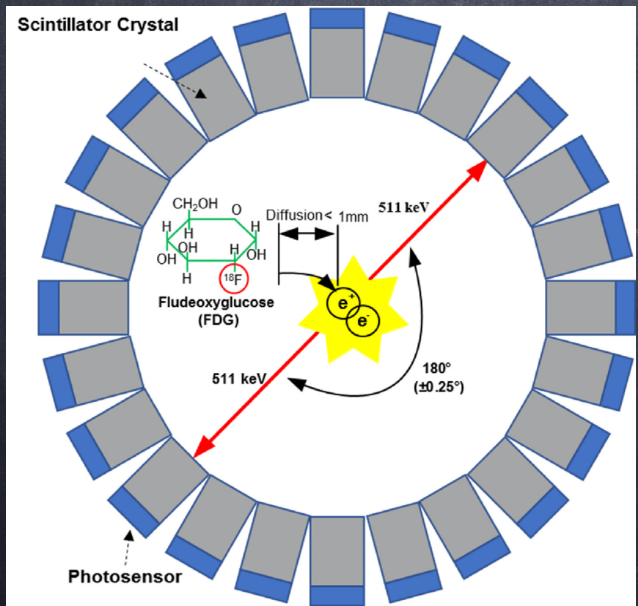


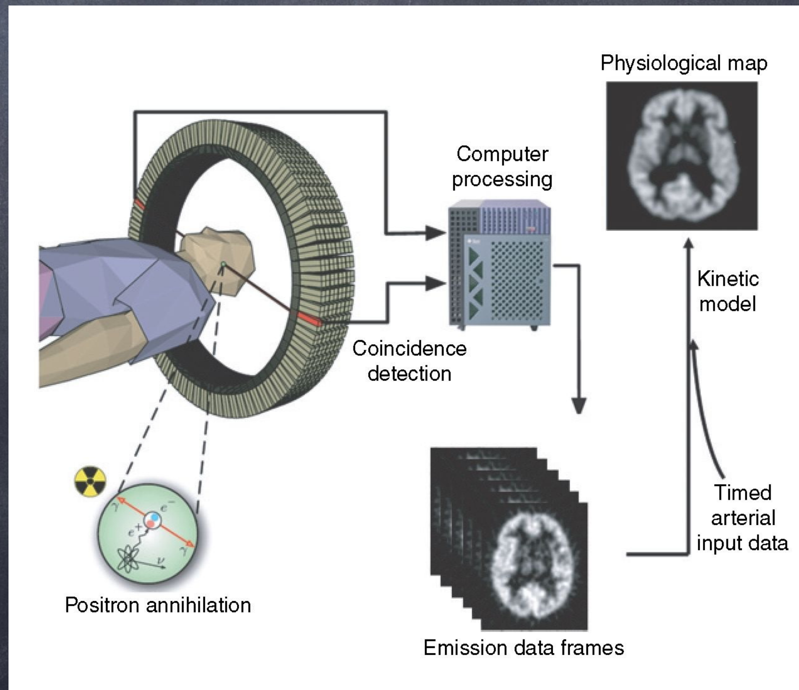
Two photons leave the body back to back. Detectors 180° apart that look for coincident arrival of 511 keV gamma rays.

By collecting data at different angles, we can reconstruct 3D images.


A
B


PET scan
^{11}C
Radioactive decay
($T_{1/2}$ ~ 20 min)
+ve
Positron
–ve
Electron
180°
Photon
Photon detected by PET camera
Photon
Photon detected by PET camera
3-D images and occupancy data
Camera


Scintillator Crystal
CH_2OH
O
H
OH
Fludeoxyglucose (FDG)
^{18}F
Diffusion< 1mm
511 keV
511 keV
e^+
e^-
180° (±0.25°)
Photosensor


Physiological map
Computer processing
Kinetic model
Coincidence detection
Timed arterial input data
Positron annihilation
Emission data frames

Spatial resolution limited to ~ 5mm, by:

1) positronium has some non-zero momentum, so the angle is not exactly 180°
2) positron can travel ~ 1 mm before it annhilates.

But PET scans can be done in real time.

By correlating images of blood flow or glucose or oxygen metabolism, & monitoring a patient stimulated in some way, biochemical events can be correlated with brain activity.
(Can reveal abnormal brain function)

PET is best used for monitoring time-dependence on metabolism of radiopharmaceuticals, but not the best technique for spatial resolution.

Supplement on Feynmann diagrams follows

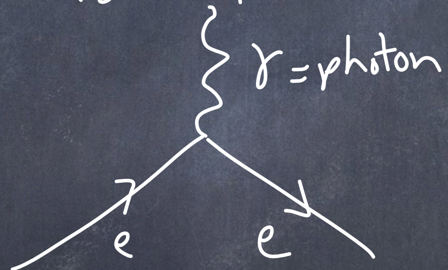
Light is an electromagnetic wave.

Light is quantized. The unit of light is a photon.

Quantum electrodynamics (QED)

All electromagnetic phenomena are ultimately reducible to the following elementary process.

(with an electron)



→ time

Time flowing horizontally to the right

This diagram reads "an electron enters, emits or absorbs a photon, and exits."

This diagram can be flipped or rotated, and the process still happens.

A particle moving backwards in time is interpreted as an antiparticle moving forwards in time.

electron = e^-

positron = e^+ antiparticle of the electron

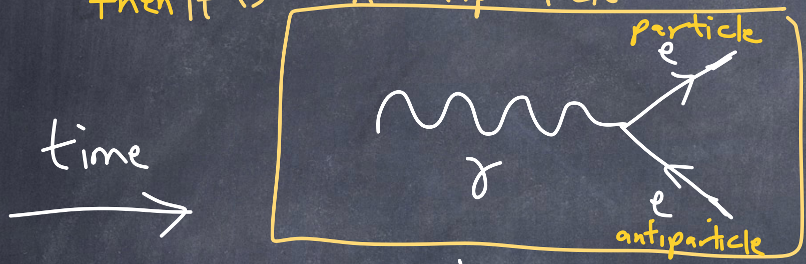
A photon does not need an arrow since it is its own antiparticle.

So this diagram reads "a positron enters, emits or absorbs a photon, and exits."

The positron was predicted in 1928 by Dirac because his formulas had 2 solutions: +, -

Discovered in 1932 by Anderson

If arrow is moving opposite to time, then it is an antiparticle.



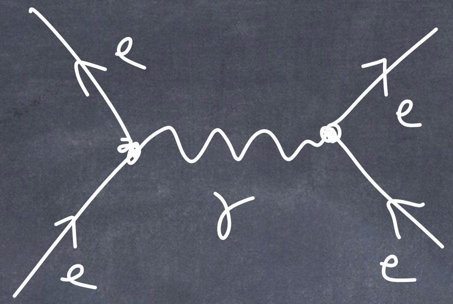
Can happen but must obey energy & momentum conservation.

"A photon enters, decays into an electron and a positron, and they exit."

These diagrams are called Feynmann diagrams.

Some people label these diagrams with e^- & e^+, but I find this dangerous.

Since a positron moving backwards in time would be an electron



→ time

Here, an electron and positron annihilate into a photon, and then the photon decays into a new electron and positron.

Note: the electric charge is conserved.

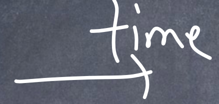
we can write this diagram as:

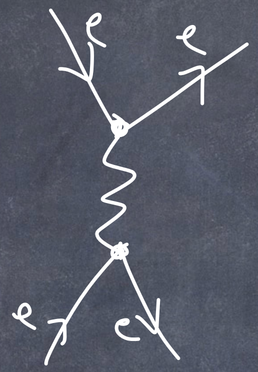
$$e^- + e^+ \rightarrow \gamma \rightarrow e^- + e^+$$

electric charge $\quad -1 + 1 = 0 = 0 = -1 + +1$

Energy + momentum are conserved in this process.

This diagram can be rotated.





Here, two electrons enter, exchange a photon and continue as electrons.

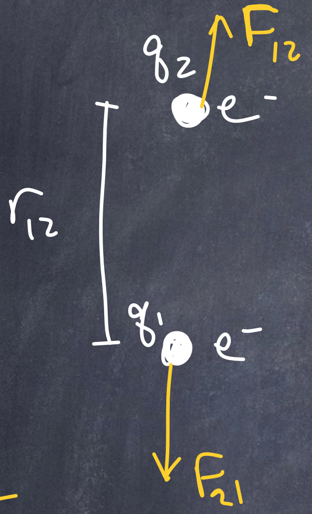
Here, the electrons repel can be seen to repel each other.

In quantum physics, forces are mediated by particles.

The photon mediates the electromagnetic force.

But does this mean classical physics is wrong?

classical physics view: (PHY 117)



F_{12} = force of q_1 on q_2

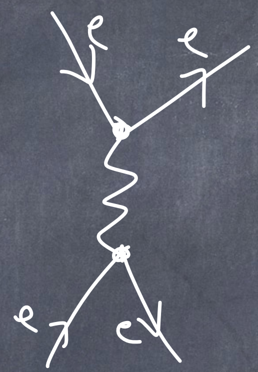
$q_1 = e = q_2$

$$-\bar{F}_{21} = \bar{F}_{12} = k\frac{q_1 q_2}{r_{12}^2}\,\hat{r}_{12}$$

Electrons are repelled by a force, which we can calculate.

quantum physics view:

time →



Here, two electrons exchange a photon and continue as electrons.

Here, the electrons repel each other.

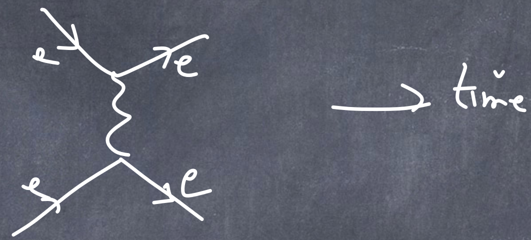
In quantum physics, forces are mediated by particles.

The photon mediates the electromagnetic force.

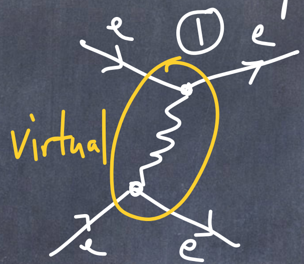
In practice, classical physics is easier to calculate for most everyday situations

What happens if a particle moves perpendicular to time?

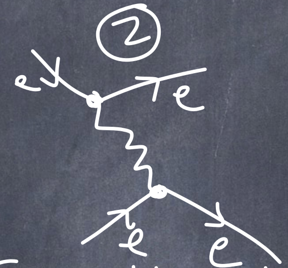
A: Here a photon moves vertically.



Really, what is happening is both of these:

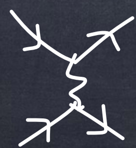


and

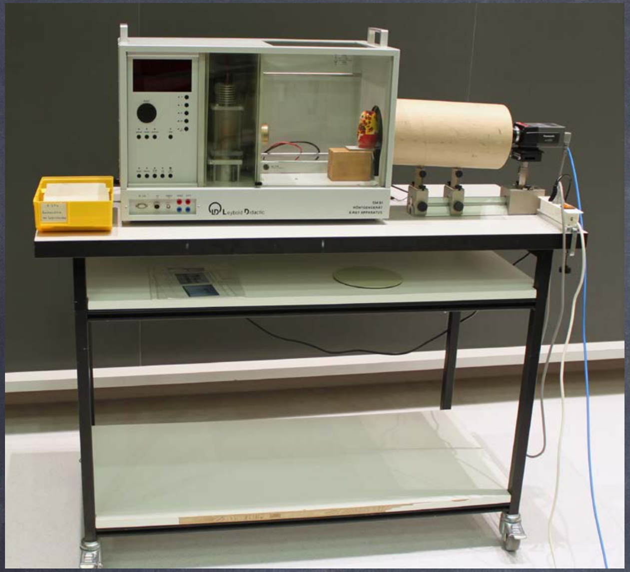


In ①, the photon is emitted from the below electron.
In ②, the photon is emitted from the above electron

The photon is not observable, we call it virtual.
we can't tell if ① or ② happens, so we use quantum mechanics to consider both.
But we draw it like this

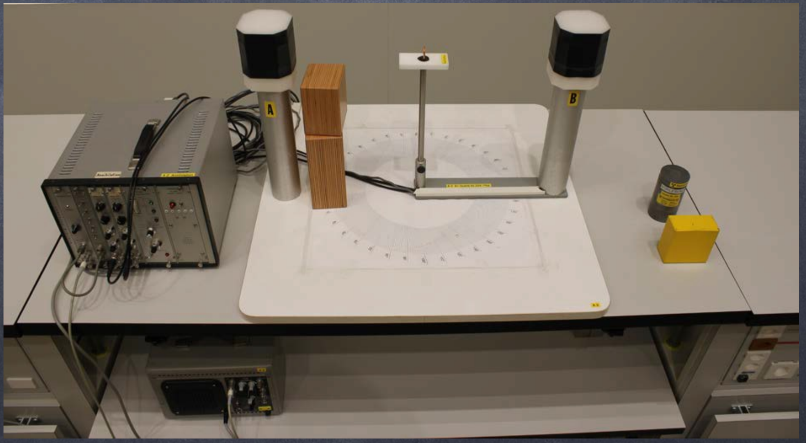


.



A24

ES90



A2