Numerical methods and applications to two-loop calculations

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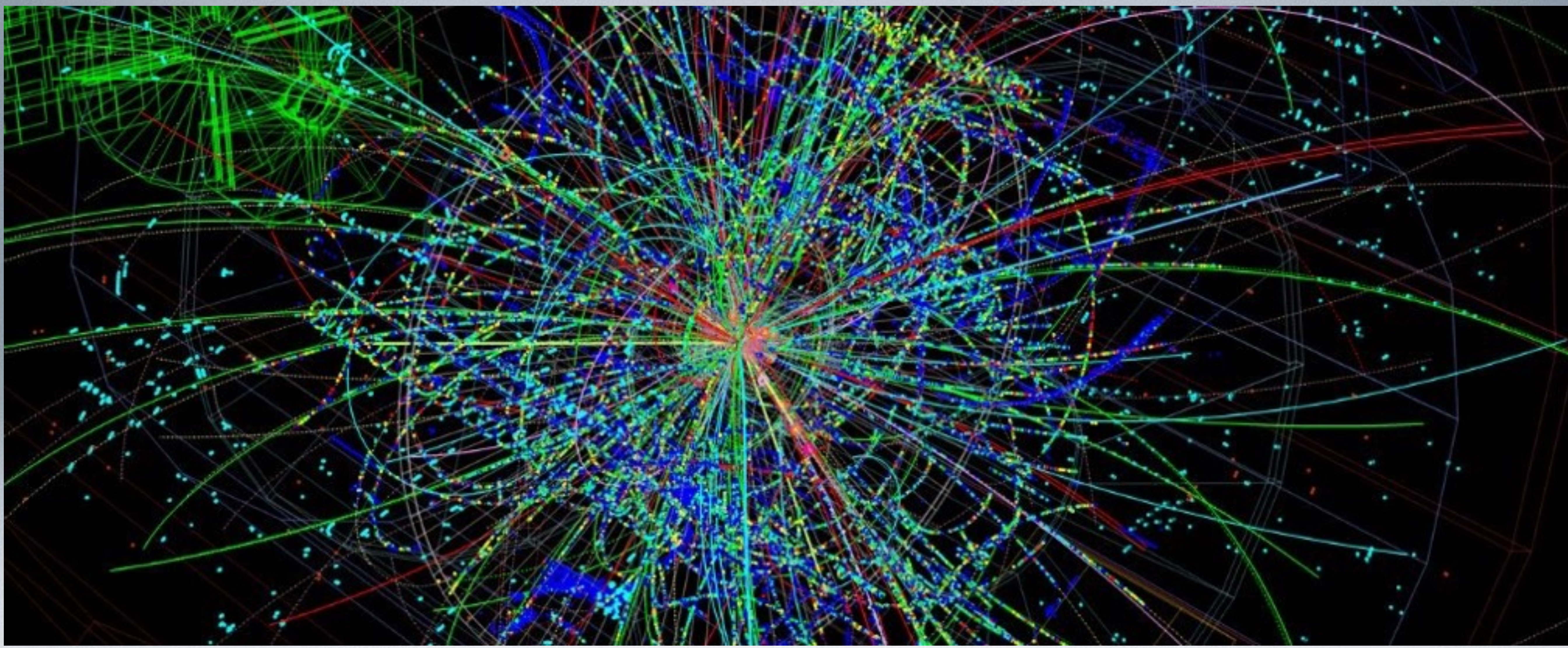


Universität Zürich, November 14, 2017

MAX-PLANCK-GESELLSCHAFT

Max-Planck-Institut für Physik
(Werner-Heisenberg-Institut)

Outline

Status of calculations beyond one loop

Numerical methods for (multi-) loop calculations

The program pySecDec

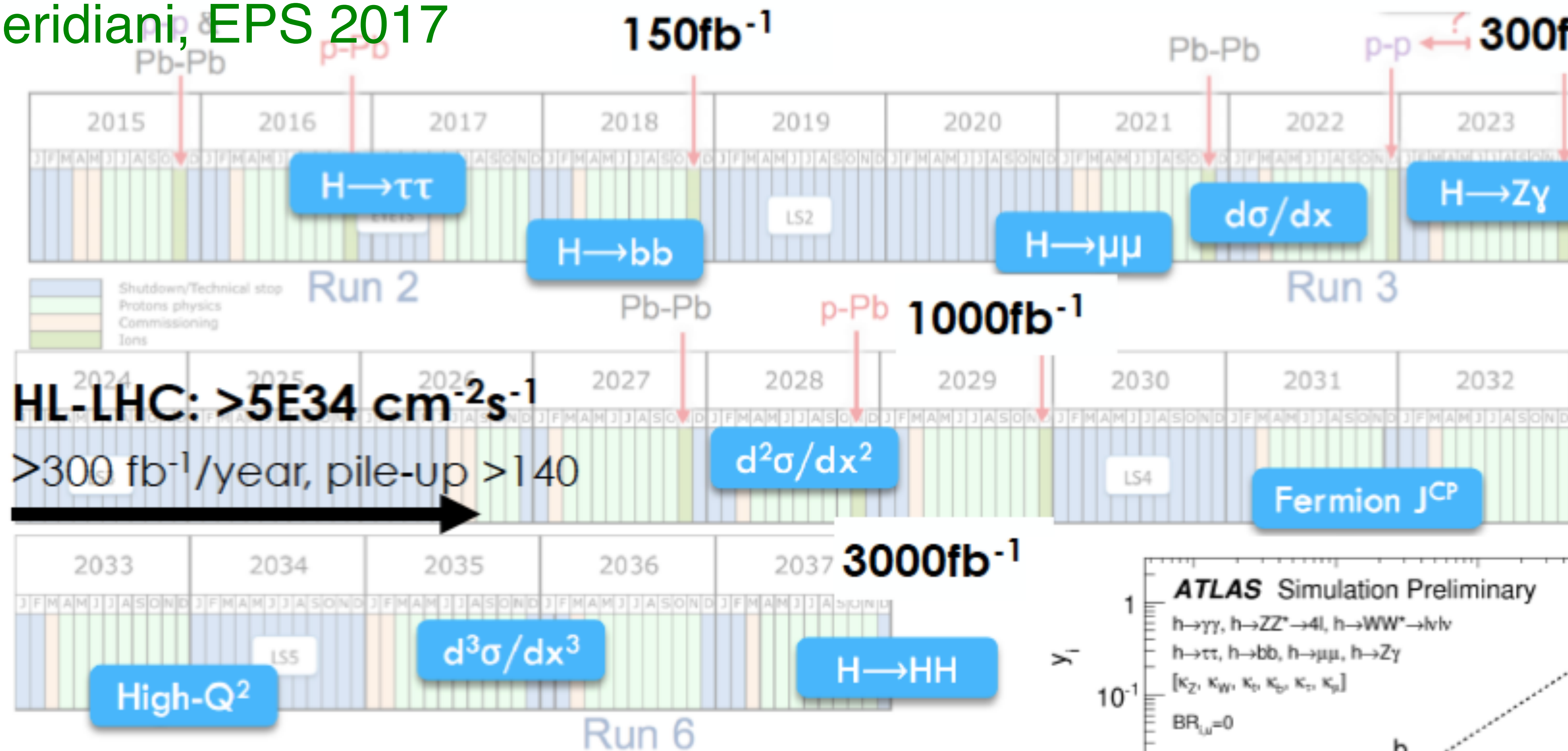
Some applications

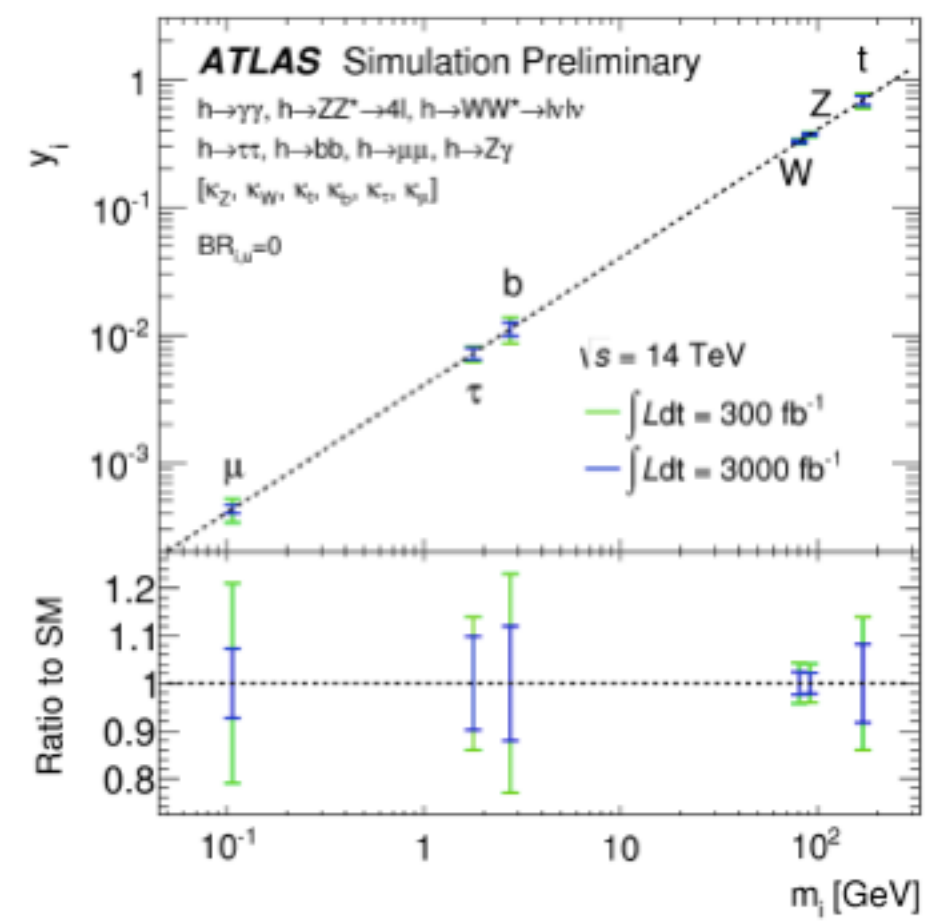
in collaboration with
S.Borowka, N.Greiner, S.Jahn, S.P. Jones,
M.Kerner, G.Luisoni, J.Schlenk, T.Zirke

TIMELINE BEYOND RUN2

Credits: A. David @ GRC 2017

P.Meridiani, EPS 2017





TIMELINE BEYOND RUN2

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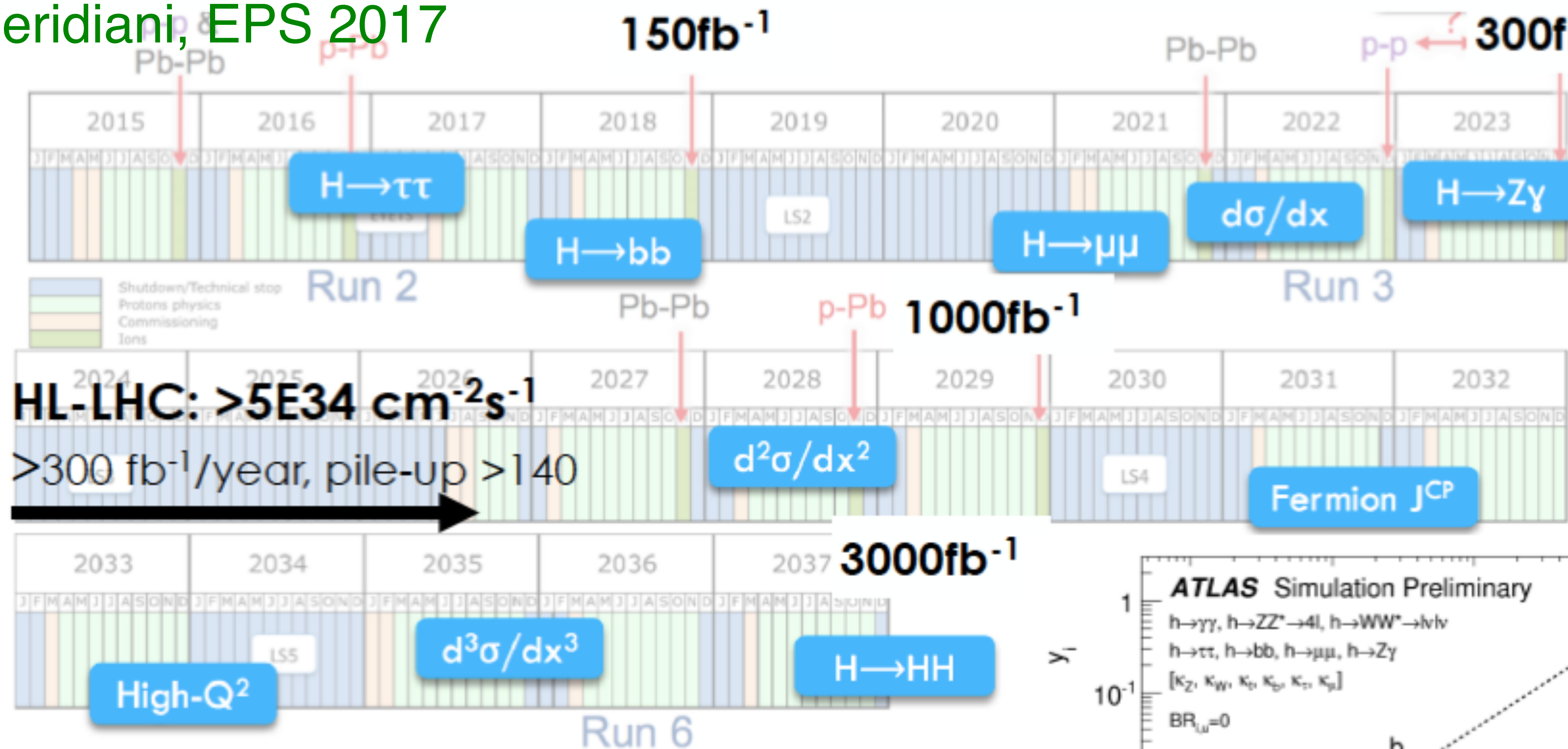
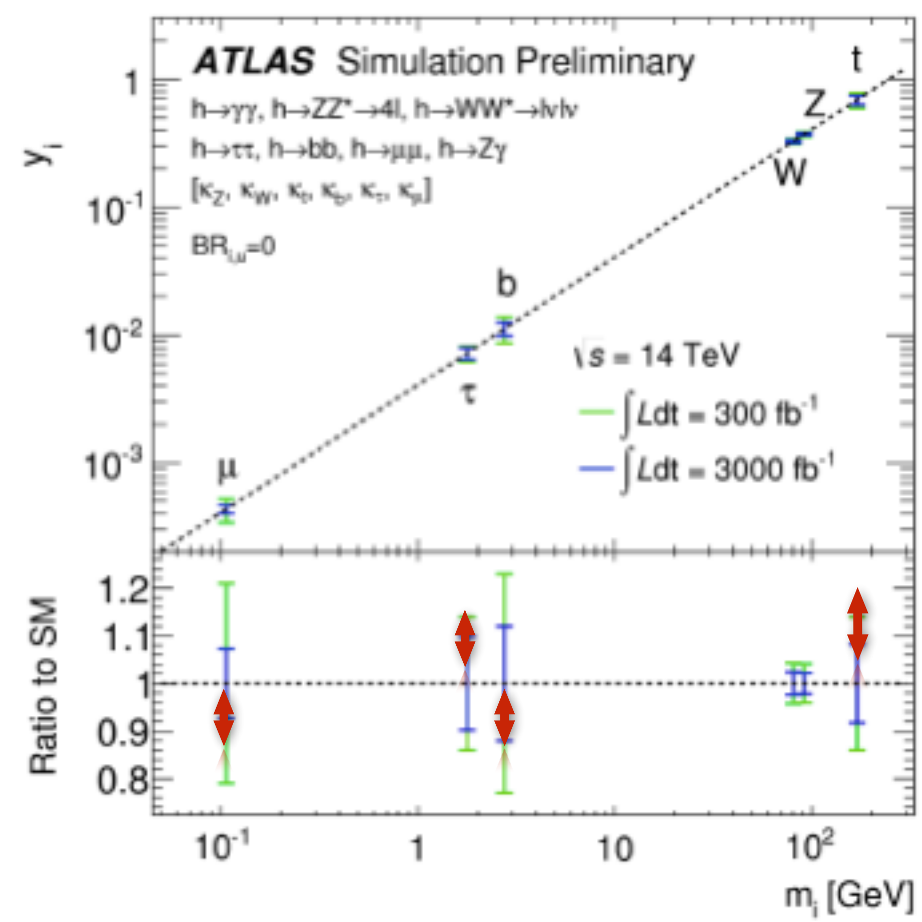


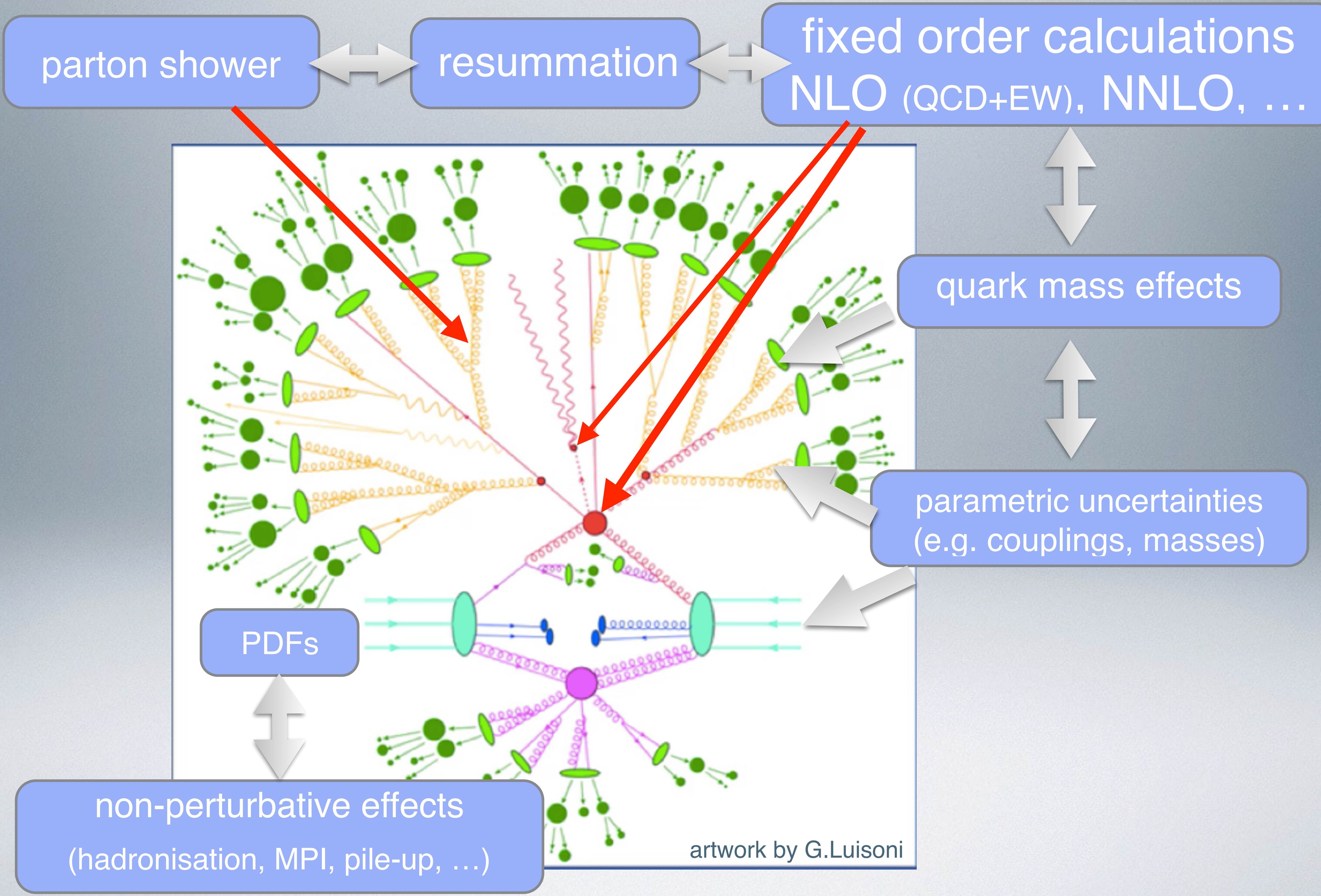
HL-LHC: >5E34 $cm^{-2}s^{-1}$

>300 fb^{-1}/year, pile-up >140



we need to be sure about the systematic uncertainties to claim something like this

The precision frontier



status of higher (fixed) order predictions involving H

	$\sigma_{\rm tot}$	$d\sigma$	bonus
$gg \to H$	N^3LO	NNLO	N^3LL threshold resum. NLO EW, mixed QCD/EW
VBF	N^3LO (DIS approx.)	NNLO	NLO EW
$WH\ /\ ZH$	NNLO	NNLO	NLO EW gg to ZH mt dep. approx.
$t\bar{t}H$	NLO	NLO	NNLL threshold resum. NLO EW
$H + \text{single top}$	NLO	NLO	
$H + \text{jets}$	1j: NNLO	1j: NNLO	NLO up to 3jets H+1jet NLO: mb dependence, high energy limit, real radiation up to 2 jets, NNLL resum.
$H\ H$	NNLO	NNLO	NLO NLL resum.

black: $m_t \to \infty$ (HEFT) blue: m_q dependence

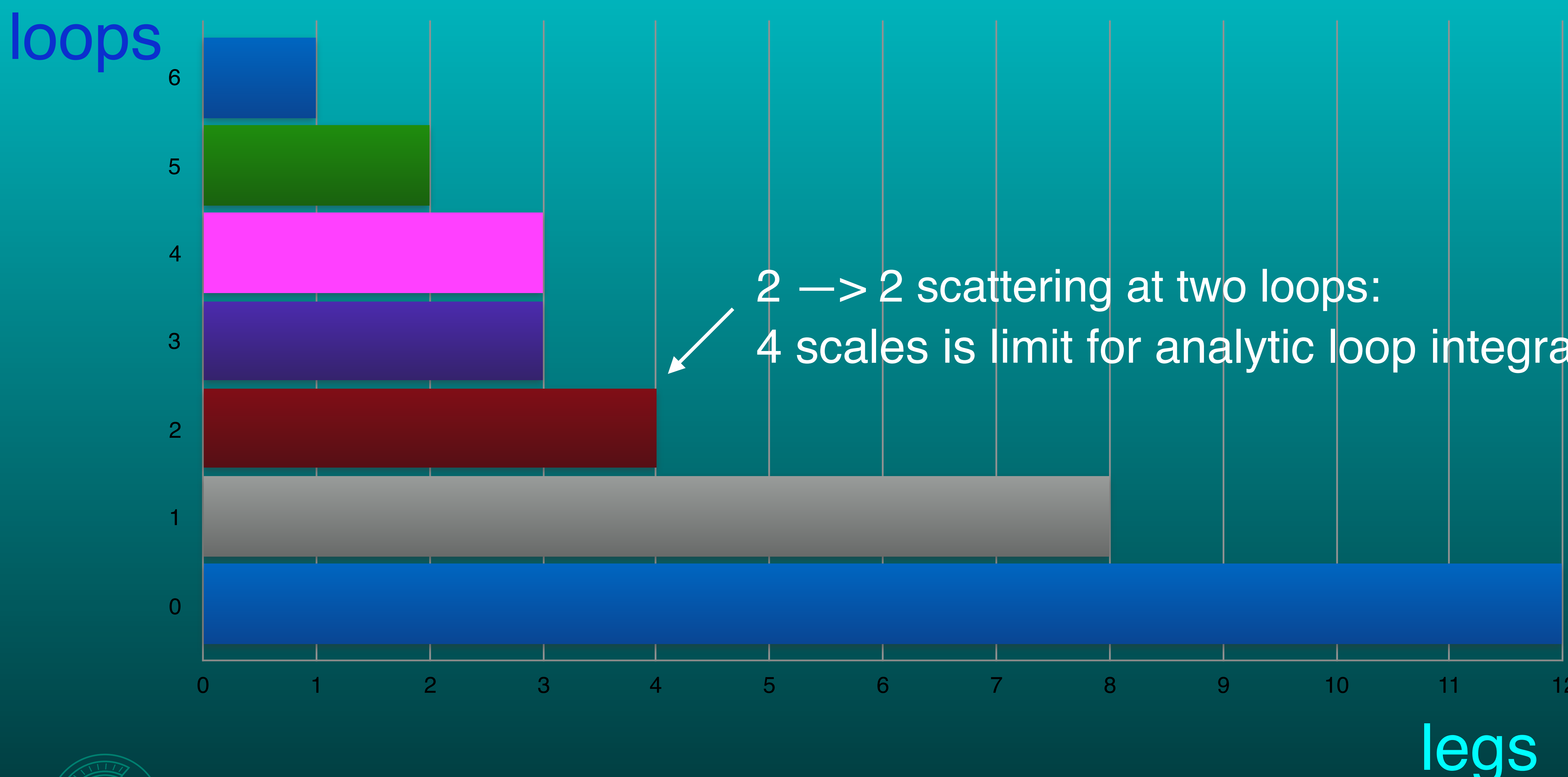
status of higher (fixed) order predictions involving H

	σ_{tot}	$d\sigma$	bonus
$gg \to H$	N^3LO Anastasiou et al '13-'15	NNLO (+NNLL) De Florian, Grazzini, Tommasini '11,'12 Dulat, Mistlberger, Pelloni '17	N^3LL threshold resum. Bizon, Monni, Re, Rottoli, Torrielli '17 NLO EW, mixed QCD/EW Degrassi, Maltoni '04; Actis, Passarino, Sturm, Uccirati '08; Anastasiou et al '08
VBF	N^3LO Bolzoni, Maltoni, Moch, Zaro '10; Dreyer, Karlberg '16	NNLO Cacciari, Dreyer, Karlberg, Salam, Zanderighi '15	NLO EW Ciccolini, Denner, Dittmaier '07
$WH\,/\,ZH$	NNLO Brein, Djouadi, Harlander '03; Brein, Harlander, Wiesemann, Zirke '11	NNLO Ferrera, Grazzini, Tramontano '11,'14	NLO EW Denner, Dittmaier, Kallweit, Mück '12 gg to ZH 1/mt Altenkamp et al '12 NNLL threshold resum. Harlander, Kulesza, Theeuwes, Zirke'14
$t\bar{t}H$	NLO Beenakker et al '01; Dawson, Reina '02; Frederix et al '14	NLO	NLO EW Frixione et al '14; (on-shell) Denner, Lang, Pellen, Uccirati '16 NNLL threshold resum. Kulesza et al '15
$H + \mathrm{single\,top}$	NLO Demartin, Maltoni, Mawatari, Zaro '15	NLO	
$H + \mathrm{jets}$	1j: NNLO Boughezal, Caola et al '15; Boughezal et al '15; Chen et al '15	1j: NNLO	NLO up to 3jets Cullen et al '13 H+1jet NLO: mb dependence Lindert, Melnikov, Tancredi, Wever '17
$H\,H$	NNLO De Florian, Mazzitelli '15; Steinhauser et al '15	NNLO De Florian et al '16	NLO Borowka, Greiner, GH, Jones, Kerner, Schlenk, Schubert, Zirke '16 NLL resum. Ferrera, Pires '16

measure of complexity

#loops + #legs + **#scales (masses, off-shellness)**

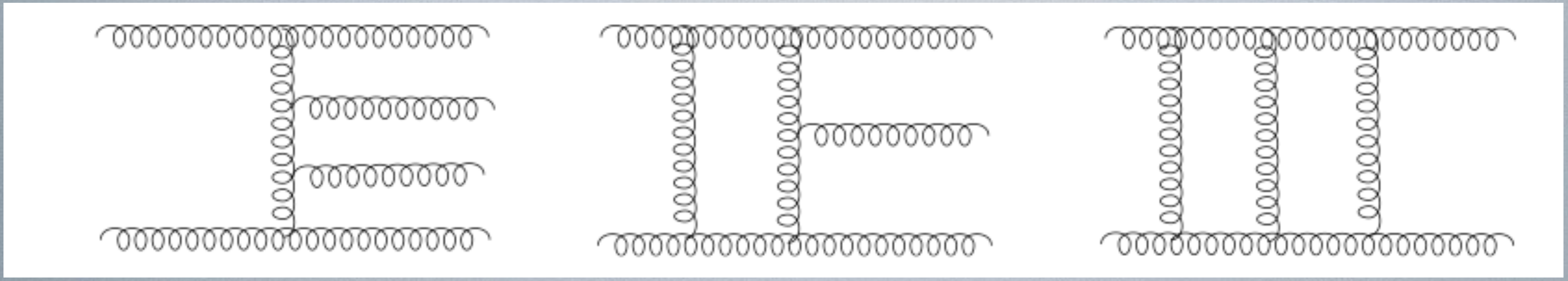
complexity does not scale linearly!



(refers to physical results, not individual integrals)

QCD corrections: building blocks

NNLO:

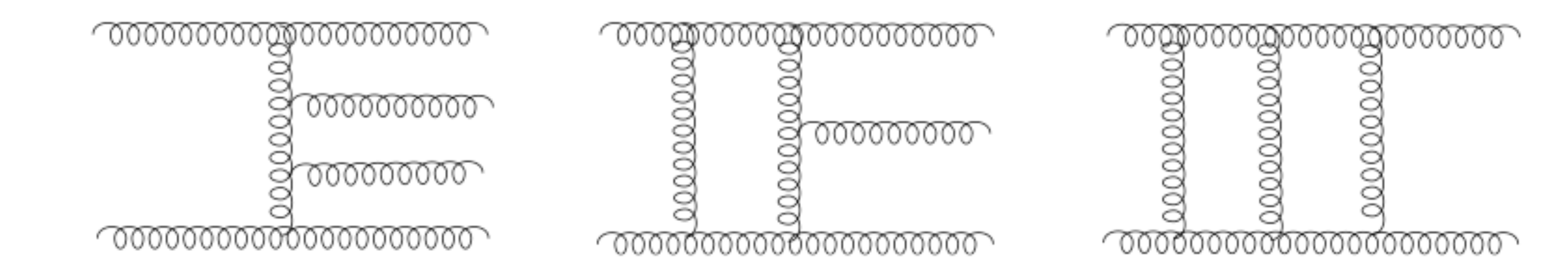


double real	1-loop virtual $\otimes$ single real	2-loop virtual
↓	↓	↓
implicit IR poles (PS integration)	explicit and implicit poles	explicit poles $1/\epsilon^{2L}$

bottlenecks: IR subtraction   two-loop integrals

QCD corrections: building blocks

NNLO:



double real

1-loop virtual
$\otimes$ single real

2-loop virtual

implicit IR poles (PS integration)

explicit and implicit poles

explicit poles $1/\epsilon^{2L}$

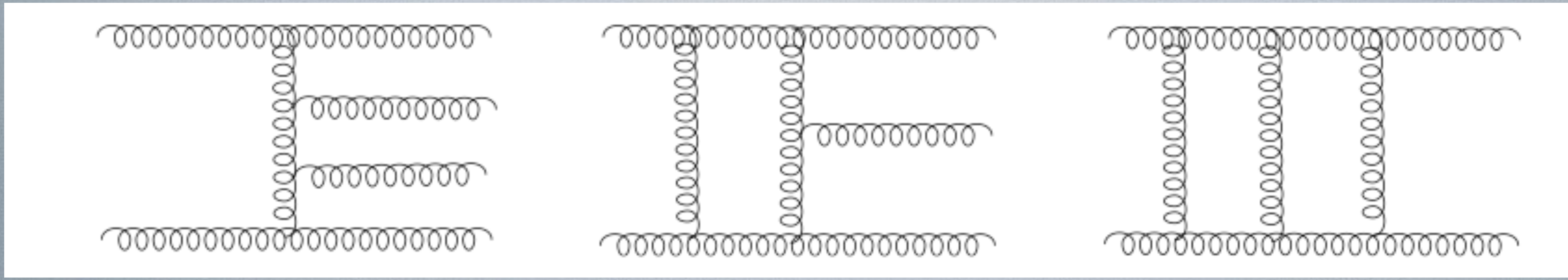
bottlenecks: IR subtraction

two-loop integrals

harder with more massless particles
(intricate IR singularity structure)

QCD corrections: building blocks

NNLO:



double real

1-loop virtual
$\otimes$ single real

2-loop virtual

↓ implicit IR poles (PS integration)

↓ explicit and implicit poles

↓ explicit poles $1/\epsilon^{2L}$

bottlenecks: IR subtraction

↑ harder with more massless particles
(intricate IR singularity structure)

two-loop integrals

↑ harder with more massive/off-shell particles
(more scales → more complicated analytic structure)

typical procedure and corresponding tools

1. automated amplitude generation

public **tools** e.g. QGRAF [P.Nogueira], FeynArts [T.Hahn et al.]

saturation of Lorentz/spin indices: helicity amplitudes or projectors to form factors

2. reduction of the loop amplitudes to coefficients $\otimes$ master integrals

reduction highly non-trivial; no unique master integral basis beyond one loop

multi-purpose **tools** e.g. Reduze [C.Studerus, A.v.Manteuffel], FIRE [A.V.Smirnov], LiteRed [R.N.Lee], Kira [Maierhöfer, Usovitsch, Uwer '17]

based on integration by parts (IBP) relations

$$\int d^D k \frac{\partial}{\partial k^\mu} v^\mu f(k, p_i) = 0$$

new: reduce complexity by construction of algebraic identities from numerical samples ("finite fields") A.v.Manteuffel, R.Schabinger '14, Peraro '16

two-loop **integrand** reduction:

very interesting new developments, but not ready for automation yet (?)

Mastrolia, Ossola '11; Badger, Frellesvig, Zhang '12; Kosower, Larsen '12, Mastrolia, Mirabella, Ossola, Peraro '12; Feng, Huang '12; Papadopoulos et al.'12; Ita '15; Larsen, Zhang '16; Mastrolia, Peraro, Primo '16, Peraro '16; Larsen, Rietkerk '17 …

typical procedure

3. calculation of the master integrals

analytically? may not always be possible

numerically? may not always be accurate/fast enough

4. subtraction of IR divergent real radiation

lots of interesting recent NNLO developments (see later)

5. stable and fast Monte Carlo program

(or if not fast: how to make results available in a flexible format)

NNLO real radiation subtraction methods

- antenna subtraction analytically integrated subtraction terms

  [Gehrmann-DeRidder, Gehrmann, Glover ’05]

- qt “subtraction” slicing, (colourless final states)

  [Catani, Grazzini ’07]

- N-jettiness slicing

  [Gaunt, Stahlhofen, Tackmann, Walsh ’15]

  [Boughezal, Focke, Liu, Petriello ’15]

- sector-improved residue subtraction numerically integrated subtraction terms

  [Czakon, Heymes, Mitov ’10; Czakon, Heymes ’14] [Boughezal et al. ’11]

  - nested subtraction [Caola, Melnikov, Röntsch ’17]

- projection to Born / structure function approach only special kinematics

  [Goa, Li, Zhu ’12] [Brucherseifer, Caola, Melnikov ’14] [Cacciari, Dreyer, Karlberg, Salam, Zanderighi ’15]

- colorful only final state colour so far

  [Del Duca, Somogyi, Trocsanyi ’05]

$\mathbf{2 \to 2}$ differential, classified by IR subtraction method

- antenna subtraction

$H + \mathrm{jet}$	[Chen, Cruz-Martinez, Gehrmann, Glover, Jaquier '14,'16]
$Z + \mathrm{jet}$	[Gehrmann-DeRidder, Gehrmann, Glover, Huss, Morgan '15,'16]
$2\,\mathrm{jets}$	[Currie, Gehrmann-DeRidder, Gehrmann, Glover, Pires '14; Currie, Glover, Pires '16]
$ep \to 2\,\mathrm{jets}$	[Gehrmann, Niehues '16; Currie, Gehrmann, Huss, Niehues, '17]

- qt subtraction (slicing)

WH	[Ferrera, Grazzini, Tramontano '07]
$\gamma\gamma$	[Catani, Cieri, De Florian, Ferrera, Grazzini '11]
$Z\gamma$	[Grazzini, Kallweit, Rathlev, Torre '13]
ZH	[Ferrera, Grazzini, Tramontano '14]
$W\gamma$	[Grazzini, Kallweit, Rathlev '15]
ZZ	[Cascioli, T.Gehrmann, Grazzini, Kallweit, Maierhöfer, Manteuffel, Pozzorini, Tancredi, Weihs '14; Grazzini, Kallweit, Rathlev '15]
WW	[T.Gehrmann, Grazzini, Kallweit, Maierhöfer, Manteuffel, Pozzorini, Rathlev, Tancredi '14; Grazzini, Kallweit, Pozzorini, Rathlev, Wiesemann '16]
WZ	[Grazzini, Kallweit, Rathlev, Wiesemann '16]
$HH(m_t \to \infty)$	[De Florian, Grazzini, Hanga, Kallweit, Lindert, Maierhöfer, Mazzitelli, Rathlev '16]

$\mathbf{2 \to 2}$ classified by IR subtraction method

- **sector-improved residue subtraction**

  $\sigma_{\rm tot} t\bar{t}$ [Czakon, Fiedler, Mitov '13]

  $H + {\rm jet}$ [Boughezal, Caola, Melnikov, Petriello, Schulze '15]

  $t\bar{t}$ [Czakon, Heymes, Mitov '15,'16]

  Drell-Yan (new variant) [Caola, Melnikov, Röntsch '17]

- **N-jettiness (slicing)**

  $H + {\rm jet}$ [Boughezal, Focke, Giele, Liu, Petriello '15]

  HW, HZ [Campbell, Ellis, Williams '16]

  $W + {\rm jet}$ [Boughezal, Focke, Liu, Petriello '15; Boughezal, Liu, Petriello '16]

  $Z + {\rm jet}$ [Boughezal, Campbell, Ellis, Focke, Giele, Liu, Petriello '15; Boughezal, Liu, Petriello '16]

  $\gamma\gamma$ [Campbell, Ellis, Li, Williams '16]

  $\gamma + {\rm jet}$ [Campbell, Ellis, Williams '17]

- **projection to Born** ("structure function approach")

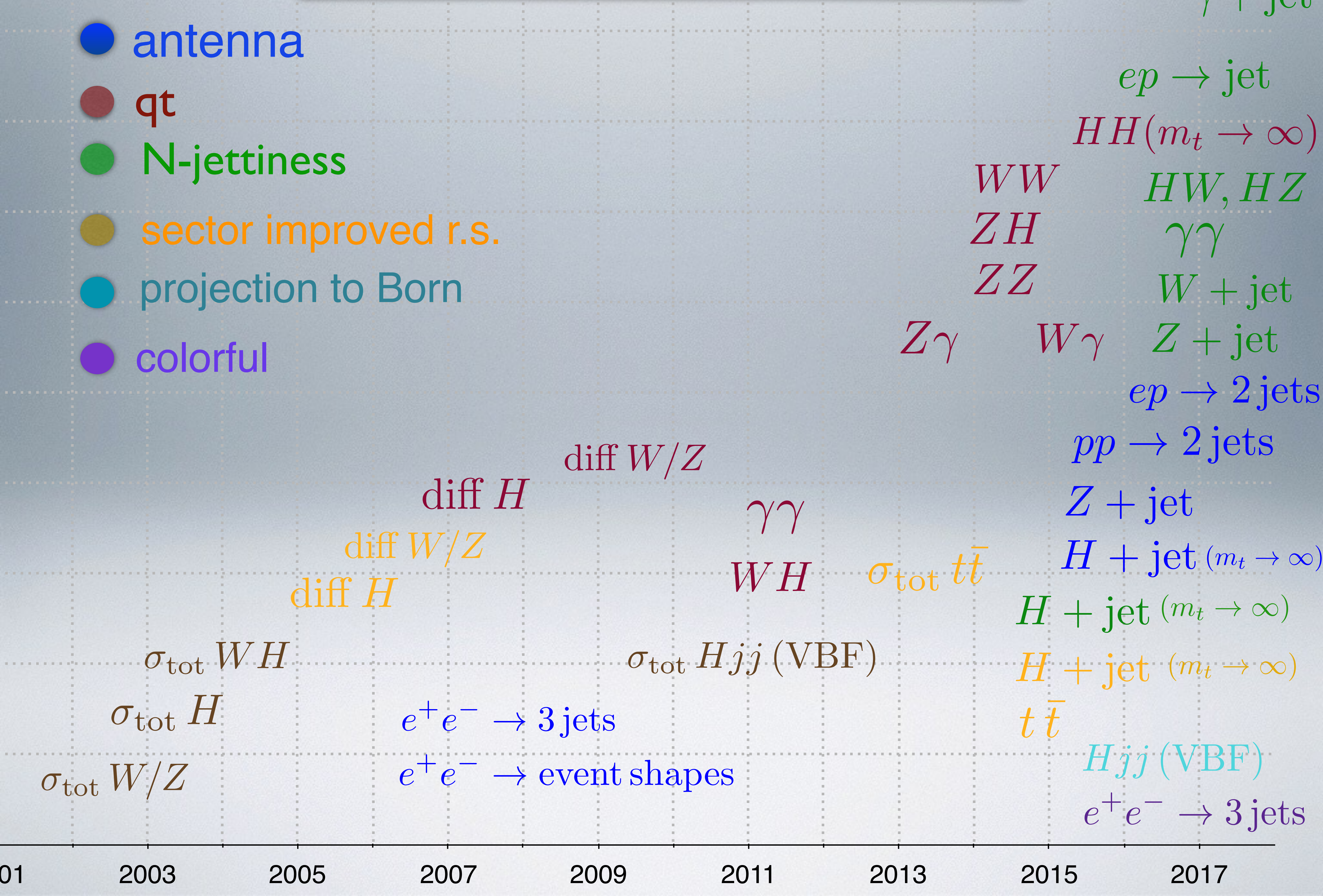
  $Hjj\,({\rm VBF})$ [Cacciari, Dreyer, Karlberg, Salam, Zanderighi '15]

  single top [Brucherseifer, Caola, Melnikov '14] [Berger, Gao, Yuan, Zhu '16]

- **colourful subtraction**

  $e^+e^- \to 3\,{\rm jets}$ [Del Duca, Duhr, Kardos, Somogyi, Szor, Trocsanyi, Tulipant '16]

NNLO



antenna
qt
N-jettiness
sector improved r.s.
projection to Born
colorful
$\gamma + \text{jet}$
$ep \to \text{jet}$
$HH (m_t \to \infty)$
WW
HW, HZ
ZH
$\gamma\gamma$
ZZ
$W + \text{jet}$
$Z\gamma$
$W\gamma$
$Z + \text{jet}$
$ep \to 2\,\text{jets}$
$pp \to 2\,\text{jets}$
diff W/Z
diff H
$\gamma\gamma$
$Z + \text{jet}$
diff W/Z
$H + \text{jet}\ (m_t \to \infty)$
WH
$\sigma_{\text{tot}}\ t\bar{t}$
diff H
$H + \text{jet}\ (m_t \to \infty)$
$\sigma_{\text{tot}}\ WH$
$\sigma_{\text{tot}}\ Hjj\ (\text{VBF})$
$H + \text{jet}\ (m_t \to \infty)$
$\sigma_{\text{tot}}\ H$
$e^+e^- \to 3\,\text{jets}$
$t\,\bar{t}$
$Hjj\ (\text{VBF})$
$\sigma_{\text{tot}}\ W/Z$
$e^+e^- \to \text{event shapes}$
$e^+e^- \to 3\,\text{jets}$
01
2003
2005
2007
2009
2011
2013
2015
2017

multi-loop integrals

processes not (yet) known precisely enough typically involve

- several mass scales (EW corrections, quark masses, BSM particles)
- more than one loop

analytic results for multi-scale two-loop integrals are sparse

numerical evaluation:

- often considered as “poor man’s solution” as long as analytic results are not available
- but easier extendible to many mass scales

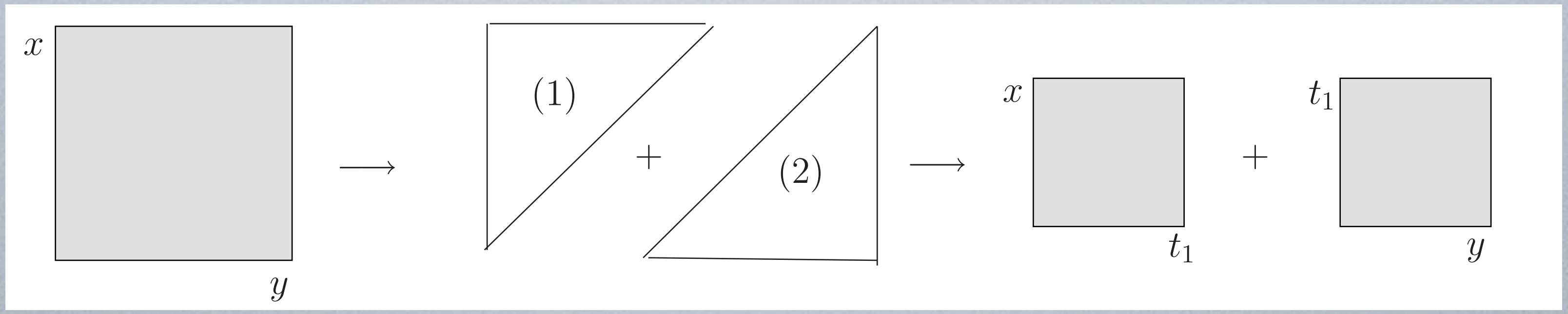
pro's and con's

	analytic	numerical
pole cancellation	exact	with numerical uncertainty
fast evaluation	☑ (mostly)	depends
control of integrable singularities	☑ control of analytic regions	difficult
extension to more scales	difficult	less difficult
automation	difficult	less difficult

some (semi-) numerical methods for loop integrals

- numerical solution of differential equations
  [Caffo, Czyz, Laporta, Remiddi ’98; Czakon, Mitov ‘07 …]
- dispersion relations [Bauberger et al ’94, Bauberger Freitas ‘17 …]
- use Bernstein-Sato-Tkachov theorem [Passarino, Uccirati et al ’01- …]
- numerical evaluation of Mellin-Barnes representations
  [Czakon ’05; … Dubovyk, Freitas, Gluza, Riemann, Usovitsch ‘16]
- numerical extrapolation [De Doncker, Yuasa, Kato, Fujimoto, Kurihara, Ishikawa, Olagbemi, Shimizu]
- direct numerical integration in momentum space
  [Soper ’99; Gong, Soper, Nagy ‘09; Weinzierl, Reuschle et al. ’10- …]
- loop-tree duality (4-dim) [Rodrigo, Buchta, Chachamis, Sborlini, Driencourt-Mangin et al. ’08- …]
- sector decomposition [Hepp ’66; Denner, Roth 96; Binoth, GH ’00; …]

sector decomposition



algorithmic procedure to factorise end-point singularities

$$I = \int_0^1 dx \int_0^1 dy\, x^{-1-\epsilon} (a_1 x + a_2 y)^{-1} [\underbrace{\Theta(x-y)}_{(1)} + \underbrace{\Theta(y-x)}_{(2)}]$$

subst. (1) $y = x z$ (2) $x = y z$ to remap to unit cube

$$I = \int_0^1 dx\, x^{-1-\epsilon} \int_0^1 dz\, (a_1 + a_2 z)^{-1} + \int_0^1 dy\, y^{-1-\epsilon} \int_0^1 dz\, z^{-1-\epsilon} (a_1 z + a_2)^{-1}$$

Feynman parameter representation of multi-loop integrals

$$G = (-1)^{N_\nu} \frac{\Gamma(N_\nu - LD/2)}{\prod_{j=1}^{N} \Gamma(\nu_j)} \int_0^\infty \prod_{j=1}^{N} dx_j \; x_j^{\nu_j - 1} \, \delta(1 - \sum_{l=1}^{N} x_l) \, \frac{\mathcal{U}^{N_\nu - (L+1)D/2}}{\mathcal{F}^{N_\nu - LD/2}}$$

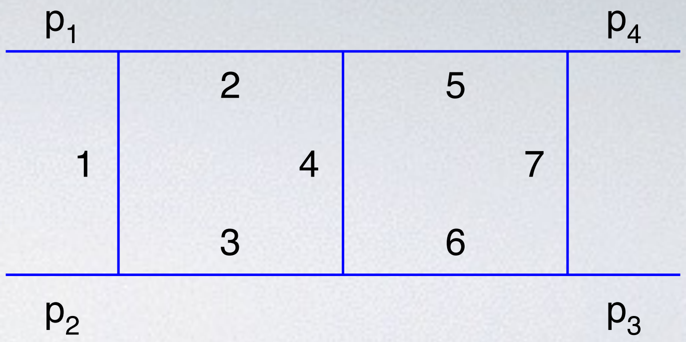
L: number of loops

$N_\nu = \sum_j \nu_j$ sum of propagator powers

$\mathcal{U}(x)$ polynomial of degree L (L loops) (first Symanzik polynomial)

$\mathcal{F}(x, s_{ij})$ polynomial of degree L+1 (second Symanzik polynomial)

example 2-loop planar box with p4 off-shell



$$\mathcal{U} = x_{123}x_{567} + x_4 x_{123567}$$

$$\mathcal{F} = (-s_{12})(x_2 x_3 x_{4567} + x_5 x_6 x_{1234} + x_2 x_4 x_6 + x_3 x_4 x_5) + (-s_{23}) x_1 x_4 x_7 + (-p_4^2) x_7 (x_2 x_4 + x_5 x_{1234}) - i\delta$$

$$x_{ijk\ldots} = x_i + x_j + x_k + \ldots$$

sector decomposition

after iterated decomposition:

in each subsector

$$G_k = \prod_{j=1}^{N-1} \left(\int_0^1 dx_j \, x_j^{a_j - b_j \epsilon} \right) \frac{\mathcal{U}_k(\vec{x})^{\mathrm{expoU}(\epsilon)}}{\mathcal{F}_k(\vec{x}, s_{ij})^{\mathrm{expoF}(\epsilon)}}$$

singularity structure $(a_j < 0)$ factored out

$$\mathcal{U}_k(\vec{x}) = 1 + u(\vec{x})$$

$$\mathcal{F}_k(\vec{x}, s_{ij}) = \mathrm{const.} + f(\vec{x}, s_{ij})$$

produce Laurent series in the regulator

subtract ("plus distribution") then expand in epsilon

$$\int_0^1 dx\, x^{-1-b\epsilon} g(x) = -\frac{1}{b\epsilon}\, g(0) + \int_0^1 dx\, x^{-b\epsilon} \left[\frac{g(x)-g(0)}{x}\right]$$

result: $G = \sum_{j=-2L}^{n} C_j(\vec{x}, s_{ij})\, \epsilon^j$

finite parameter integrals
to be integrated numerically

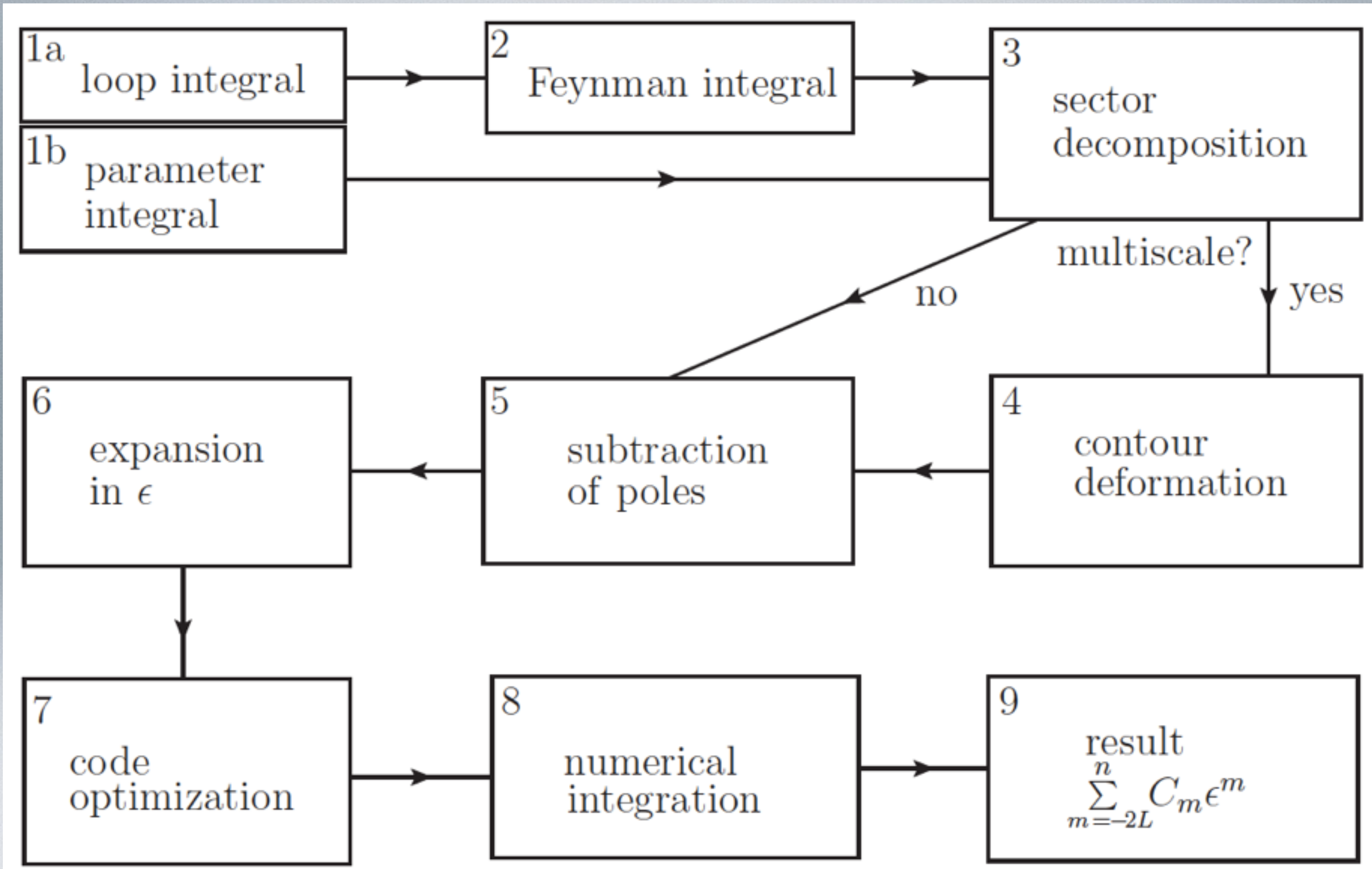
so far only endpoint singularities (UV/IR) have been treated.

if the integral also has singularities of threshold type
$(\mathcal{F}(\vec{x}, s_{ij}) = 0$ with $x_i \neq 0$)

$\Rightarrow$ **contour deformation** (in Feynman parameter space)

[Soper '99; Binoth et al '05; Nagy, Soper '06; Anastasiou, Beerli, Daleo '07; Borowka, Carter, GH '12]

basic workflow

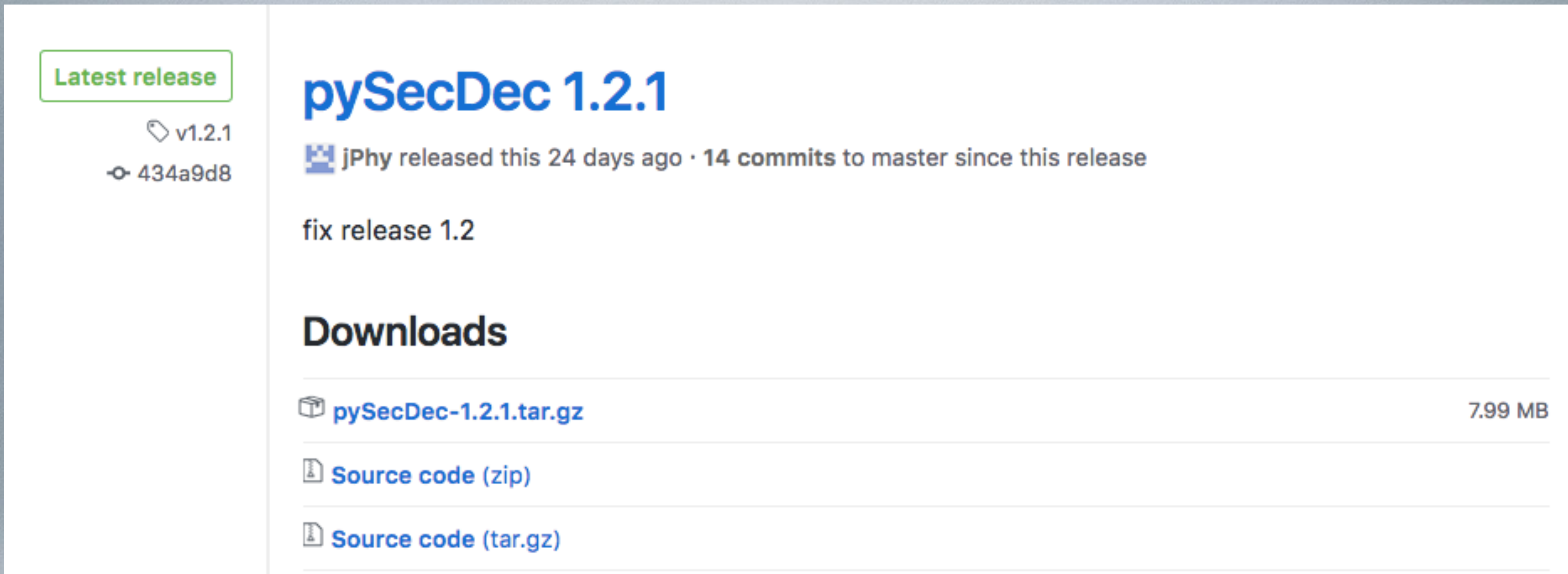


uses FORM [Vermaseren, Ruijl,Ueda]

uses CUBA library [T.Hahn]

The program SecDec

http://secdec.hepforge.org https://github.com/mppmu/secdec/releases



algorithm: T. Binoth, GH '00

version 1.0: J. Carter, GH '10

version 2.0: S.Borowka, J. Carter, GH '12

version 3.0: S.Borowka, GH, S.Jones, M.Kerner, J.Schlenk, T.Zirke '15

pySecDec: **S.Borowka, GH, S.Jahn, S.Jones, M.Kerner, J.Schlenk, T.Zirke '17**

other programs based on sector decomposition:

- **sector_decomposition** (uses Ginac) (only Euclidean region)

  [Bogner, Weinzierl ’07]

  supplemented with **CSectors**

  for construction of integrand in terms of Feynman parameters

  [Gluza, Kajda, Riemann, Yundin ’10]

- FIESTA (versions 1,2,3,4) (uses Mathematica, C++)

  [A.Smirnov, V.Smirnov, Tentyukov, ’08,’09,’13,‘15]

- FORM implementation of

  Fujimoto, Kaneko and Ueda ’08,’10

  (not public)

  uses a decomposition algorithm based on computational geometry, guaranteed to stop

pySecDec new features

- algebraic part in form of python modules
- optimised C++ code generated with FORM [Vermaseren, Ruijl, Ueda]
- automatic creation of a static and a dynamic library
- can act on (almost) any polynomial, not only loop integrals
- any number of regulators is possible
  (e.g. ϵ, α needed in SCET analytic regularisation)
- treatment of numerators much more flexible
  (Lorentz tensors, inverse propagators)
- symmetry finder can detect isomorphisms between sectors
- procedure to detect and remap spurious singularities where contour deformation vanishes
- diagrams can be drawn

pySecDec timing examples

(algebraic part, numerical part)

	pySecDec time (s)	SecDec 3 time (s)	Fiesta 4.1 time (s)
triangle2L	(40.5, 9.6)	(56.9, 28.5)	(211.4, 10.8)
triangle3L	(110.1, 0.5)	(131.6, 1.5)	(48.9, 2.5)
elliptic2L_euclidean	(8.2, 0.2)	(4.2, 0.1)	(4.9, 0.04)
elliptic2L_physical	(21.5, 1.8)	(26.9, 4.5)	(115.3, 4.4)
box2L_invprop	(345.7, 2.8)	(150.4, 6.3)	(21.5, 8.8)

with default settings of pySecDec (epsrel=0.01)

[four-core Intel(R) Core(TM) i7-4770 CPU @ 3.40GHz]

pySecDec

- download:

  https://github.com/mppmu/secdec/releases

- installation:

```
tar -xzvf pySecDec-<version>.tar.gz
cd pySecDec-<version>
make
```

- documentation: https://secdec.readthedocs.io

- included in distribution:

  FORM [Kuipers, Ueda, Ruijl, Vermaseren] catch [https://github.com/philsquared/Catch]

  Cuba [T. Hahn] nauty [McKay, Piperno] GSL [Galassi et al.]

- not included in distribution: (needed for geometric decomposition method)

  normaliz [https://www.normaliz.uni-osnabrueck.de] pySecDec — pySecDec 1.2 d

secdec.readthedocs.io/en/stable/

pySecDec
stable

Search docs

Read the Docs v: stable

pySecDec

pySecDec [PSD17] is a toolbox for the calculation of dimensionally regulated parameter integrals using the sector decomposition approach [BH00]; see also [Hei08], [BHJ+15].

pySecDec usage

$$\int_0^1 dx \int_0^1 dy (x+y)^{-2+\epsilon} = \frac{1}{\epsilon} + (1-\log(2)) + O(\epsilon) \approx \frac{1}{\epsilon} + 0.306853 + O(\epsilon)$$

Step 1: write input files

generate_easy.py

```python
from pySecDec import make_package

make_package(

name = 'easy',
integration_variables = ['x','y'],
regulators = ['eps'],

requested_orders = [0],
polynomials_to_decompose = ['(x+y)^(-2+eps)'],

)
```

integrate_easy.py

```python
from pySecDec.integral_interface \
    import IntegralLibrary

# load c++ library
easy_integral = \
    IntegralLibrary('easy/easy_pylink.so')

# integrate
_, _, result = easy_integral()

# print result
print('Numerical Result:')
print(result)
```

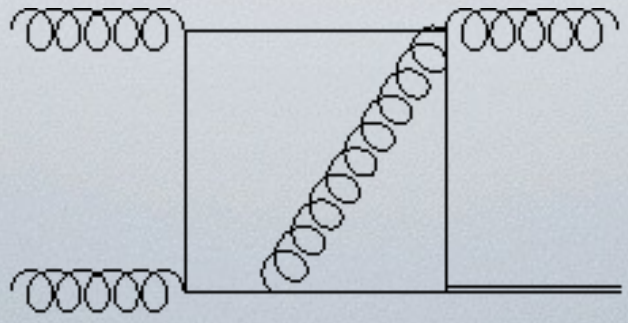
Step 2: run pySecDec

```
$ python generate_easy.py && make -C easy && python integrate_easy.py
<skipped some output>
Numerical Result:
 + (1.00015897181235158e+00 +/- 4.03392522752491021e-03)*eps^-1 + (3.06903035514056399e-01 +/- 2.82319349818329918e-03) + O(eps)
```

Stephan Jahn

pySecDec examples

elliptic integrals



[Bonciani, Del Duca, Frellesvig, Henn, Moriello, V.A. Smirnov '16]

[see also Tancredi, Primo '16]

$$f_{66}^{A} = (-s/m^2)^{\frac{3}{2}} \; I_{110111100}$$

$$I_{a_1\ldots a_9} = \int \frac{d^D k_1}{i\pi^{\frac{D}{2}}} \frac{d^D k_2}{i\pi^{\frac{D}{2}}} \frac{D_8^{-a_8} D_9^{-a_9}}{[D_1]^{a_1}[D_2]^{a_2}[D_3]^{a_3}[D_4]^{a_4}[D_5]^{a_5}[D_6]^{a_6}[D_7]^{a_7}}$$
$$D_1 = k_1^2 - m^2, D_2 = (k_1+p_1+p_2)^2 - m^2, D_3 = k_2^2 - m^2,$$
$$D_4 = (k_2+p_1+p_2)^2 - m^2, D_5 = (k_1+p_1)^2 - m^2, D_6 = (k_1-k_2)^2,$$
$$D_7 = (k_2-p_3)^2 - m^2, D_8 = (k_2+p_1)^2, D_9 = (k_1-p_3)^2.$$

analytic result (Euclidean) at $s=-4/3, t=-16/5, p_4^2=-100/39, m=1$

$$f^{A}_{66,\text{analytic}} = 0.247074199140732131068066$$

pySecDec with $\texttt{epsrel} = 10^{-5}, \texttt{maxeval} = 10^{7}$

$$f_{66}^{A} = 0.2470743601 \pm 6.9692 \times 10^{-6}$$ [time: ~3.5 s]

physical point: $s = 90, t = -2.5, p_4^2 = 1.6, m^2 = 1$

$$\left(\frac{-s}{m^2}\right)^{-\frac{3}{2}} f_{66}^{A} = -0.04428874 + i\,0.01606818 \pm (2.456 + i\,2.662) \times 10^{-5})$$

[time: ~1m40s, epsrel=1e-4]

pySecDec examples

4-photon amplitude (1-loop)

$$\mathcal{M}^{++--} = -8\left\{1 + \frac{t^2+u^2}{s} I_4^{D+2}(t,u) + \frac{t-u}{s}\left(I_2^D(u) - I_2^D(t)\right)\right\}$$

examples/4photon1L_amplitude contains

Makefile amp.cpp yyyy_box6Dim.py yyyy_bubble.py

```
#! /usr/bin/env python
import pySecDec as psd
from pySecDec.loop_integral import loop_package

# 4-photon amplitude M++-- :
#Amp4y_ppmm=-8*( 1 + (t**2+u**2)/s *Box6dim(t,u) +  (t-u)/s*(  BubbleD(u)-BubbleD(t) ) );

li = psd.loop_integral.LoopIntegralFromGraph(
internal_lines = [['0',[1,2]],[0,[2,3]],[0,[3,4]],[0,[4,1]]],
external_lines = [['p1',1],['p2',2],['p3',3],['p4',4]],

replacement_rules = [
                        ('p1*p1', 0),
                        ('p2*p2', 0),
                        ('p3*p3', 0),
                        ('p4*p4', 0),
                        ('p3*p2', 'u/2'),
                        ('p1*p2', 't/2'),
                        ('p1*p4', 'u/2'),
                        ('p1*p3', '-u/2-t/2'),
                        ('p2*p4', '-u/2-t/2'),
                        ('p3*p4', 't/2')
                    ],
dimensionality= '6-2*eps'
)

Mandelstam_symbols = ['t','u']
mass_symbols = []

loop_package(

name = 'yyyy_box6Dim',

loop_integral = li,

real_parameters = Mandelstam_symbols + mass_symbols,

# the highest order of the final epsilon expansion
# (change this value to whatever you think is appropriate)
requested_order = 0,

# the optimization level to use in FORM (can be 0, 1, 2, 3)
form_optimization_level = 2,

# the WorkSpace parameter for FORM
form_work_space = '100M',

# the method to be used for the sector decomposition
# valid values are ``iterative`` and ``geometric``
decomposition_method = 'iterative',

)
```

pySecDec examples

4-photon amplitude (1-loop)

amp.cpp:

generated automatically

```
#include "yyyy_bubble/yyyy_bubble.hpp"
#include "yyyy_box6Dim/yyyy_box6Dim.hpp"
```

… definition of bubble, box6Dim …

```
/*
 * numerical amplitude using pySecDec Master Integrals
 */
secdecutil::Series<secdecutil::UncorrelatedDeviation<std::complex<double>>> yyyy_numerical(double s, double t, double u)
{
    return -8.*( 1. + (t*t + u*u)/s * box6Dim(t,u) +  (t-u)/s*( bubble(u)-bubble(t) ) );
}
```

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“real life” application: Higgs boson pair production in gluon fusion

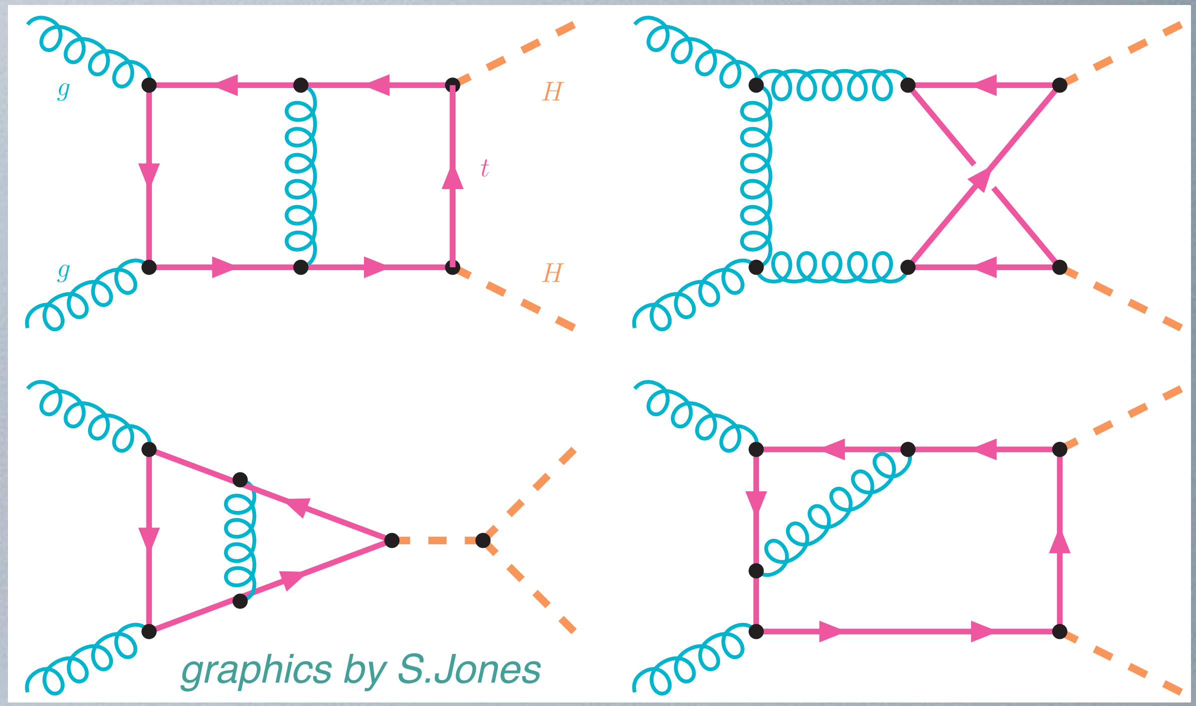
NLO calculation with full top quark mass dependence

Borowka, Greiner, GH, Jones, Kerner, Schlenk, Schubert, Zirke ‘16

4 independent scales s12, s23, mH, mt

- analytic results for master integrals mostly unknown

- all integrals calculated **numerically** with **SecDec**



graphics by S.Jones

- total number of integrals:

  before reduction: ~10000, after reduction ~330,
  after sector decomposition 11244 (3086 non-planar)

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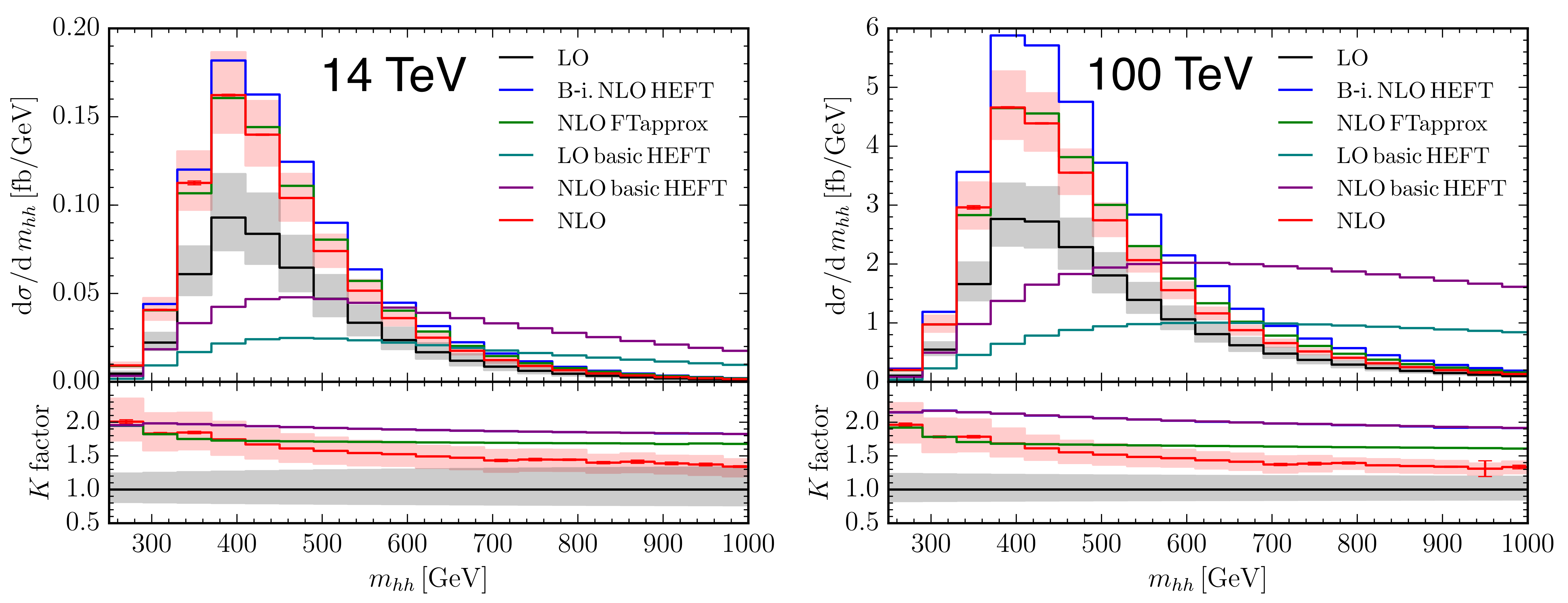
Higgs boson pair invariant mass



for large invariant masses:

Born-improved NLO HEFT overestimates by about 50%, FTapprox by about 40%
(at 14 TeV, worse at 100 TeV)

top quark loops resolved → HEFT has wrong scaling behaviour at high energies

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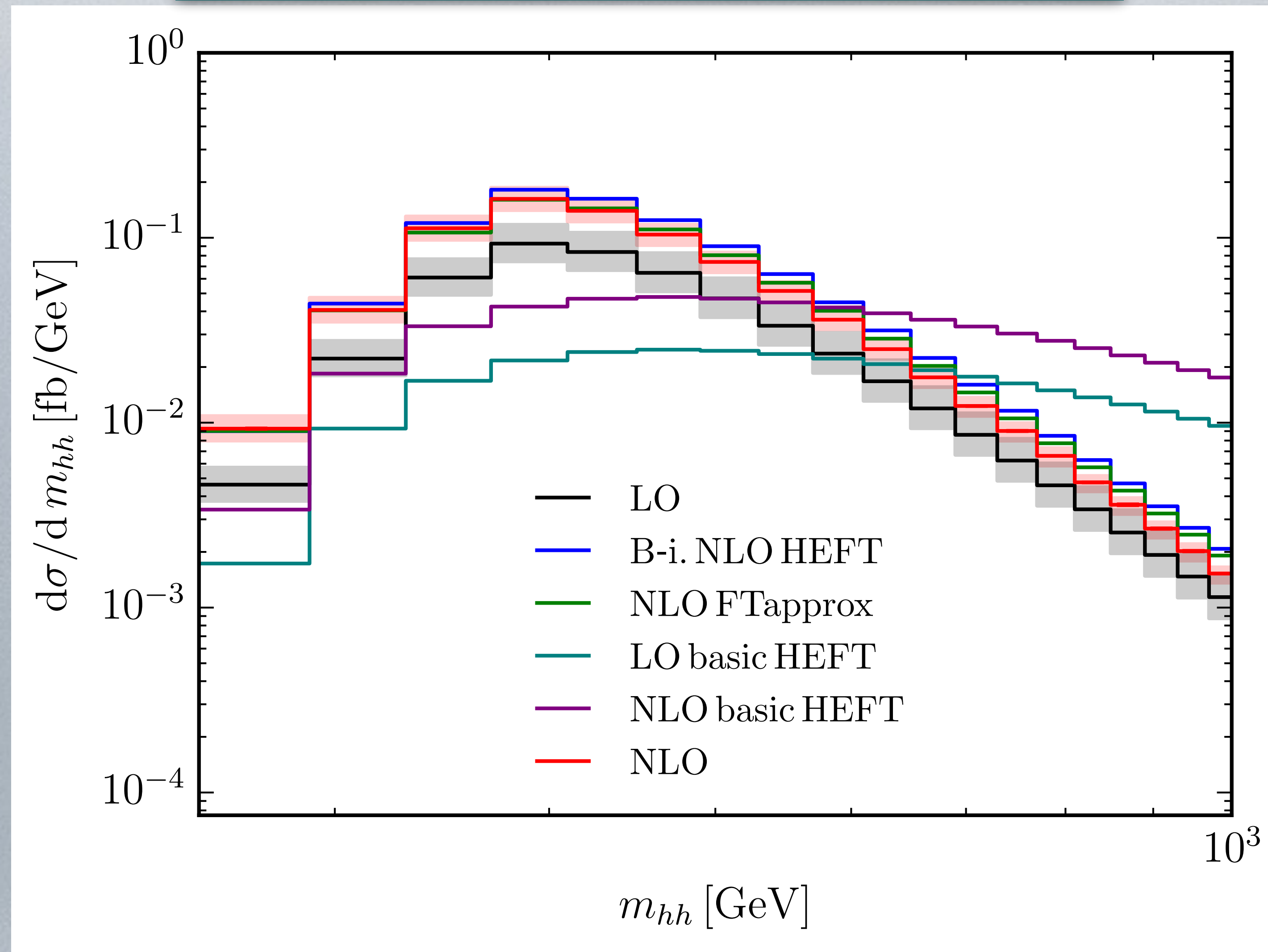
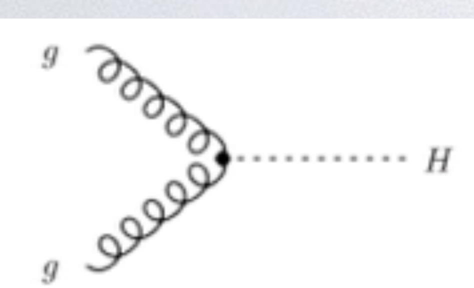
scaling behaviour



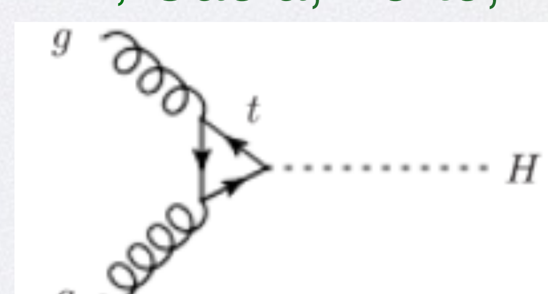
$$\frac{d\hat{\sigma}}{dm_{hh}} \sim m_{hh}^{-3}$$ i.e. partonic cross section scales as $\hat{s}^{-1}$

HEFT approximation: $$\frac{d\hat{\sigma}}{dm_{hh}} \sim m_{hh}$$ i.e. $\hat{\sigma} \sim \hat{s}$

similar for H+jets: Greiner, Höche, Luisoni, Schönherr, Winter '16
see also Marzani et al. '08; Caola, Forte, Marzani, Muselli, Vita '16



$d\sigma/dp_{T,h}^2 \rightarrow (p_{T,h}^2)^{-1}$



$d\sigma/dp_{T,h}^2 \rightarrow (p_{T,h}^2)^{-2}$

combination with parton showers

GH, S.Jones, M.Kerner, G.Luisoni, E.Vryonidou '17

- avoid evaluation of two-loop amplitude for each phase space point
- two-loop amplitude depends only on $\hat{s}, \hat{t}$ $(m_t, m_H \text{ fixed})$

→ construct 2-dim grid

- variable transformation to achieve more uniform distribution

$$x = f(\beta(\hat{s})), \quad c_\theta = |\cos\theta| = \left| \frac{\hat{s} + 2\hat{t} - 2m_H^2}{\hat{s}\beta(\hat{s})} \right| \qquad \beta(\hat{s}) = \sqrt{1 - 4m_H^2/\hat{s}}$$

combination with POWHEG and MadGraph5_aMC@NLO

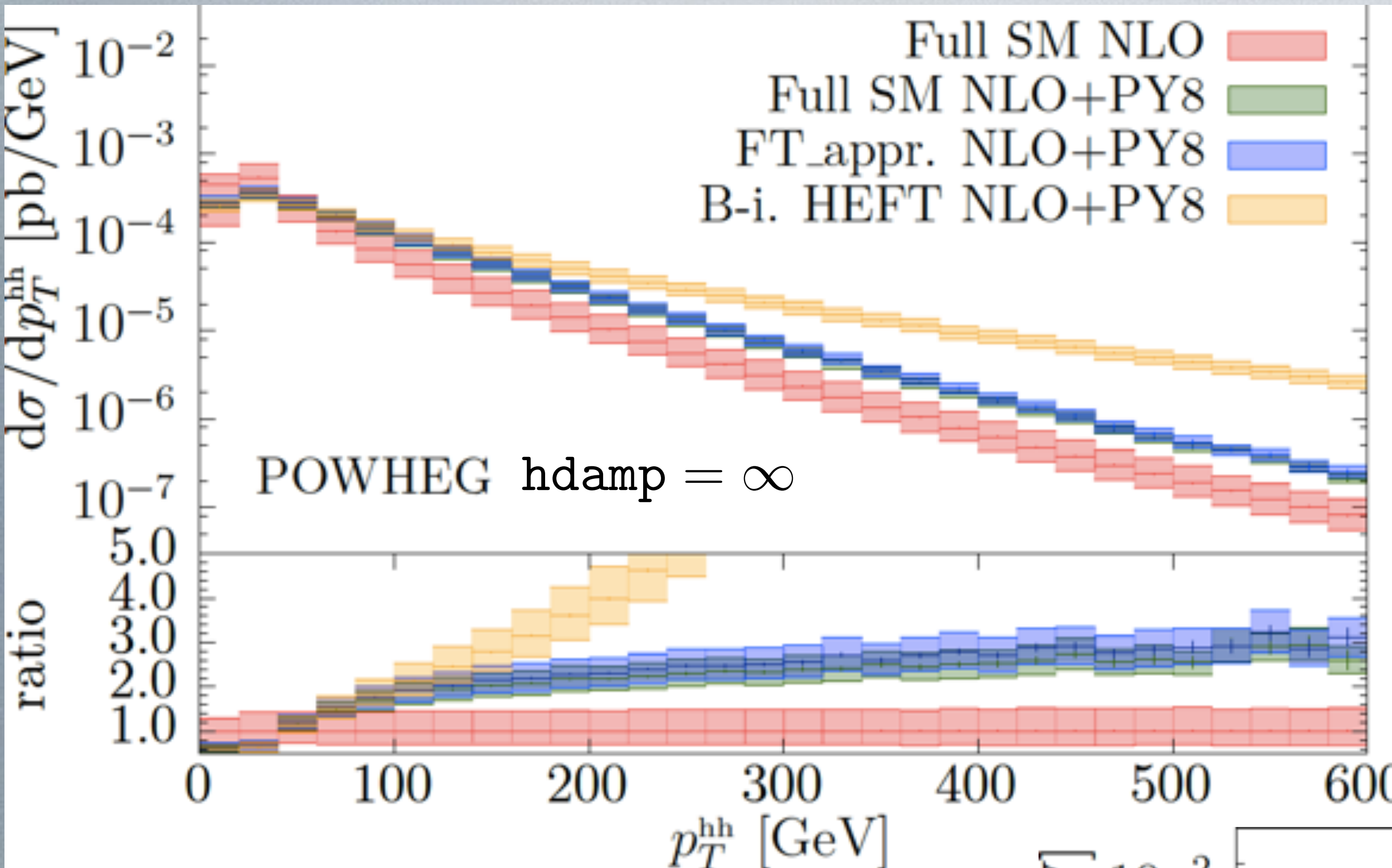
POWHEG-BOX-V2: User-Process-V2/ggHH

New!

and Sherpa

S. Jones, S. Kuttimalai '17

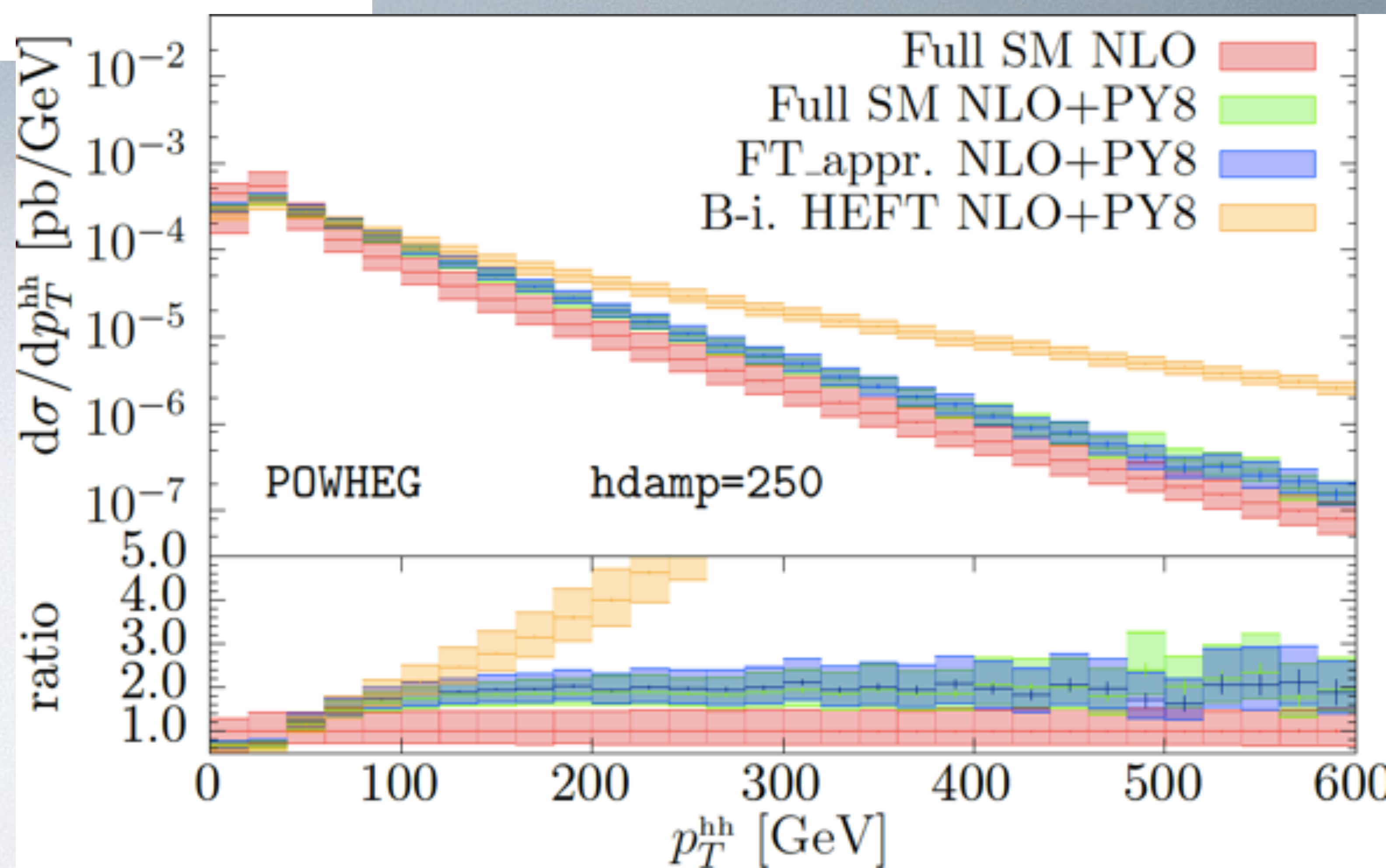
dependence on shower parameters



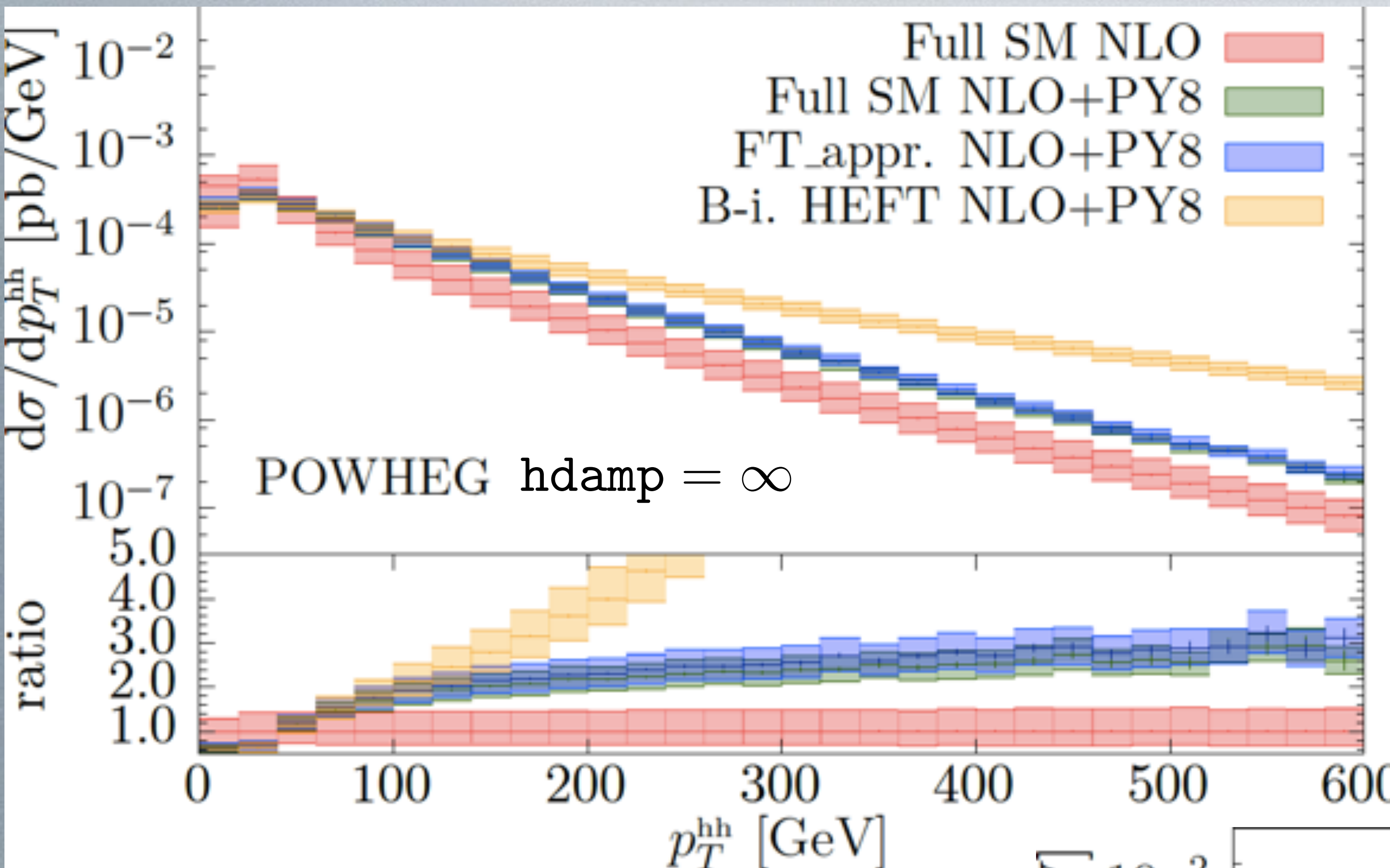
hdamp=h limits amount of exponentiated hard radiation

$$R_{\text{sing}} = R \times F \,,$$
$$R_{\text{reg}} = R \times (1 - F)$$

$$F = \frac{h^2}{(p_T^{\text{hh}})^2 + h^2}$$



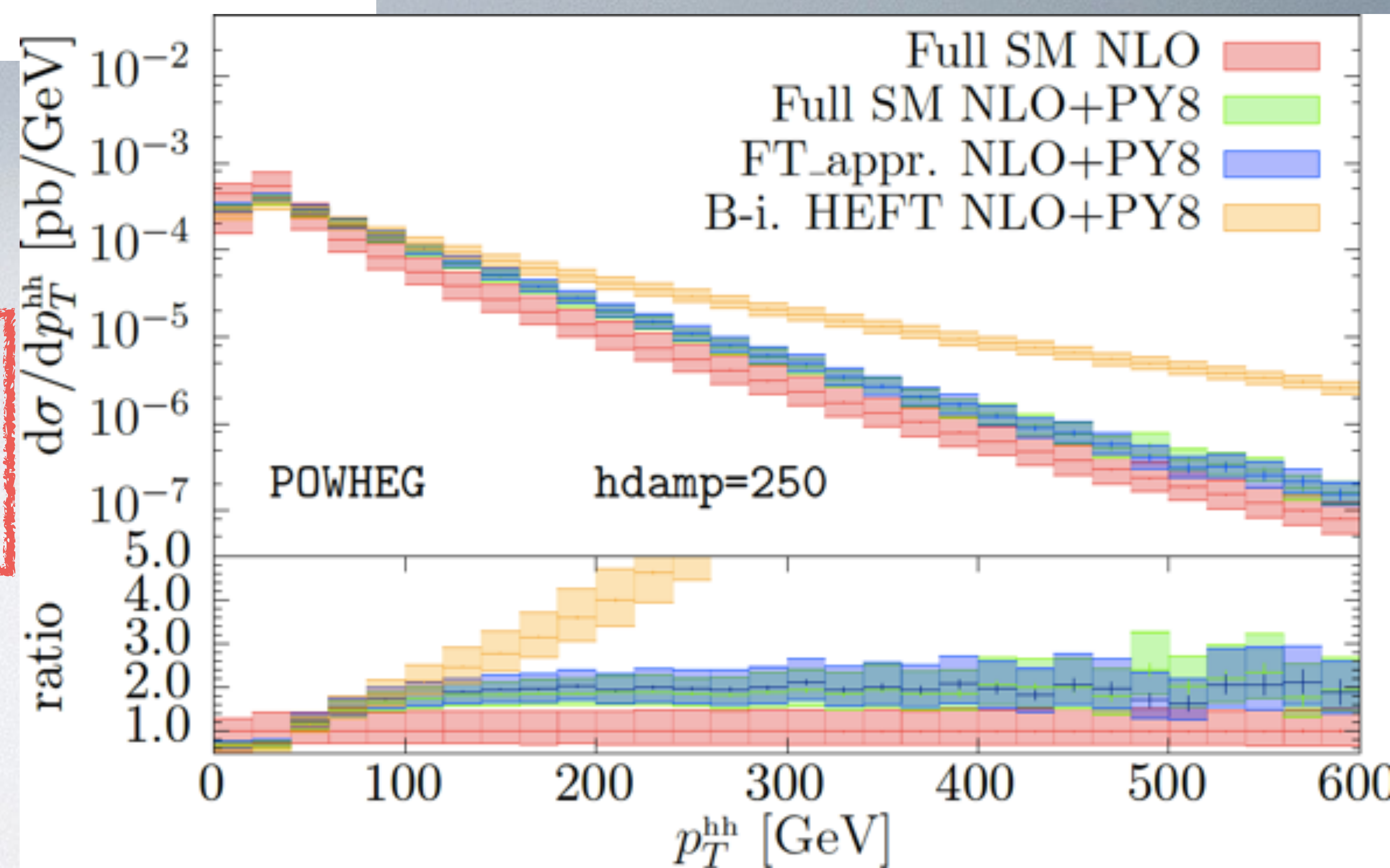
dependence on shower parameters



hdamp=h limits amount of exponentiated hard radiation

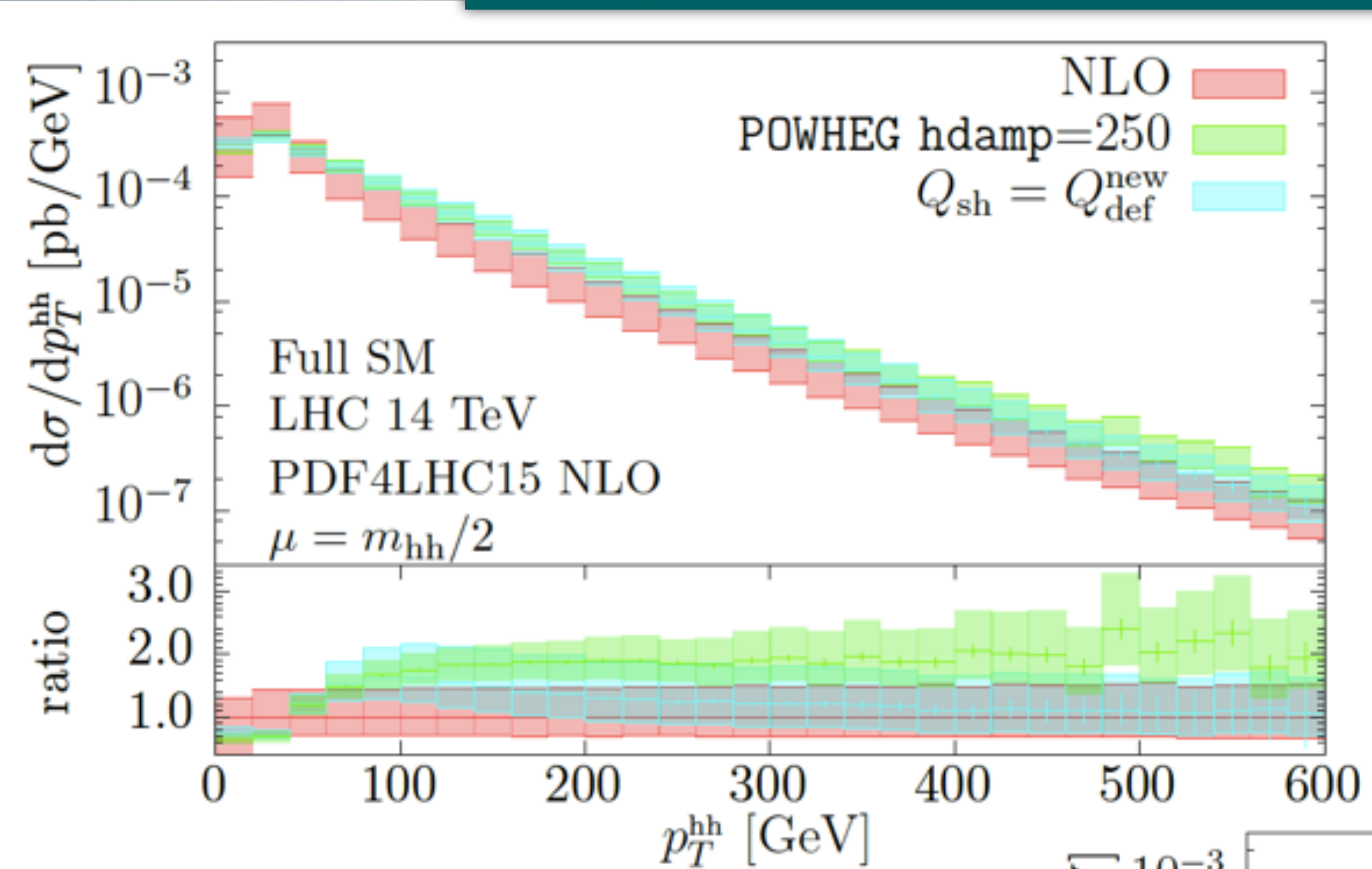
$$R_{\rm sing} = R \times F\,,$$
$$R_{\rm reg} = R \times (1 - F)$$

$$F = \frac{h^2}{(p_T^{\rm hh})^2 + h^2}$$

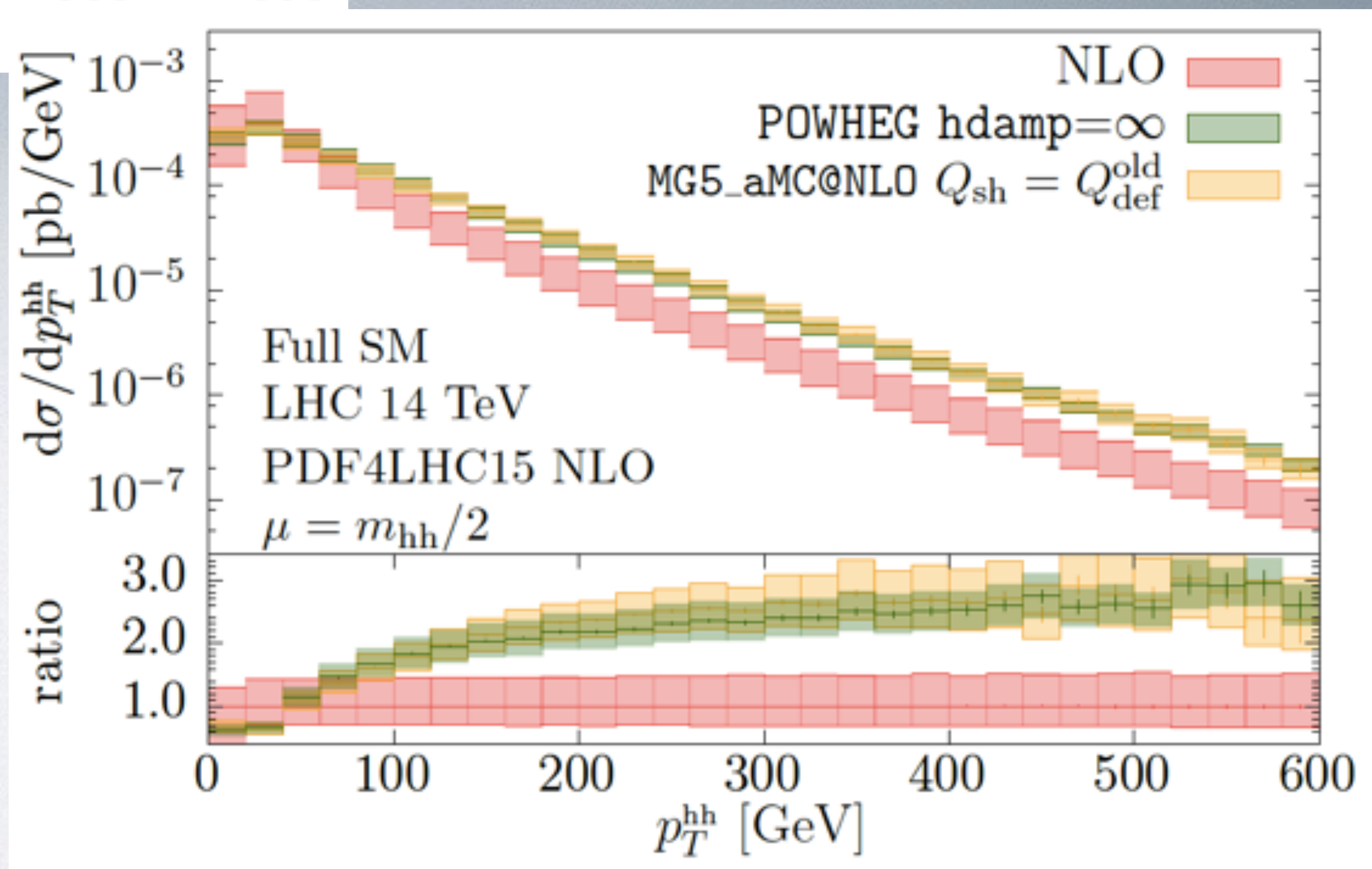


shower effects large but order(s) of magnitude smaller than difference to Born-improved HEFT

compare POWHEG and MG5_aMC@NLO



new default shower starting scale matches onto NLO fixed order at large `pThh`



combination with Sherpa

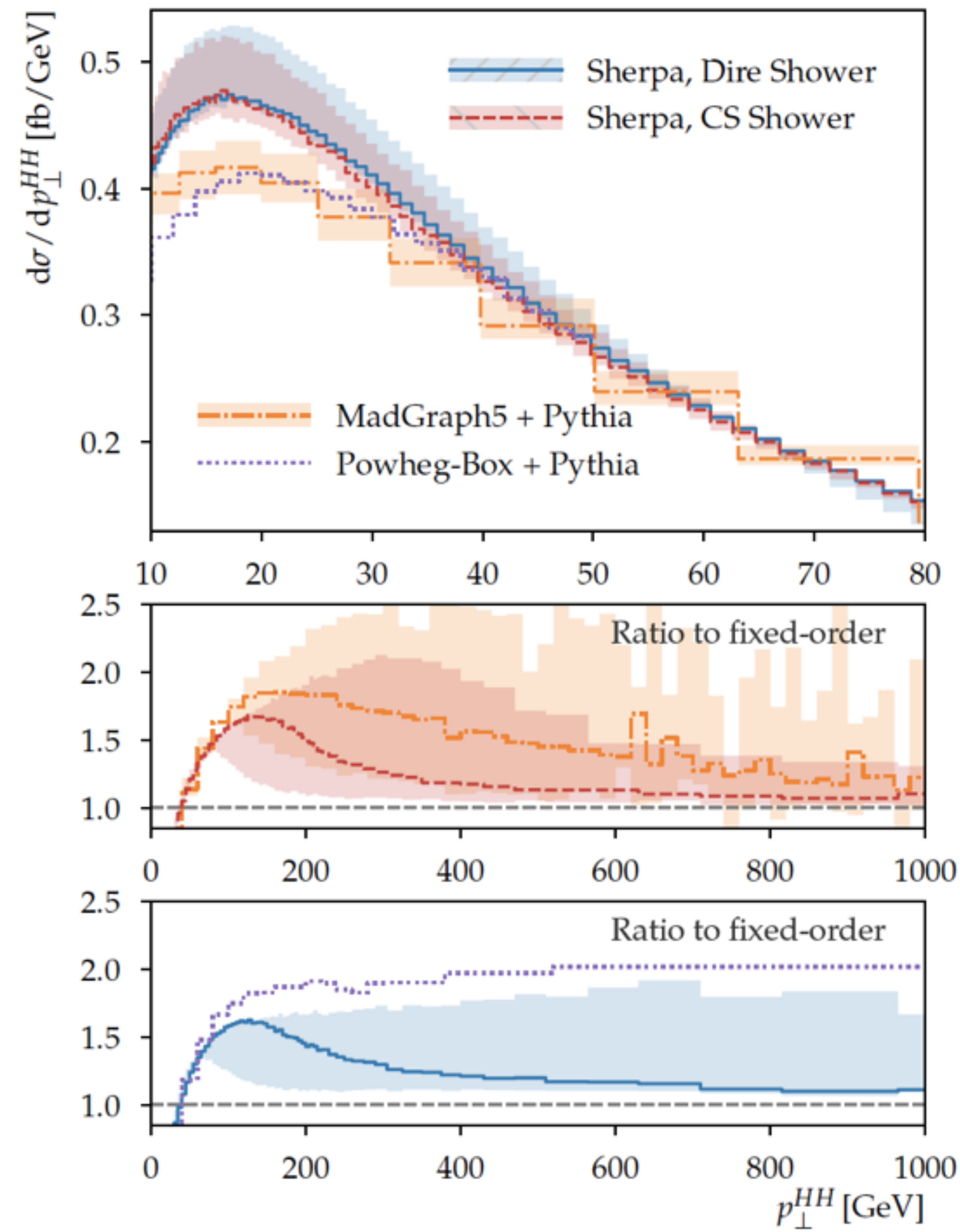
S. Jones,
S. Kuttimalai '17

- differences in peak region due to different matching algorithms

- large pThh region:

MG5_aMC@NLO results within (large) uncertainty bands

Powheg with hdamp=250 not within μ_{PS} variation band; vary hdamp to obtain Powheg uncertainty band



Summary

- The LHC entered the phase of precision measurements, requiring precision calculations

- **numerical** methods for 2-loop integrals can prove useful in cases where many mass scales are present

- new SecDec version **pySecDec:**
  - interface to reduction programs and SECT integrals facilitated (inverse propagators, multiple regulators, …)
  - flexible usage (library of master integrals)

- applications: numerical evaluation of multi-scale two-loop integrals (e.g. elliptic integrals, HH 2-loop amplitude)

- method extendible to other processes/more scales

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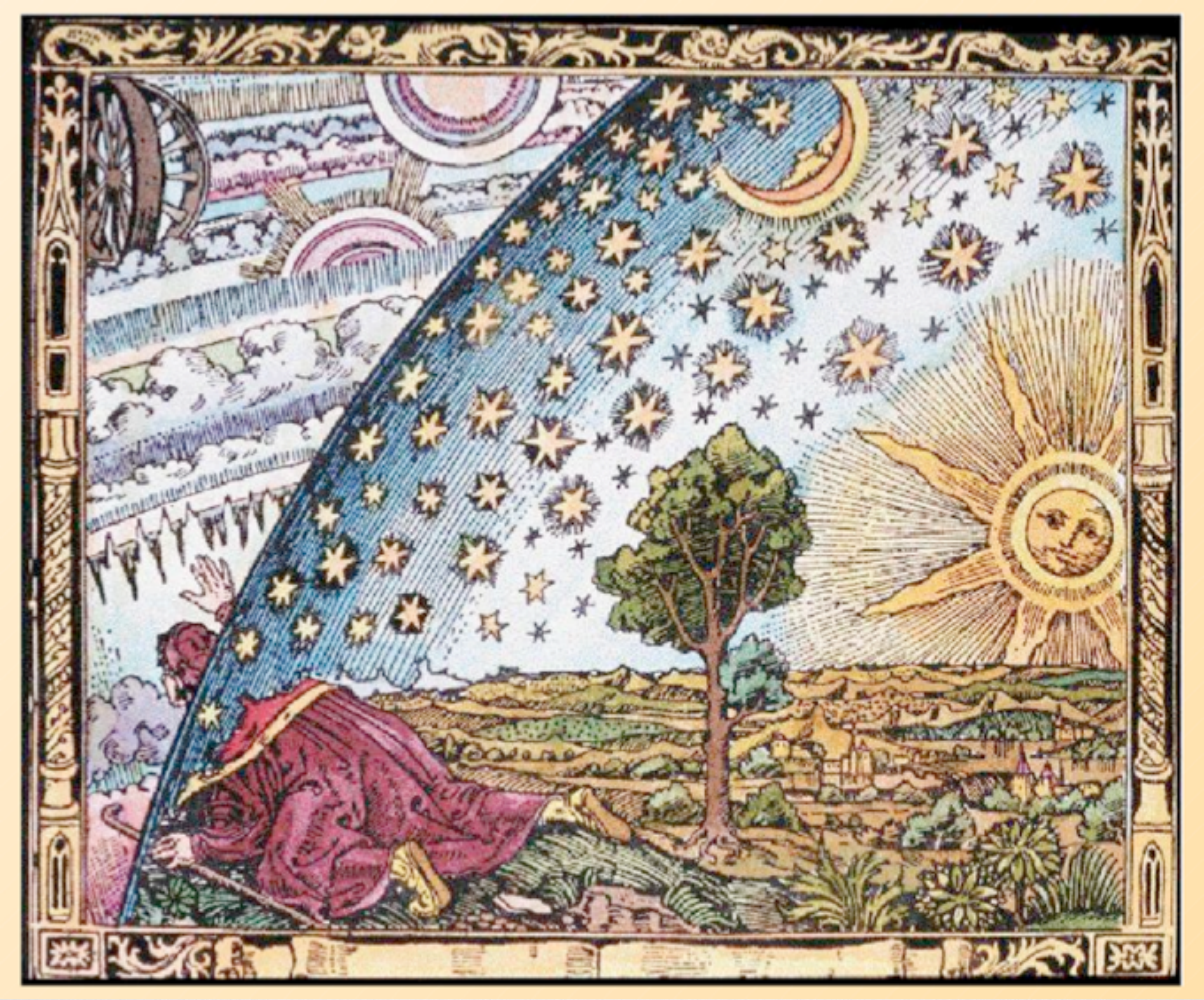

BACKUP SLIDES

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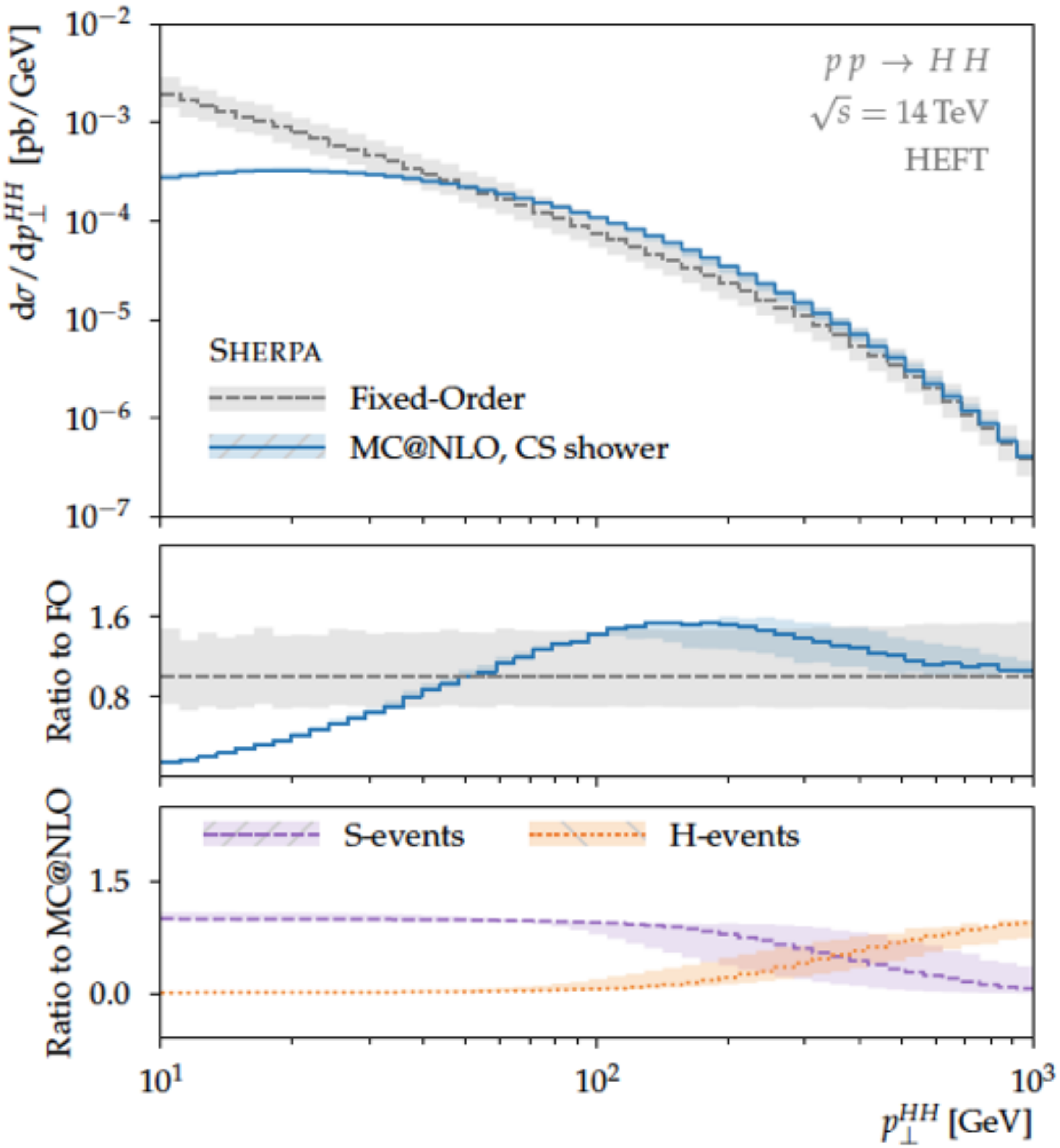
combination with Sherpa



HEFT approximation

- μ_{PS} cancellation between S- and H-events
- PS uncertainties reduced
- Recover fixed-order in tail

$$(\bar{B} - B) \times P \ll R$$

$$\bar{B}(\phi_B)\Delta(\mu_{PS}^2, t)P(\phi_1)\Theta(\mu_{PS}^2 - t)\,d\phi_B\,d\phi_1$$

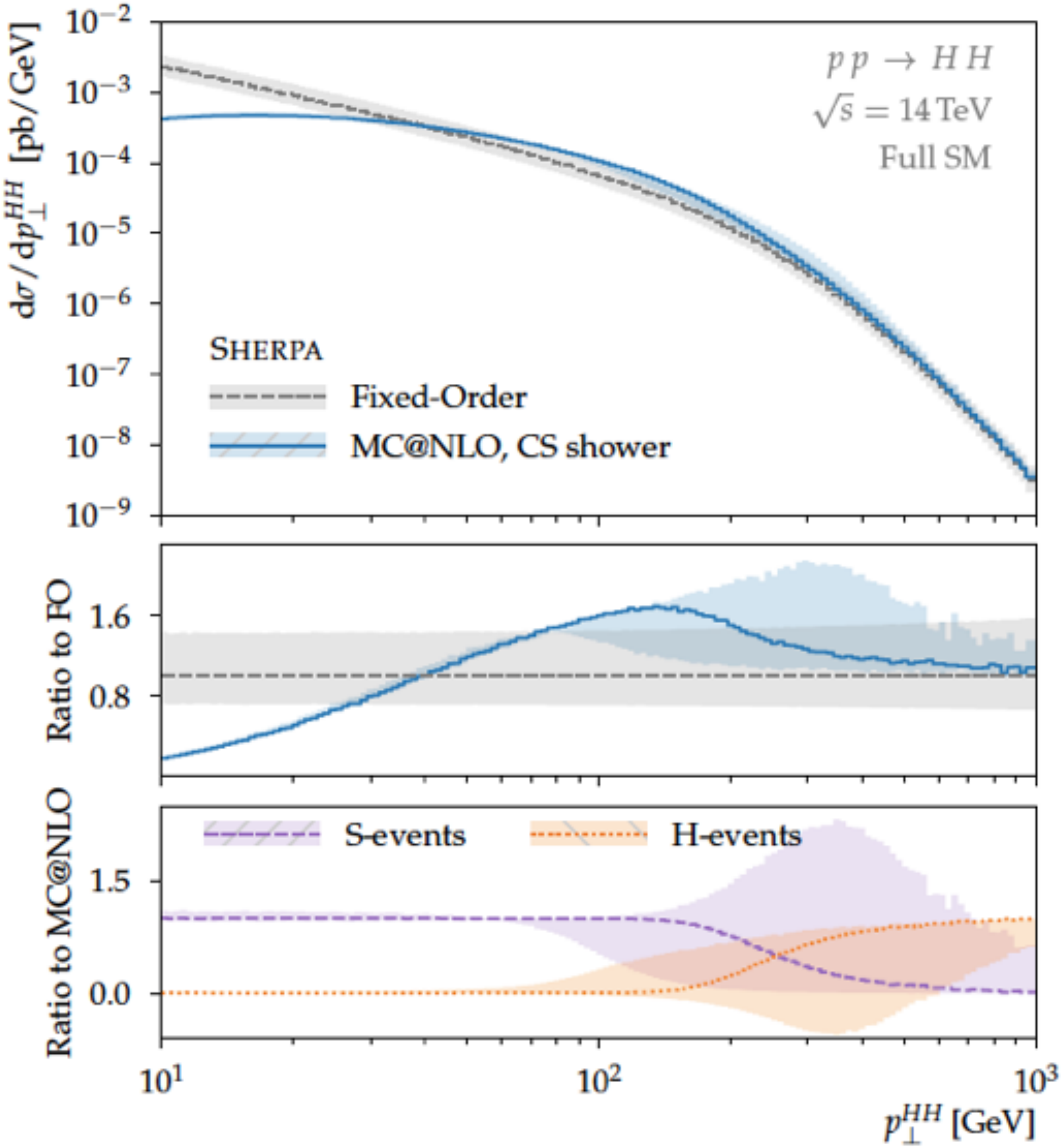
S-events

$$\left[R(\phi_R) - B(\phi_B)P(\phi_1)\Theta(\mu_{PS}^2 - t)\right] d\phi_R$$

H-events

S. Kuttimalai, Higgs Couplings 2017

combination with Sherpa



Full SM

- much larger uncertainties
- PS effects extend into tail
- large $\mu_{PS} \Rightarrow$ don't recover fixed order

$$(\bar{B} - B) \times P \sim R$$

$$\bar{B}(\phi_B)\Delta(\mu_{PS}^2, t)P(\phi_1)\Theta(\mu_{PS}^2 - t)\, d\phi_B\, d\phi_1$$

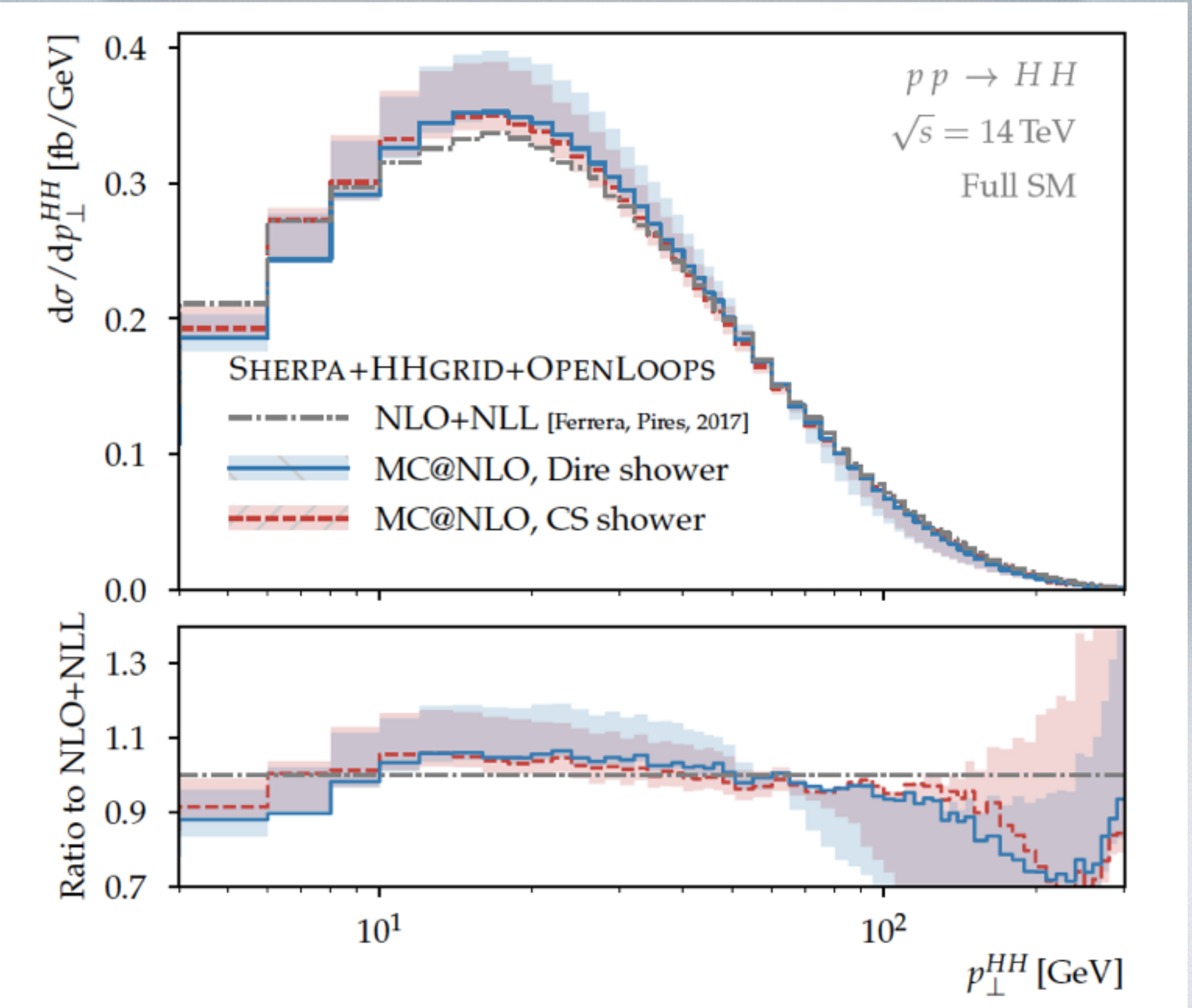
S-events

$$\left[R(\phi_R) - B(\phi_B)P(\phi_1)\Theta(\mu_{PS}^2 - t)\right] d\phi_R$$

H-events

S. Kuttimalai, Higgs Couplings 2017

comparison to analytic resummation



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top mass effects

total cross sections at 14 TeV $\mu_0 = m_{HH}/2$

	$\sigma_{\mathrm{LO}}[\mathrm{fb}]$	$\sigma_{\mathrm{NLO}}[\mathrm{fb}]$	$\sigma_{\mathrm{NNLO}}[\mathrm{fb}]$
HEFT	$17.07^{+30.9\%}_{-22.2\%}$	$31.93^{+17.6\%}_{-15.2\%}$	$37.52^{+5.2\%}_{-7.6\%}$
B-i. HEFT	$19.85^{+27.6\%}_{-20.5\%}$	$38.32^{+18.1\%}_{-14.9\%}$	
$\mathrm{FT}_{\mathrm{approx}}$	$19.85^{+27.6\%}_{-20.5\%}$	$34.26^{+14.7\%}_{-13.2\%}$	
full m_t dep.	$19.85^{+27.6\%}_{-20.5\%}$	$32.91^{+13.6\%}_{-12.6\%}$	

PDF4LHC15_nlo_30_pdfas

m_H=125 GeV, m_t=173 GeV

uncertainties: $\mu_{R,F} \in [\mu_0/2, 2\,\mu_0]$ (7-point variation)

HXSWG: $\sigma'_{NNLL} = \sigma_{NNLL} + \delta_t\, \sigma^{\mathrm{HEFT}}_{NLO} = 39.64^{+4.4\%}_{-6.0\%}$

NLO-improved NNLO HEFT

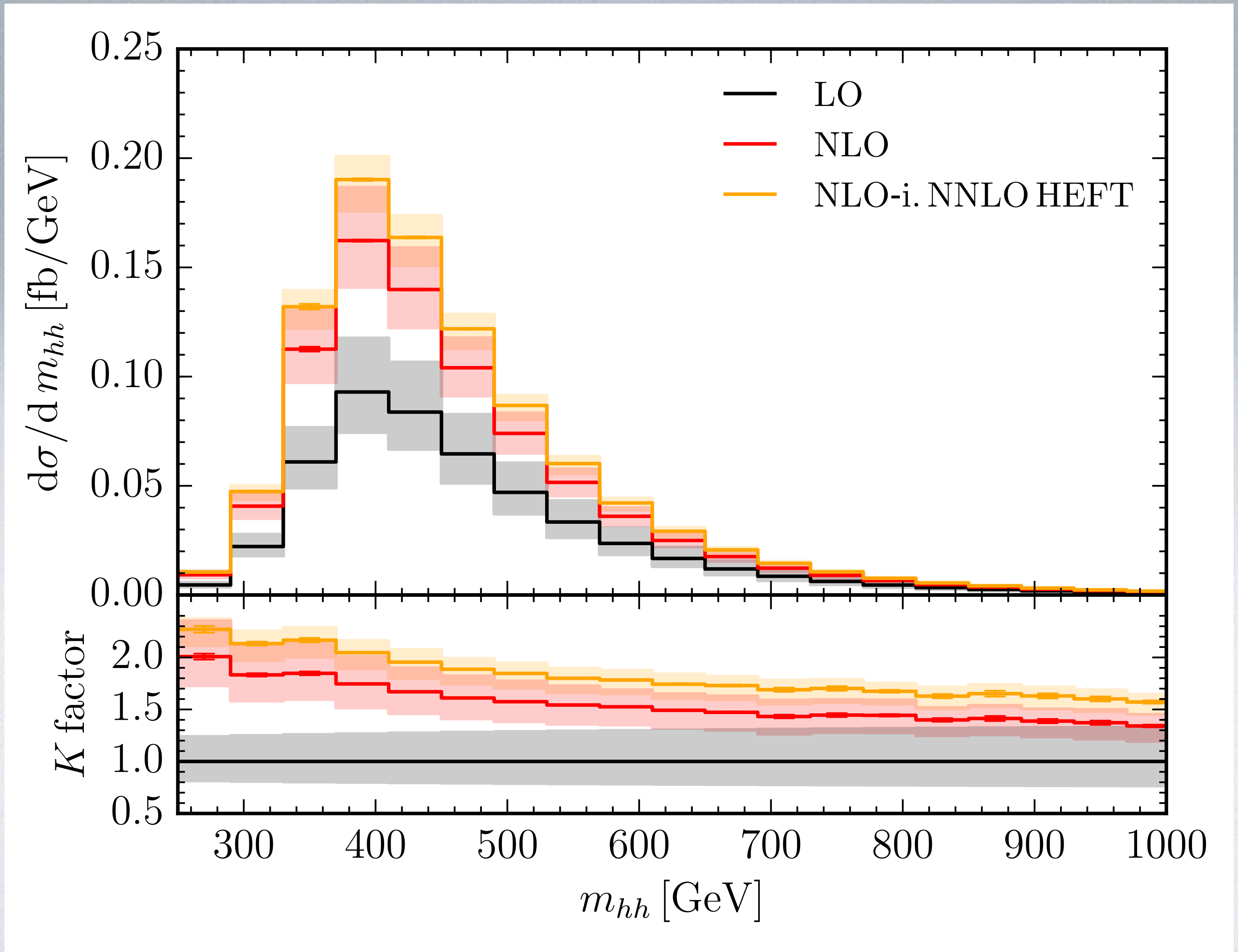
NNLO HEFT:

De Florian, Grazzini, Hanga, Kallweit, Lindert, Maierhöfer, Mazzitelli, Rathlev 1606.09519

"NLO-improved NNLO HEFT": [Borowka, Greiner, GH, Jones, Kerner, Schlenk, Zirke 1608.04798]

$$\frac{d\sigma^{\mathrm{NLO-i.NNLO\,HEFT}}}{dm_{hh}} = \frac{d\sigma_{\mathrm{NLO}}}{dm_{hh}} \times \frac{d\sigma^{\mathrm{HEFT}}_{\mathrm{NNLO}}/dm_{hh}}{d\sigma^{\mathrm{HEFT}}_{\mathrm{NLO}}/dm_{hh}}$$

bin-by-bin rescaling at observable level by NNLO HEFT K-factor



would lead to

$\sigma' = 38.56\ fb$

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