PHY127 FS2023

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Lecture 12

May 26th, 2023

$$\Delta E = h\nu = 2\mu_z B_z$$

This frequency of precession (or resonance frequency) is written as:

$$\omega_{Larmor} = 2\pi f_{Larmor} = \gamma \cdot B_z$$

γ: gyromagnetic ratio $= \dfrac{qg}{2m}$

m: mass of resonating nucleus

q: charge of nucleus

g: strength factor

$$\gamma = \frac{(2\pi)(42.58\ MHz)}{T}$$

T: magnetic field

For ^1H in a 1.5T magnetic field

$$f_{Larmor} = \frac{\gamma \cdot B_z}{2\pi} = \left(\frac{42.58\ MHz}{T}\right)\left(\frac{\cancel{2\pi}}{\cancel{2\pi}}\right)(1.5\,T)$$

$$= 63.87\ MHz$$

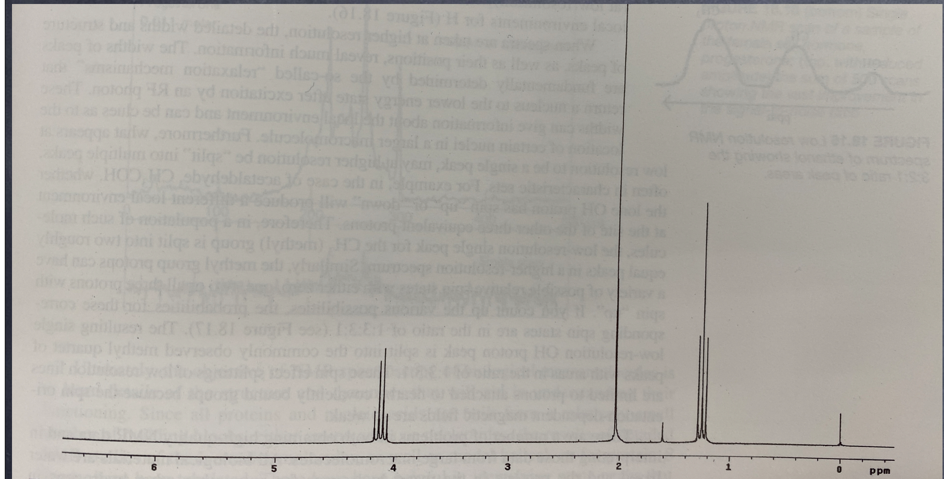


FIGURE 18.15 *The proton NMR spectrum of ethyl acetate with eight protons in three different environments. Zero ppm is defined by the peak due to TMS and the other small peak is due to a small amount of contaminating water.*

← position of peaks measured as a chemical shift, δ.

ppm

$$\delta = \frac{f_{sample} - f_{ref}}{f_{ext}} \cdot 10^6$$

δ (has units in ppm)

$f_{ext} = \nu$ where $\Delta E = h\nu$
Frequency of RF radiation

f_{sample}: measured frequency spectrum of the sample

f_{ref}: reference sample, ~~typcl~~ typically used TMS "tetra methylsilane"

TMS : $(CH_3)_4Si$ chemically inert and has a strong signal of 12 protons.

δ features : number of peaks
position of peaks
size (or area) of peaks

area under peaks → strength of the peaks is proportional to the relative numbers of hydrogen in the local chemical group.

Ethanol : $\underbrace{CH_3}\,\underbrace{CH_2}\,\underbrace{OH}$

we expect to see 3 peaks, areas should be in the ratio
3 : 2 : 1

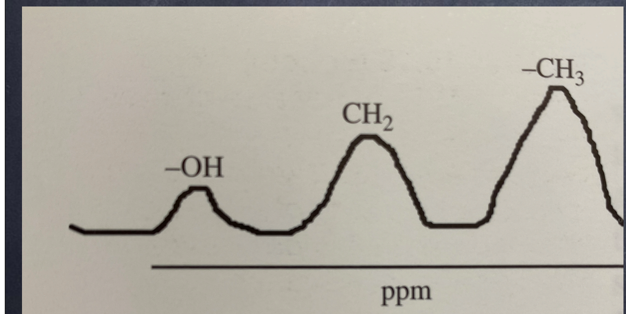
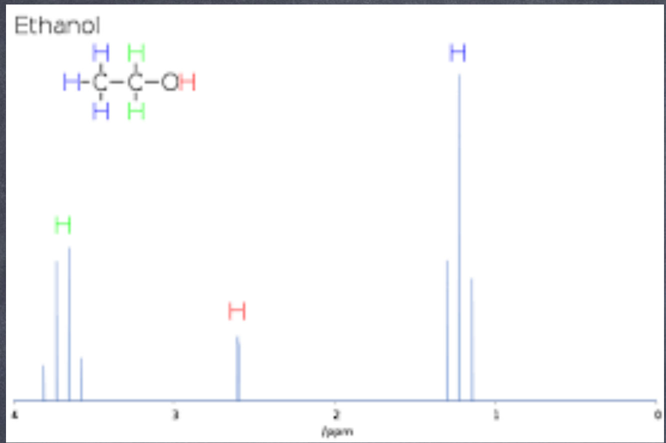


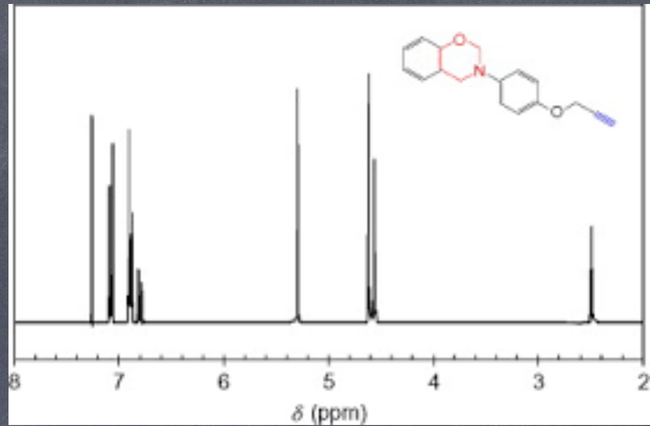
FIGURE 18.16 *Low resolution NMR spectrum of ethanol showing the 3:2:1 ratio of peak areas.*

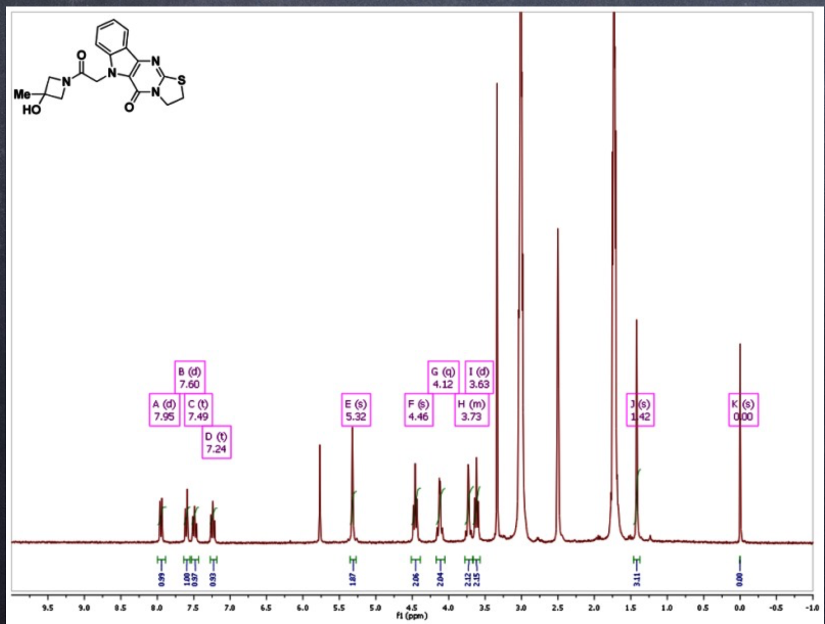
← Zoom in on NMR

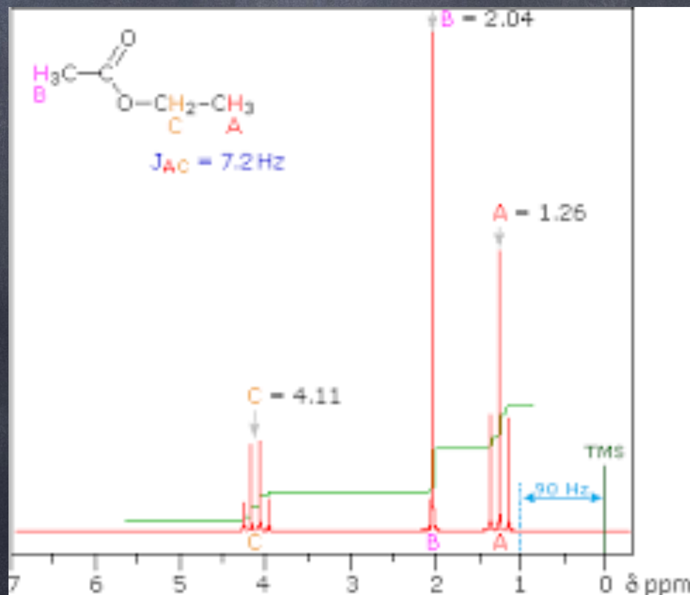
widths of the peaks provide structural information on location of nuclei in the larger molecule.

Examples of NMR spectra (x-axis is δ (ppm))









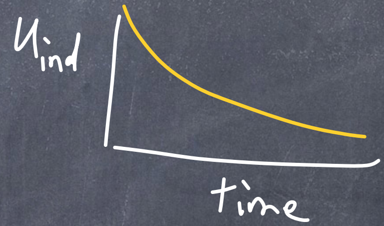
when we put a pulse of RF radiation into our sample, the magnetization of the sample changes, but ~~then~~ then the magnetization relaxes to equilibrium.

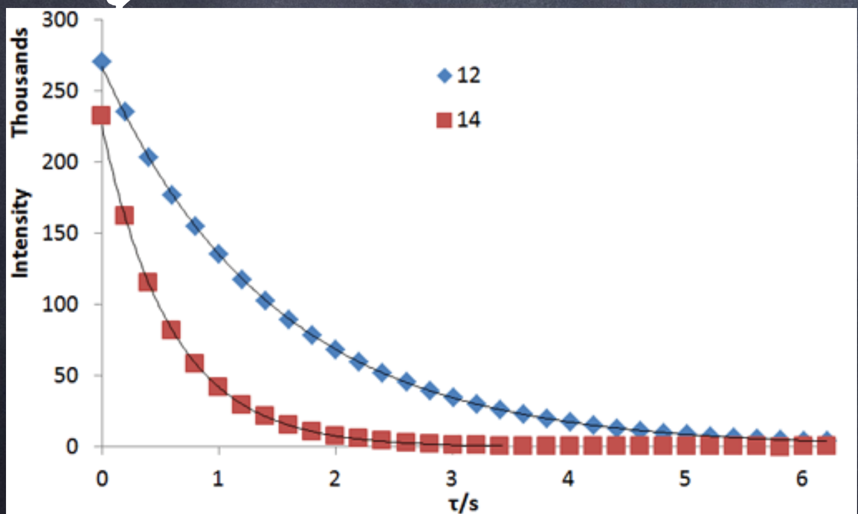
$$U_{ind} = U_{ind}^{max} e^{\left(-t/T_2\right)}$$

induced energy in the measuring solenoidal magnet coil.

max. value

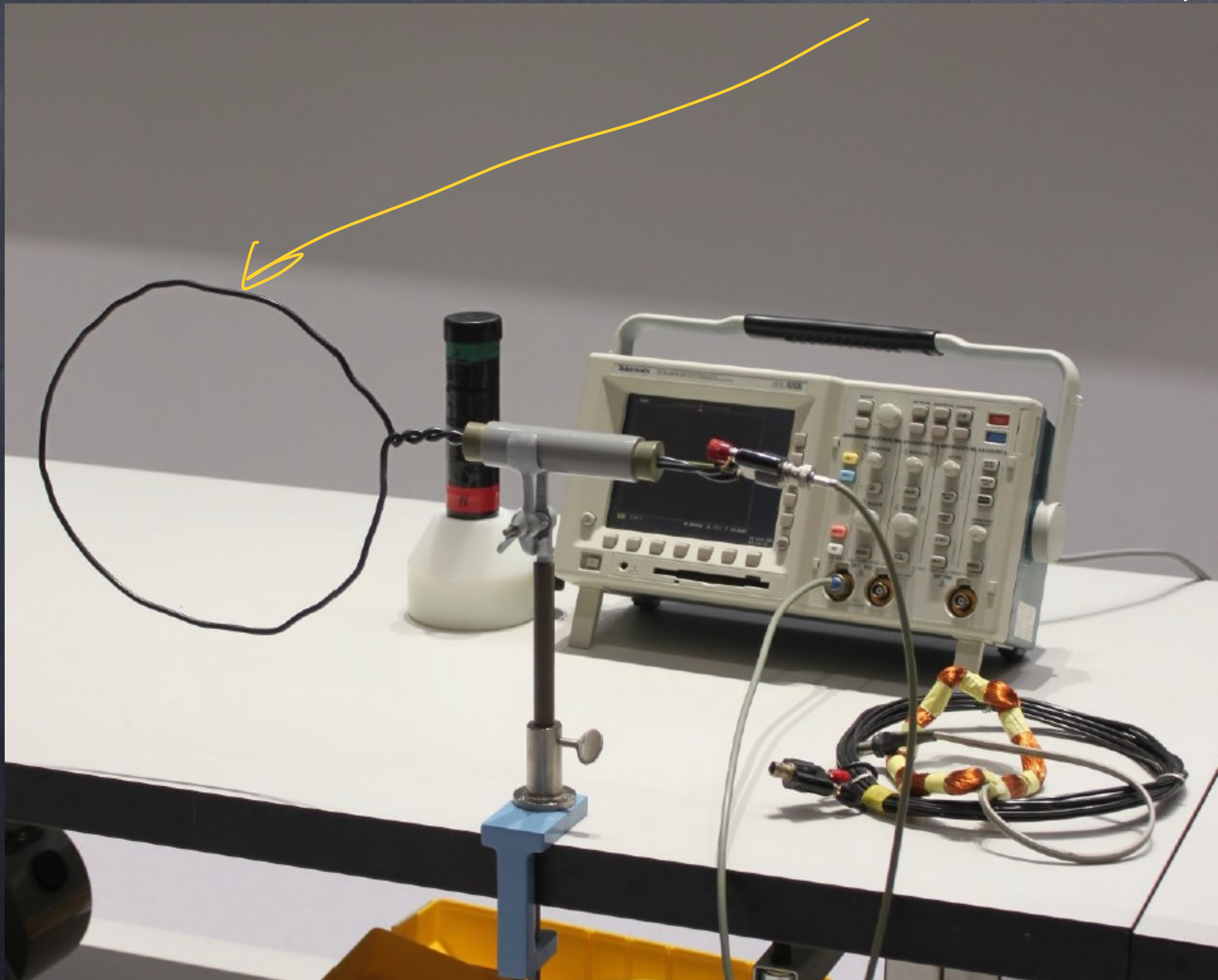
T_2: spin-relaxation time



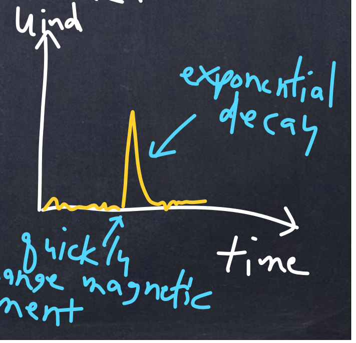


← shows T_2 for 2 different ^{t}butyl protons from 12,14-di^{t}butyl benzo[g]chrysene

In class, we put a magnet in a loop (our solenoid), then we change the magnetic moment of the magnet (using another solenoid). Then we observe an induced current in our loop.



we see also the relaxation time:



In our class experiment, we use a solenoid to change the magnetic moment of our magnet.

In NMR, we instead use a radio frequency pulse tuned to the resonance frequency f_{Larmor} to change the magnetic moment of our sample.

Now, we talk about MRI (magnetic resonance imaging)

Our goal is to do a 3-D scan of a human body.

we use NMR to map out the location of hydrogen (protons) in the human body.

If the magnetic field were constant over the body, there would be no way to tell where the NMR signal is coming from.

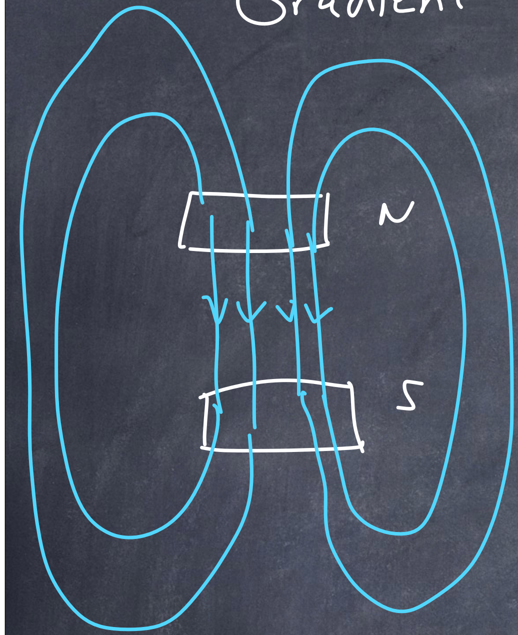
so we apply a magnetic field gradient to our body in 3 dimensions. So the magnetic field is different as a function of $x, y, \& z$

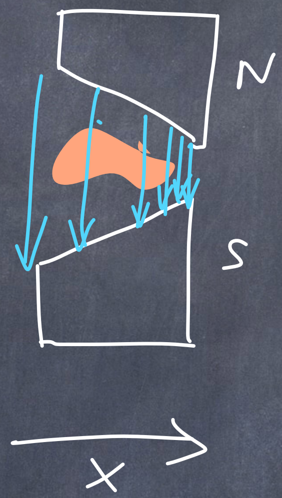
Gradients are typically 10^{-2} T/m are used so that the resonance conditon $\Delta E = 2\mu B = h\nu$ will vary along some direction according to the value of B.

If an RF pulse matches the resonance condition of a particular slice (or plane), then only these protons will be detected.

The x, y, z positions of the body are encoded in the RF frequency.

Gradient magnetic field





$$\bar{B}_x = \bar{B} + \bar{B}_{gr}(x)$$

nominal value

$$W_{Larmor} = \gamma \cdot \left(B + B_{gr}(x) \right)$$

W_{Larmor} frequency varies with x.

The level of the NMR signal at a frequency W_i is a measure of the density of hydrogen nuclei at the location x.

we would have gradients in $x, y, \& z$

$$\bar{B}_y = \bar{B} + \bar{B}_{gr}(y)$$

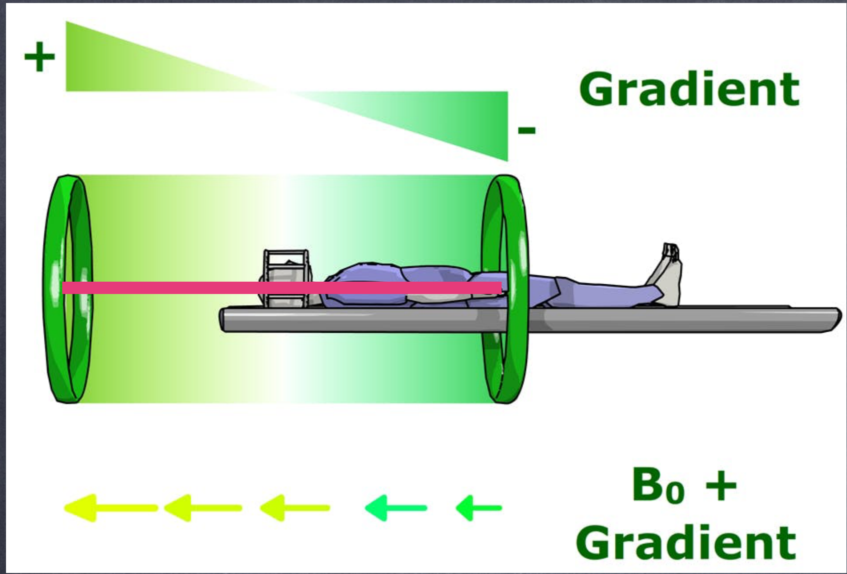
static value

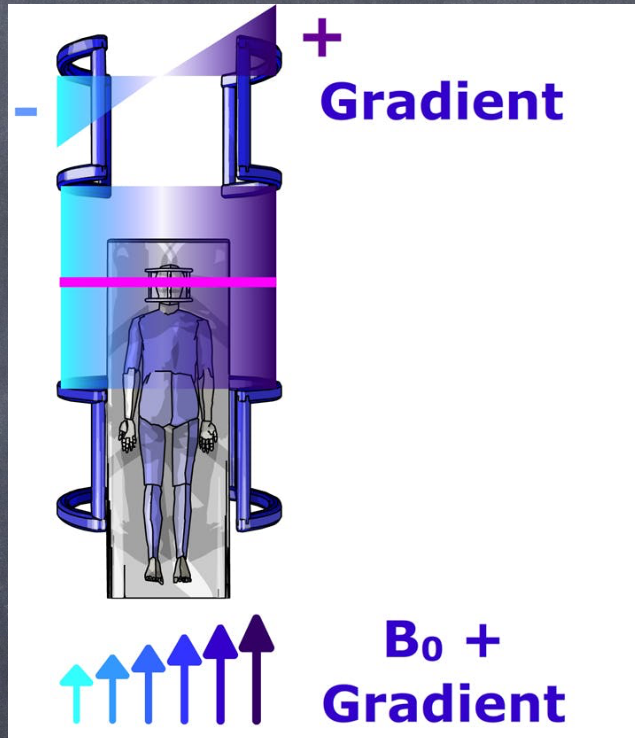
$$B_z = \bar{B} + \bar{B}_{gr}(z)$$

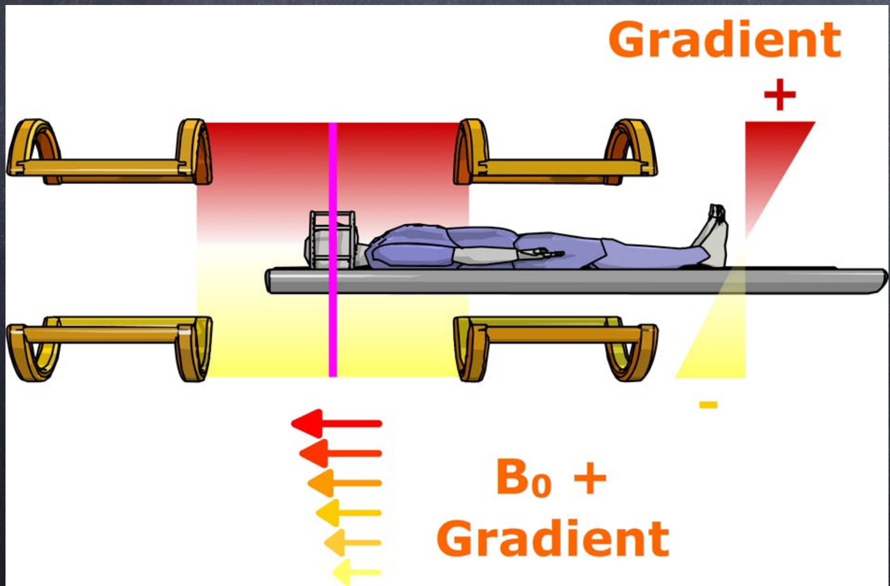
Applying the gradients sequentially in 3 directions. Transmit our RF pulses, then we measure the induced current (in our solenoid), determines the magnetization at a specific location → density of hydrogen at that location.

(This measured signal is proportional to the # hydrogen nuclei at a specific x, y, z)

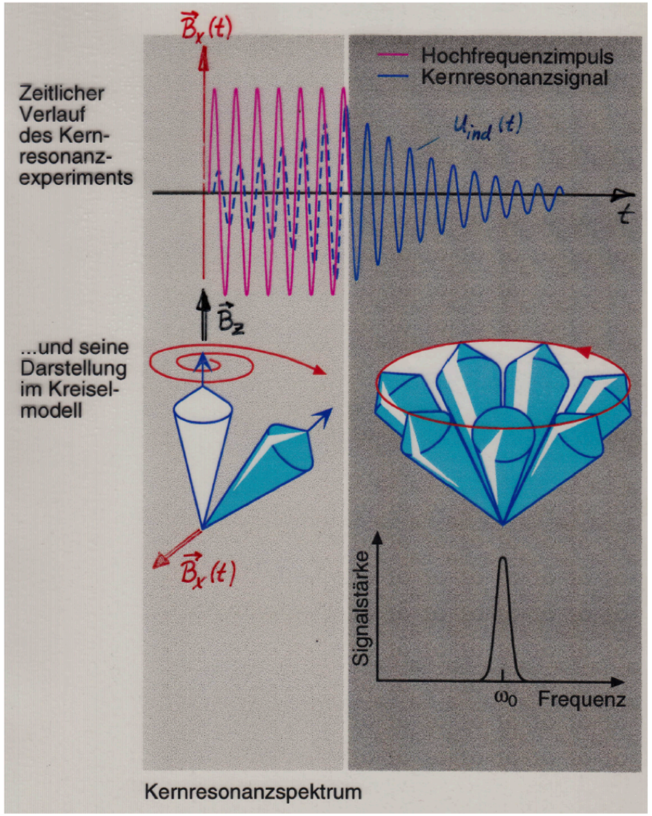
Then a computer reconstructs this 3D image.

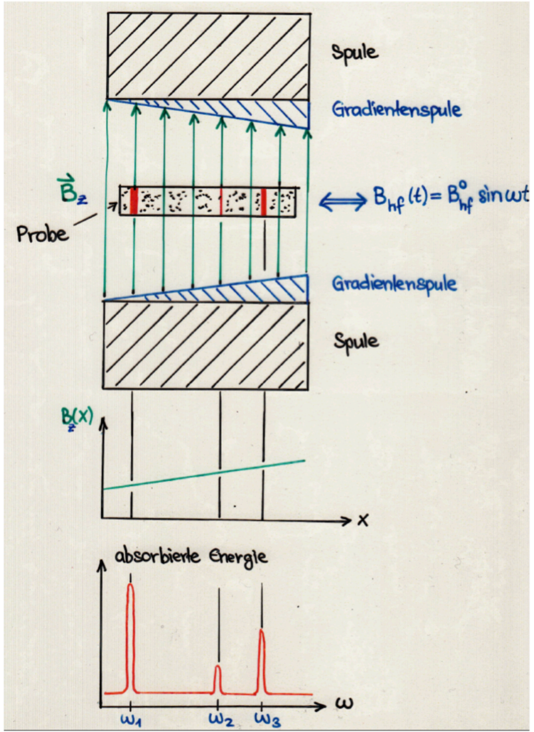

+
Gradient
-
B_0 +
Gradient


+
Gradient
-
B_0 +
Gradient


Gradient
+
-
B_0 +
Gradient

Magnetische Resonanz-Spektroskopie

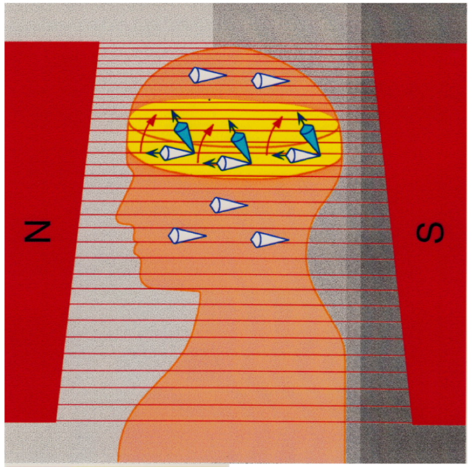


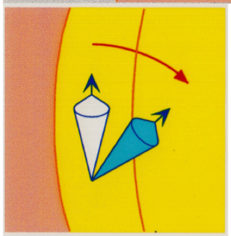


Die Ortsinformation, von woher ein Signal stammt, wird über die bekannte Ortsabhängigkeit des B-Feldes kodiert, welche in die Resonanz-frequenz eingeht.

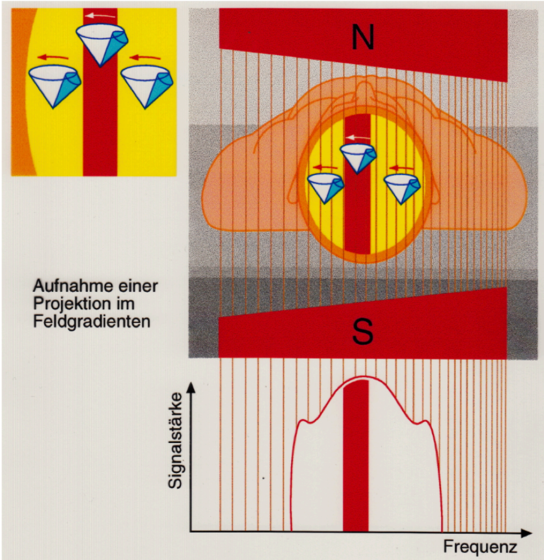
von Hugo Keller

Magnetische Resonanz-Tomographie





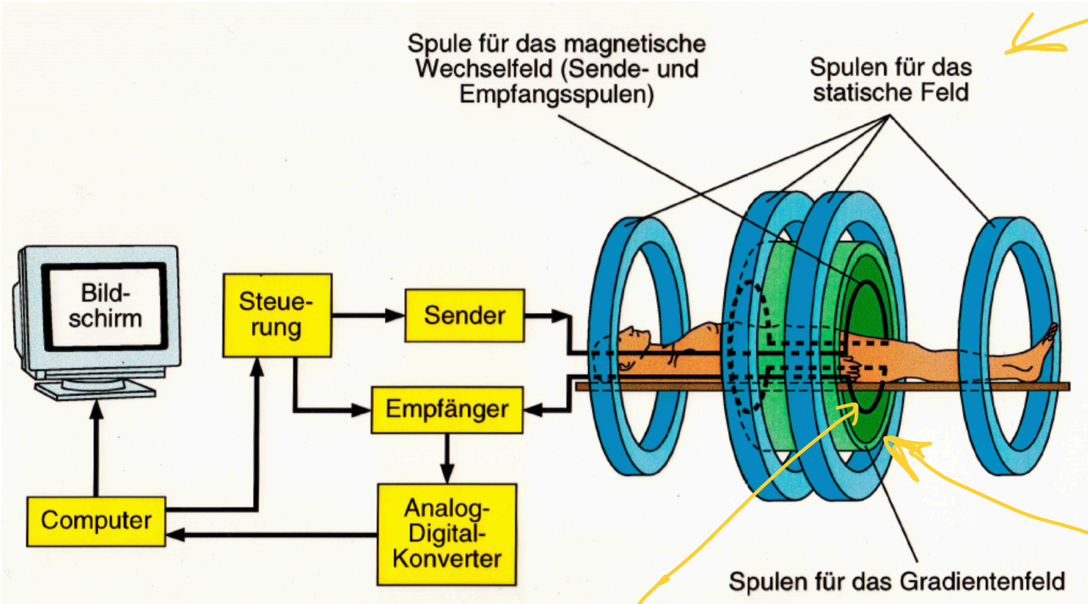
Selektive
Anregung
einer Schicht



Die verschiedenen Schnitte und Projektionen werden im Computer zu einem drei-dimensionalen Bild zusammengefügt, woraus man dann Schnitte in beliebigen Richtungen generieren kann. (altgriechisch ‘tome’ bedeutet Schnitt)

von Hugo Keller

Aufbau einer MRT-Anlage

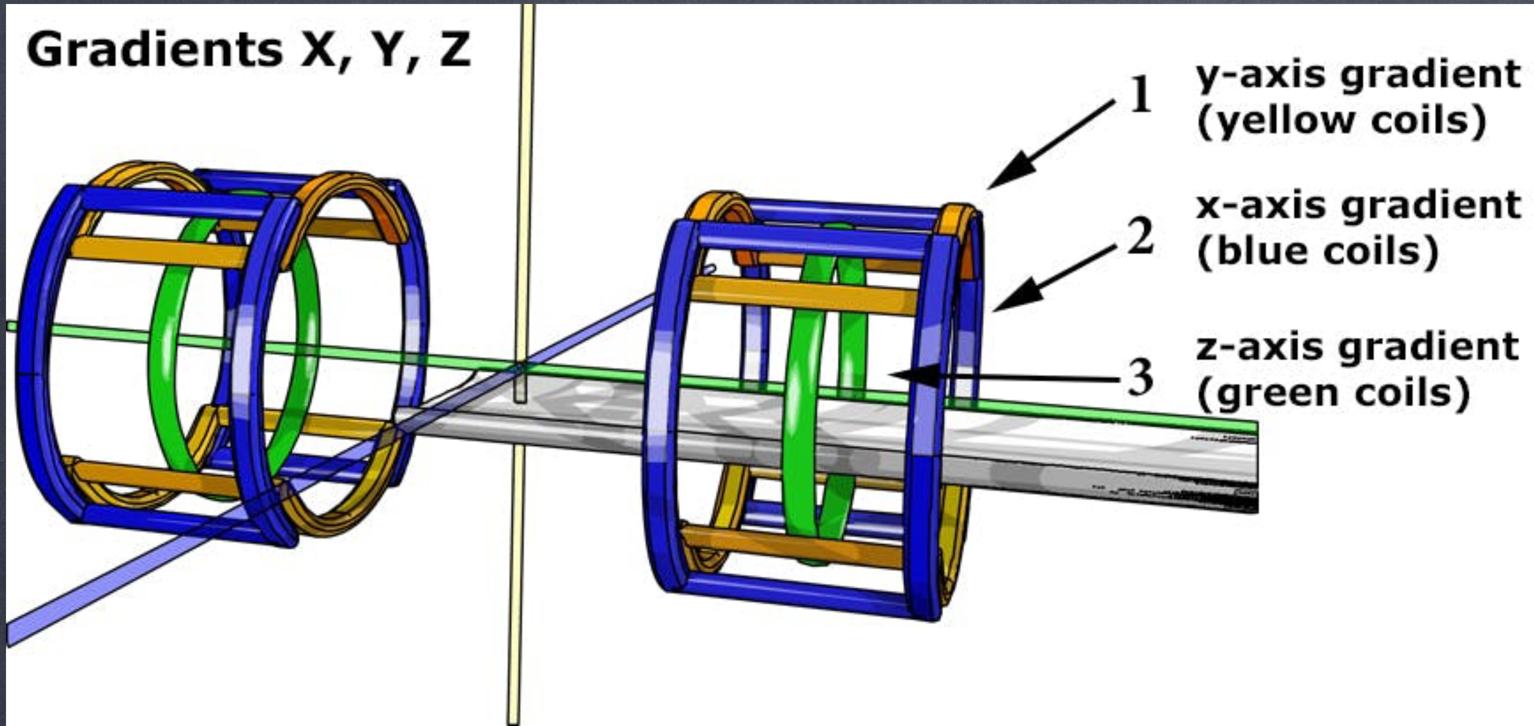


coil for measuring the changes

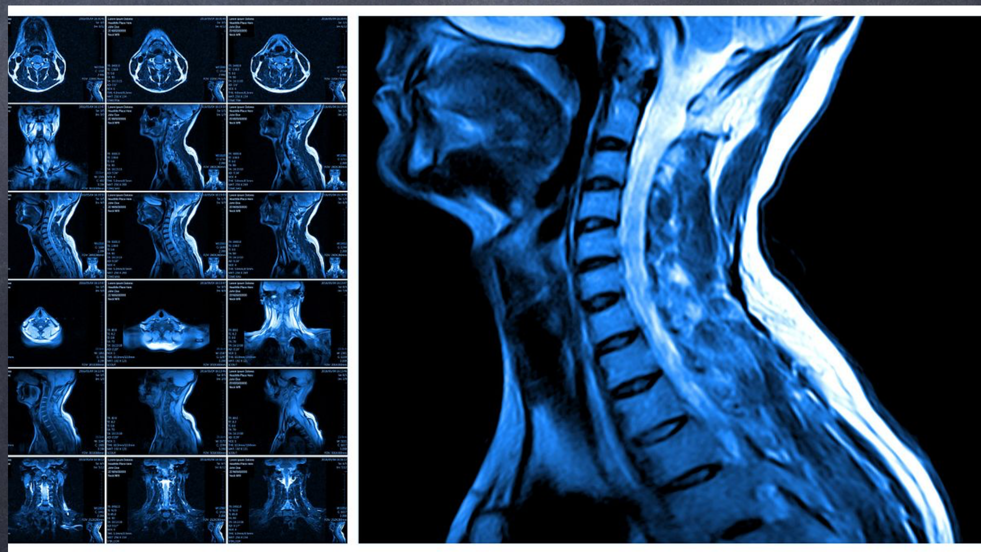
von Hugo Keller

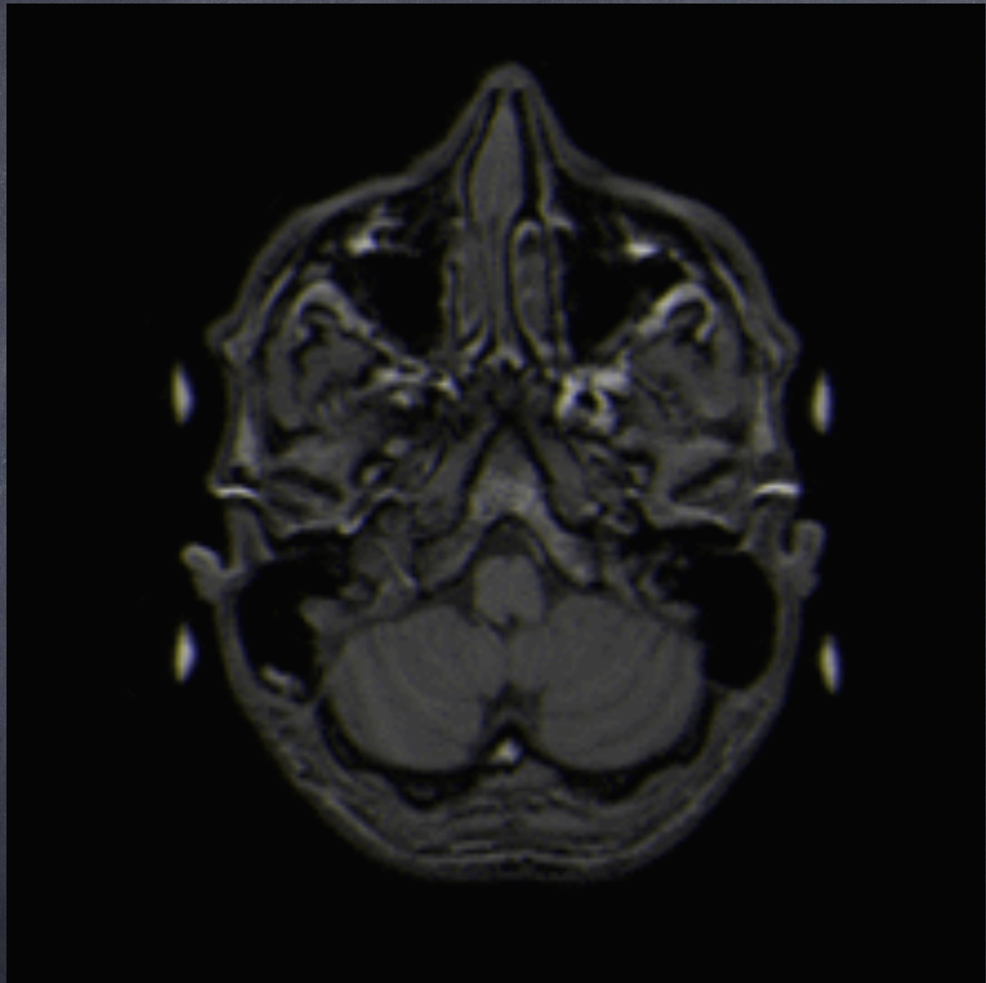
$\bar{B}$ is static

green: $\bar{B}(x)$ gradient field


Gradients X, Y, Z
1
y-axis gradient
(yellow coils)
2
x-axis gradient
(blue coils)
3
z-axis gradient
(green coils)

non-destructive
slices





Contrast between tissues in MRI comes from two processes. One is that different tissues have different water content (H_2O)

The second differentiation is from the relaxation time, T_2

TABLE 23.5

Relaxation time of hydrogen nuclear spins in an external magnetic field as a function of tissue with and without tumours

	Relaxation time T_{relax} (s)	
Tissue	Healthy	With tumour
Breast	0.37	1.08
Skin	0.62	1.05
Muscle	1.02	1.41
Liver	0.57	0.83
Stomach	0.77	1.24
Lung	0.79	1.10
Bone	0.55	1.03
Water	3.6	—

Table 18.2 *Water Content of Normal Human Tissue*

Tissue	*% water*
Brain (white matter)	84
Kidney	81
Myocardium	80
Skeletal muscle	79
Brain (gray matter)	72
Liver	71
Nerve	56
Bone (cortex)	12
Teeth	10

Nuclear physics

Atomic sizes ~ 0.1 nm $(10^{-10}$ m$)$

Nuclear sizes $\sim 10^{-15}$ m

If an atom was the size of a football field, the nucleus would be the size of a pin head (1mm)

proton mass $\sim 1800 *$ electron mass

Elements: $A = Z + N$

atomic number — number of protons — number of neutrons

Notation for element, E : ${}^{A}_{Z}E \rightarrow {}^{A}E$

units $1\,u \equiv \frac{1}{12}\,C^{12}$ atom

example ${}^{13}C \equiv {}^{13}_{6}C$

Nuclide refers to a particular Z and N combination.
Nuclides with the same Z but different N are called isotopes.

Some isotopes are stable, and some are radioactive.

Structure of isotopes has been determined by scattering electrons on the isotope.

$\lambda = \frac{h}{p}$ for relativistic electrons $\rightarrow \boxed{p \sim \frac{E}{c}}$

from $E^2 = (cp)^2 + \underbrace{(mc^2)^2}_{\text{can be neglected if } E \gg m}$

For 200 MeV electron,

$$(cp)^2 = E^2 - (m_e c^2)$$

$$cp = \sqrt{\underbrace{(200\ \mathrm{MeV})^2}_{200^2} - \underbrace{\left(0.511\ \frac{\mathrm{keV}}{c^2}\, c^2\right)^2}_{2E-7}}$$

$$cp \cong 200\ \mathrm{MeV}$$

will show this (mc^2) is small

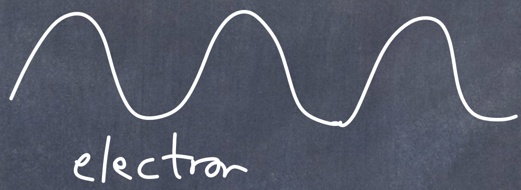
units
E [eV]
p $\left[\frac{eV}{c}\right]$
m $\left[\frac{eV}{c^2}\right]$

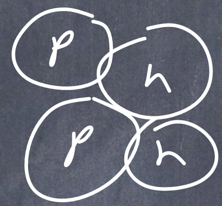
For 200 MeV electron, $p = 200 \ \frac{MeV}{c}$ $\qquad c = 3E8 \ \frac{m}{s}$

$$\lambda = \frac{h}{p} = \frac{4.1357E-15 \ eV \cdot s}{200 \ \frac{MeV}{c}} = \frac{4.1357E-15 \ \cancel{eV} \cdot \cancel{s}}{200 \ \cancel{eV} E6 \ \cancel{eV}} \cdot 3E8 \ \frac{m}{\cancel{s}}$$

$$\lambda = 6E-15 \ m = 6 \ fm$$

This is small enough to probe nuclear sizes.





Nuclei are mostly spherical, but some are elipsoidal

size $R \cong R_0 A^{1/3}$ $R_0 \sim 1.2$ fm

density is almost the same for all nuclei

density of a proton : $\rho = \dfrac{m_p}{\frac{4}{3}\pi r^3} = \dfrac{1.67\text{E}-27\,\text{kg}}{\frac{4}{3}\pi(1.2\text{E}-15\,\text{m})^3}$

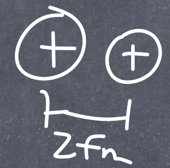
$$\rho = 2\text{E}17\ \text{kg}/\text{m}^3$$

ρ is 10^{14} times larger than the density of an atom.

Why is the nucleus ever stable?

Densely packed with protons that repel each other.

Coulomb force: $F = \frac{e^2}{4\pi\varepsilon_0 r^2}$



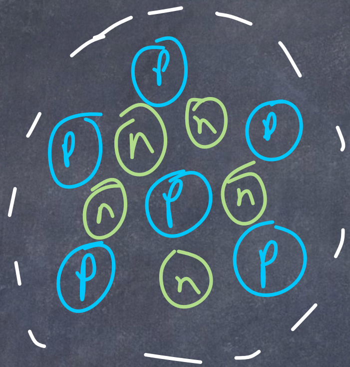
For $r = 2$ fm, 2 protons repel each other with a force of $F = 60$ N. (equivalent to a 6 kg weight)

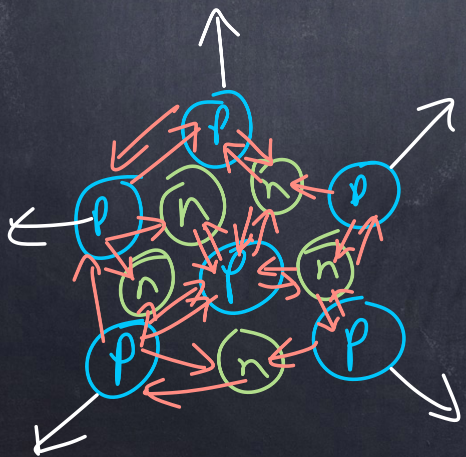
Answer: the nucleus is held together by the strong nuclear force. This is an attractive force more than 100 times stronger than the Coulomb force repulsion.

only when the protons are close to each other (as in a nucleus) nucleon = proton or a neutron

Nucleons are all attracted by the strong nuclear force.

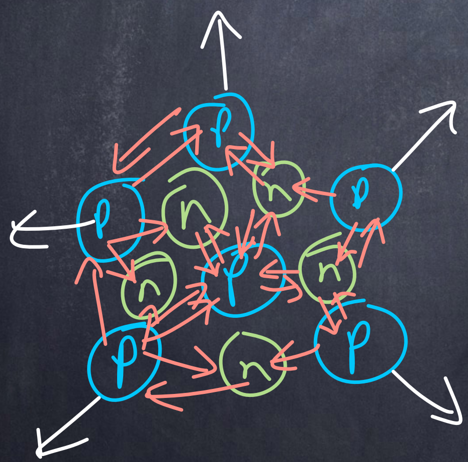
Nucleus





strong force

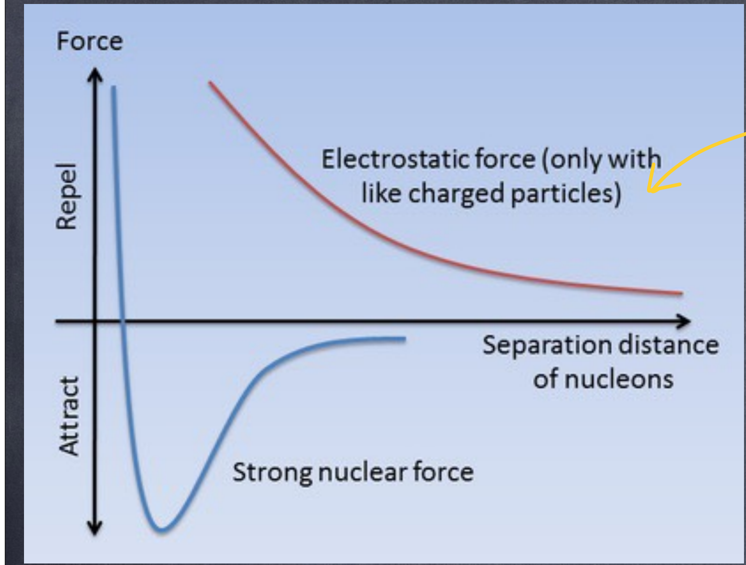
electrostatic repulsion



strong force

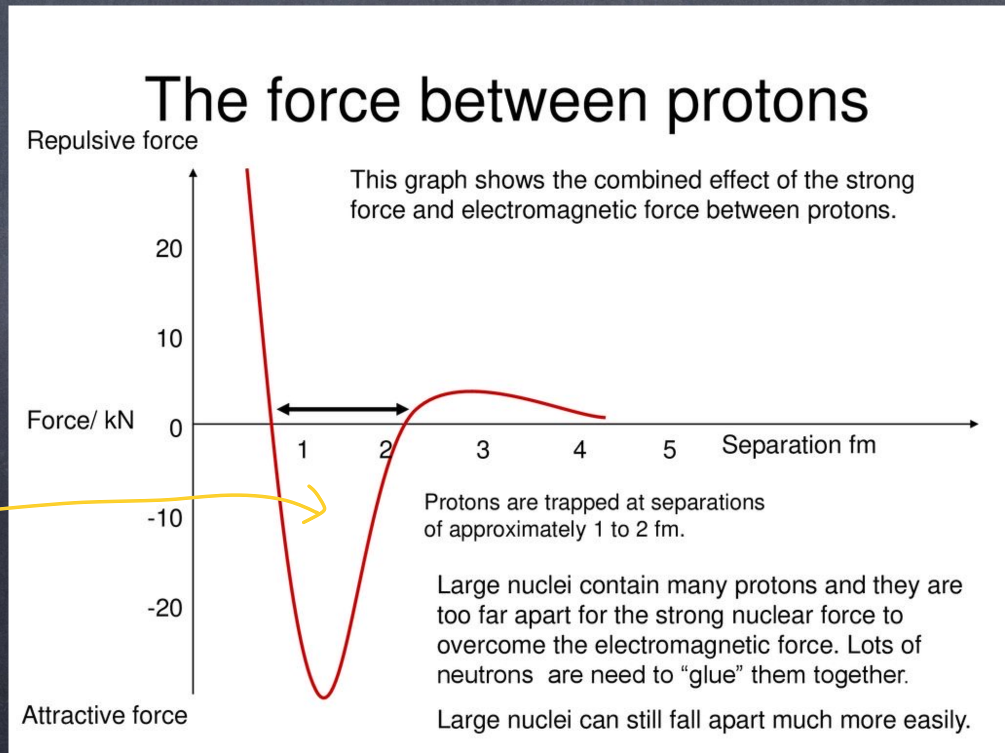
electrostatic repulsión

force between protons ← Two separate forces

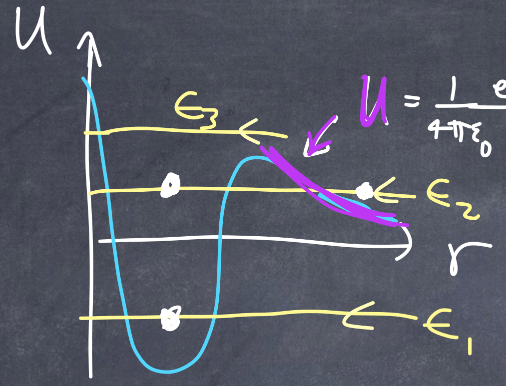


coulomb force

the combined effect of the two forces is here



where protons would be trapped in the potential



$U = \frac{1}{4\pi\epsilon_0}\frac{e^2}{r} = \frac{ke^2}{r}$

U: potential energy of a proton at a given radius.

So, imagine a proton approaching a nucleus, it needs enough energy to overcome the barrier, then it can be trapped in the potential well.

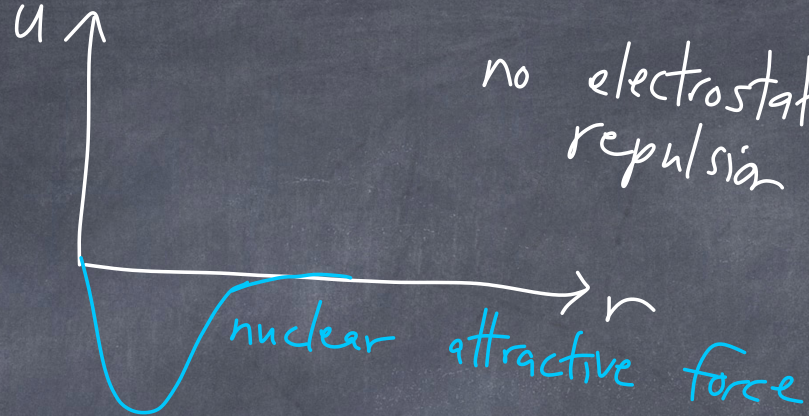
total energy of proton in well

! ϵ_1: stuck in the potential, not enough energy to escape (stable nuclei)

ϵ_2: meta-stable, quantum mechanics lets the proton tunnel out of the well, since it has enough energy to exist outside the nucleus.

ϵ_3: unstable, immediately escapes the nucleus.

For a neutron



no electrostatic repulsion

Easier for a neutron to get captured by a nucleus (than a proton)

The nucleus is often thought of as a liquid drop.

nucleus	drop
both are incompressible and have uniform density	
held together by the nuclear force	held together by surface tension

drop can break into smaller drops

The total energy of a nucleus is the sum of the kinetic energy & potential energy of the constituents (nucleons). The potential energy is negative and larger than the kinetic energy, so the total energy is negative.

The total energy of the individual constituents (nucleons) is larger than the energy of the assembled nucleus. The difference is due to the binding energy of the nucleus.

$$\text{Nuclear binding energy} = Z(m_p c^2) + N(m_n c^2) - mc^2$$

m_p : mass of proton

m_n : mass of neutron

m : mass of the assembled nucleus.

The binding energy is about 1% of the mass

$$\frac{8\ \text{MeV}}{1\ \text{u}} = \frac{8\ \text{MeV}}{930\ \text{MeV}} \sim 1\%$$