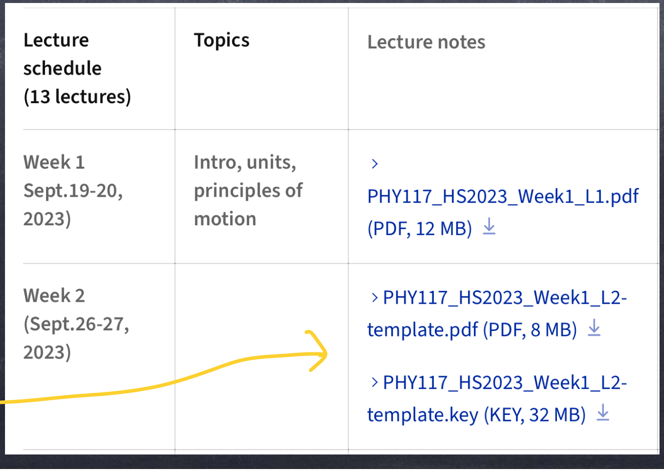
PHY117 HS2023

Prof. Ben Kilminster
Sept. 20, 2023
Week 1, Lecture 2

https://www.physik.uzh.ch/de/lehre/PHY117/HS2023.html



Lecture schedule (13 lectures)	Topics	Lecture notes
Week 1 Sept.19-20, 2023	Intro, units, principles of motion	› PHY117_HS2023_Week1_L1.pdf (PDF, 12 MB)
Week 2 (Sept.26-27, 2023)		› PHY117_HS2023_Week1_L2-template.pdf (PDF, 8 MB) › PHY117_HS2023_Week1_L2-template.key (KEY, 32 MB)

Template for notes here

Q: what are these: ?

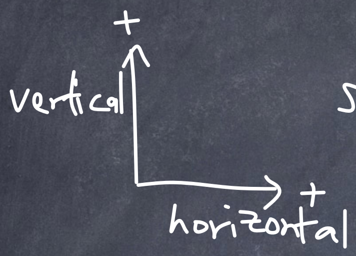
$\hat{x}$, $\hat{y}$, $\hat{z}$

A: They are special vectors with length 1 called unit vectors

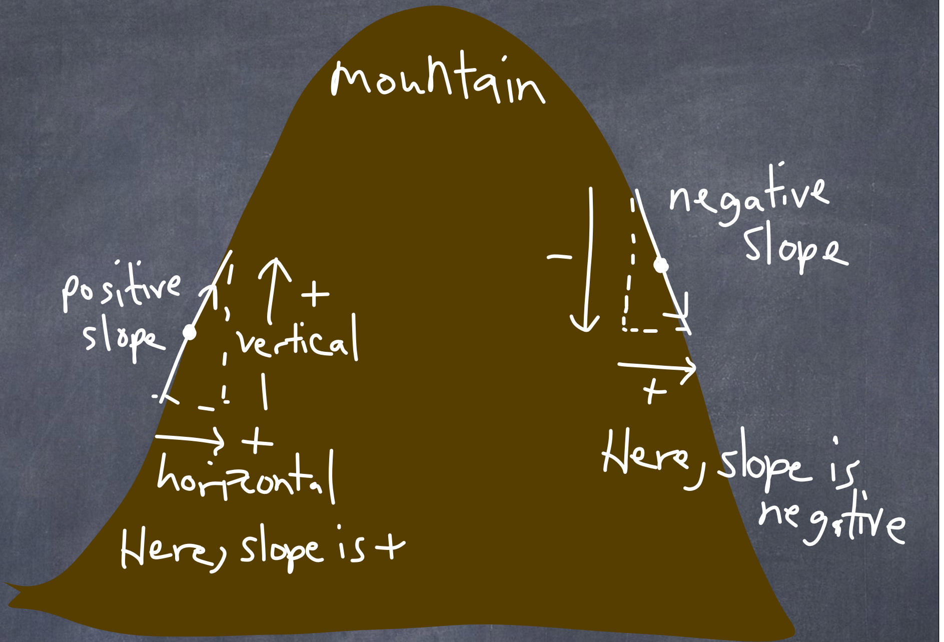
the "^" is called a "hat."

Unit vectors give us a coordinate system to define vectors.

Q: what is a slope?



$$\text{slope} = \frac{\text{vertical}}{\text{horizontal}}$$



Q: what is the difference between velocity + speed?

A: velocity is a vector
speed is a scalar

$$\text{velocity} = \bar{v} \left(\begin{matrix}\text{magnitude} \\ + \\ \text{direction}\end{matrix}\right)$$

$$\text{speed} = v = |\bar{v}| \left(\begin{matrix}\text{magnitude,} \\ \text{no direction}\end{matrix}\right)$$

$$\text{Example:} \begin{cases} \bar{V} = 3\frac{m}{s}\,\hat{x} \\ V = 3\frac{m}{s} \end{cases}$$

Q:

Reminder on derivatives:

$$f(x) = \sin(x)$$

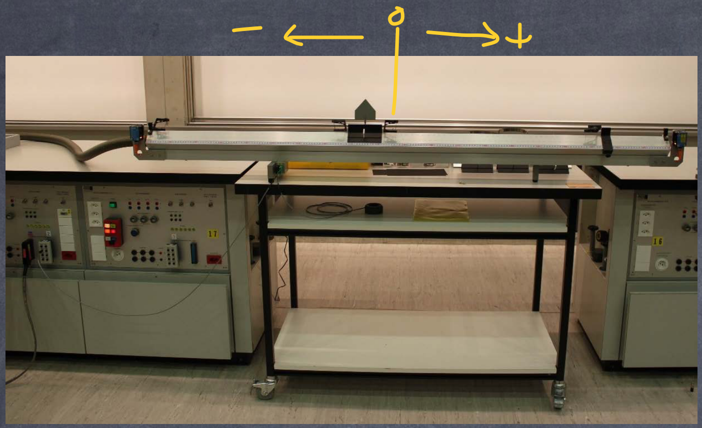
$$\frac{df(x)}{dx} = f'(x) = \cos(x)$$

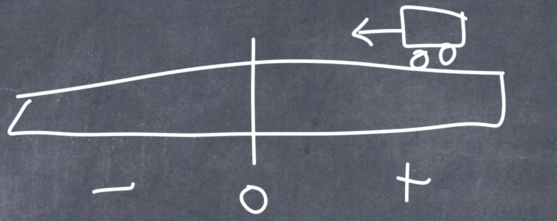
chain rule means we multiply by $\frac{d(kx)}{dx} = k$

$$f(x) = \sin(kx)$$

$$f'(x) = \left[\cos(kx)\right](k)$$

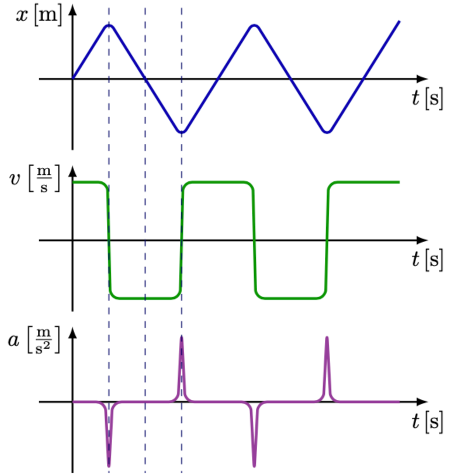
Explanation of yesterday's experiment:



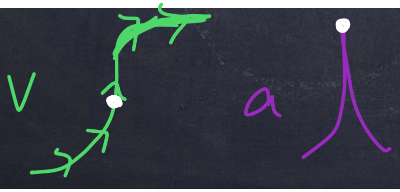


velocity is the slope of x vs. t

acceleration is the slope of v vs. t



(f) Train bouncing back and forth with constant speed. The acceleration peaks when it quickly changed directions.



velocity is +: moving in + direction

-: moving in - direction

$a = 0$: when velocity is constant

$a = (-)$: when velocity is decreasing

Here: acceleration is not constant. It has a peak structure.

we will discuss this later when we talk about "impulse".

Can we make our air-car accelerate more smoothly?

we can by making use of a "simple harmonic oscillator"

One example is a spring.

The position changes according to the formula

$$x(t) = x_0 \cos(\omega t)$$

x_0 ↑ starting position

↑ ω is the angular frequency

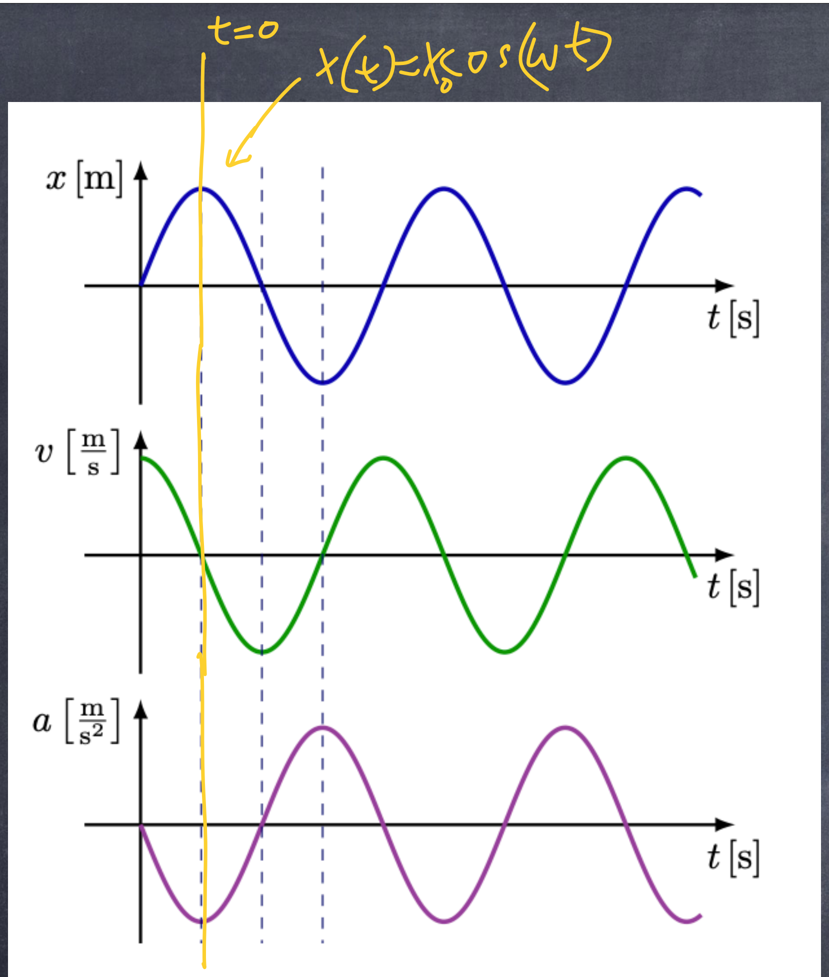
$\omega = \frac{2\pi}{T}$, where

T is the "period", the time it takes to do one cycle.

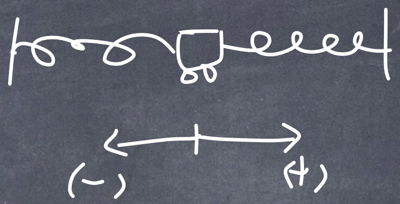
Then: $v(t) = \frac{dx}{dt} = \underbrace{-x_0\omega}_{v_0} \sin(\omega t)$

$$a(t) = \frac{dv}{dt} = -x_0 \omega^2 \cos(\omega t)$$

Notice $a(t) = x(t)(-\omega^2)$ ↙ This we will come back to.



(g) Periodic one dimensional motion of a mass on a spring moving back and forth. Velocity is largest when $x = 0 = a$.

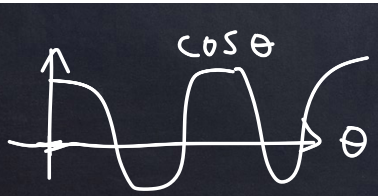


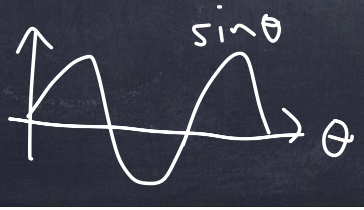
$x(t) = x_0 \sin(\omega t)$

$v(t) = V_0 \cos(\omega t)$

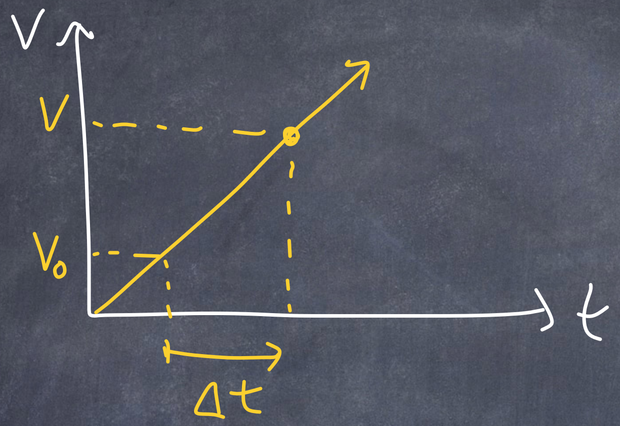
$a(t) = -a_0 \sin(\omega t)$

Notice that depending on when we start $t=0$, $x(t)$ can either be $x_0 \sin(\omega t)$ or $x_0 \cos(\omega t)$





Motion with constant acceleration:



a = slope of v vs. t
= constant

$$a = \frac{dv}{dt}$$

we can determine V from a starting V_0 and the slope, a.

$$\boxed{V = V_0 + at}$$

final velocity

starting velocity

$$a \cdot t = \left[\frac{m}{s^{\cancel{2}}}\right]\left[\cancel{s}\right] = \frac{m}{s}$$

Average velocity is half-way between V_0 + V:

$$\boxed{V_{av} = \frac{1}{2}(V_0 + V)}$$

Some other useful formulas for constant acceleration

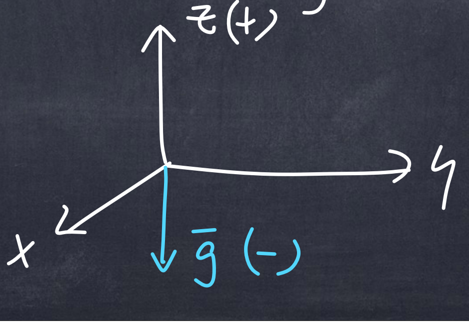
$$\boxed{\Delta x = \frac{1}{2}(v_0 + v)t}$$

$$\boxed{\underset{\substack{\uparrow\\ \text{final}\\ \text{position}}}{x} = \underset{\substack{\uparrow\\ \text{starting}\\ \text{position}}}{x_0} + \underset{\substack{\uparrow\\ \text{starting}\\ \text{velocity}}}{v_0} t + \frac{1}{2}\underset{\substack{\uparrow\\ \text{acceleration}\\ \text{(constant)}}}{a} t^2}$$

$$\boxed{v^2 = v_0^2 + 2a\Delta x}$$

Most of the time, on earth, the acceleration is due to gravity.

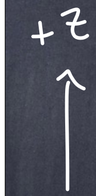
$$\bar{a} = -g\hat{z}$$

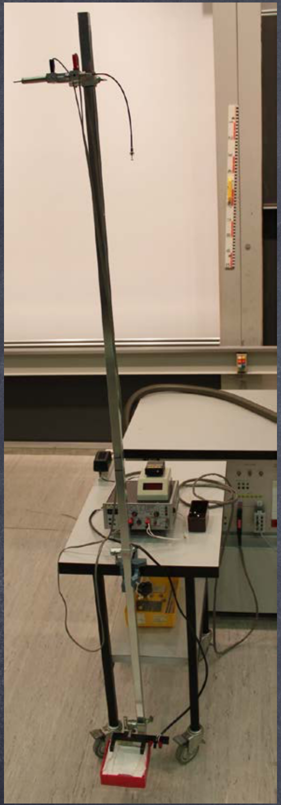


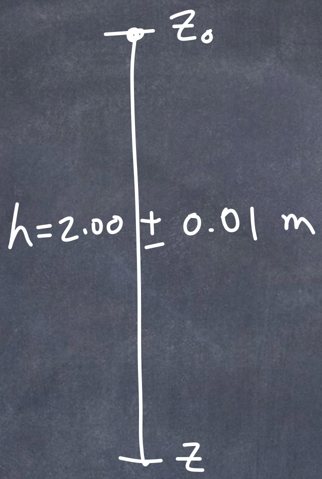
Does gravity accelerate everything the same?

yes (but sometimes other forces like air resistance)

what is g ? $\left[\frac{m}{s^2}\right]$







$$\bar{a} = -g\hat{z}$$

$$z = z_0 + v_{0_z} t + \frac{1}{2} a t^2$$

$$\underbrace{z - z_0}_{(-)} = \cancelto{0}{v_{0_z}} t - \frac{1}{2} g t^2$$

$$\underbrace{z_0 - z}_{(+)} = \frac{1}{2} g t^2$$

$$h = \frac{1}{2} g t^2 \rightarrow g = \frac{2h}{t^2}$$

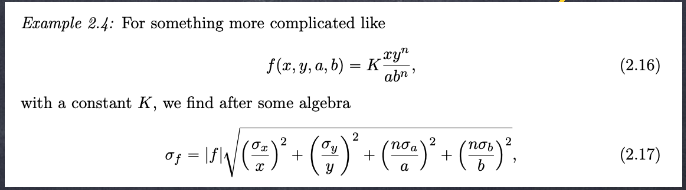
measure the time :

0.6396 s

0.6395 s

$t = 0.6395 \pm 0.0001$ s

$$g = \frac{2(2.00\,m)}{(0.6395\,s)^2} = \boxed{9.78 \pm 0.05\,\frac{m}{s^2}}$$



Example 2.4: For something more complicated like

$$f(x, y, a, b) = K\frac{xy^n}{ab^n}, \tag{2.16}$$

with a constant K, we find after some algebra

$$\sigma_f = |f|\sqrt{\left(\frac{\sigma_x}{x}\right)^2 + \left(\frac{\sigma_y}{y}\right)^2 + \left(\frac{n\sigma_a}{a}\right)^2 + \left(\frac{n\sigma_b}{b}\right)^2}, \tag{2.17}$$

Bern:

$g = 9.8084\ \frac{m}{s^2}$

$$\sigma_g = g\sqrt{\left(\frac{0.01\,m}{2.00\,m}\right)^2 + \left(\frac{2(0.0001\,s)}{0.6395\,s}\right)^2}$$

$$\sigma_g = 9.78\,\frac{m}{s^2}\left(\sqrt{2.5\mathrm{e}{-5} + 9.8\mathrm{e}{-8}}\right.$$

$$\sigma_g = 0.05\,\frac{m}{s^2}$$

Assume we throw a ball up at 20 m/s, what is its position in 4s?

$$z = \underset{\substack{\uparrow \\ 1\,m}}{z_0} + \underset{\substack{\uparrow \\ +20\frac{m}{s}}}{V_{0z}}\,\underset{\substack{\uparrow \\ 4s}}{t} + \underset{\substack{\uparrow \\ a=-10\frac{m}{s^2}}}{\frac{1}{2}a}\,\underset{\substack{\uparrow \\ (4s)^2}}{t^2}$$

$$Z = 1_m + 80\,m\frac{\cancel{s}}{\cancel{s}} + \frac{1}{2}\left(-10\frac{m}{s^2}\right)(4s)^2$$

$z = 1\,m$ (same as initial position)

How fast is it moving?

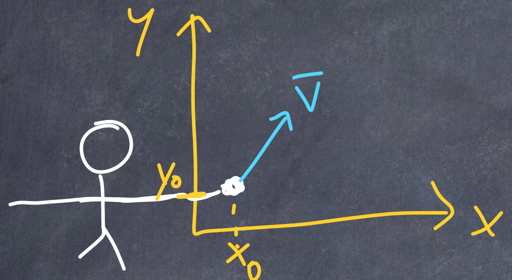
$$V_z = V_{0z} + at = 20\frac{m}{s} + \left(-10\frac{m}{s^2}\right)(4s)$$

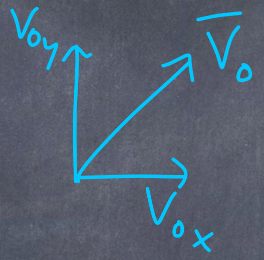
$$V_z = -20\frac{m}{s}$$

(opposite initial velocity

Motion in 2-D:

When we throw an object it moves in a parabola.





Initial velocity:

$$\bar{V}_0 = V_{0x}\hat{x} + V_{0y}\hat{y}$$

(Neglect air resistance)

x-direction:

$$\boxed{V_x = V_{0x}} + \overset{0}{\cancel{a}}t$$

$$\boxed{x = x_0 + V_{0x}t} + \frac{1}{2}\overset{0}{\cancel{a}}t^2$$

velocity in x-direction is constant

y-direction:

$$\boxed{V_y = V_{0y} + at} \Rightarrow V_y = V_{0y} - gt$$

$$y = y_0 + V_{0y}t + \frac{1}{2}at^2 \Rightarrow$$

$$\boxed{y = y_0 + V_{0y}t - \frac{1}{2}gt^2}$$

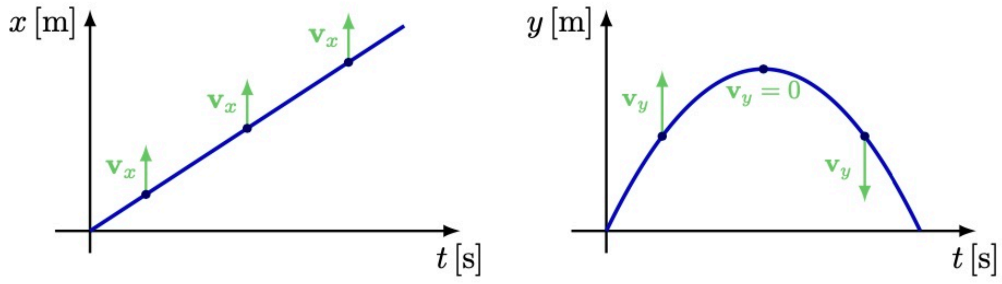
$V_x = V_{0x}$

$x = x_0 + V_{0x} t$

$V_y = V_{0y} - gt$

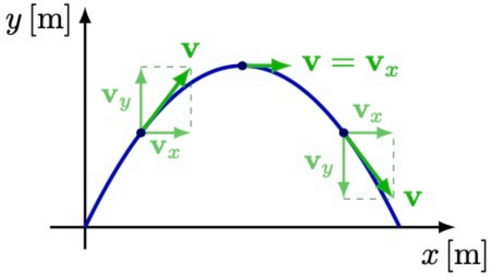
$y = y_0 + V_{0y} t - \frac{1}{2} g t^2$

$x = x_0$

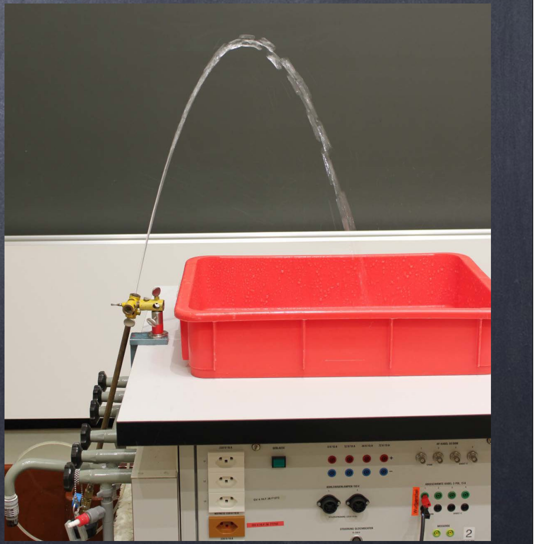


(a) x-t diagram. **(b)** y-t diagram.

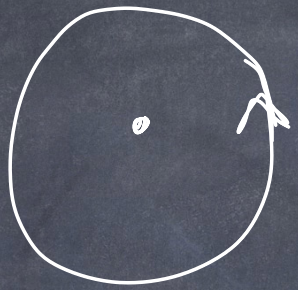
What we see:



(c) y-x diagram.



Motion in a circle:

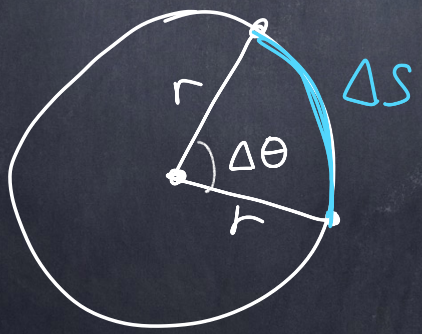


circle has $360^\circ = 2\pi$ radians

$90^\circ = \frac{\pi}{2}$ radians

Calculator: "deg" / "rad"

Circumference of a circle is $C = 2\pi r$

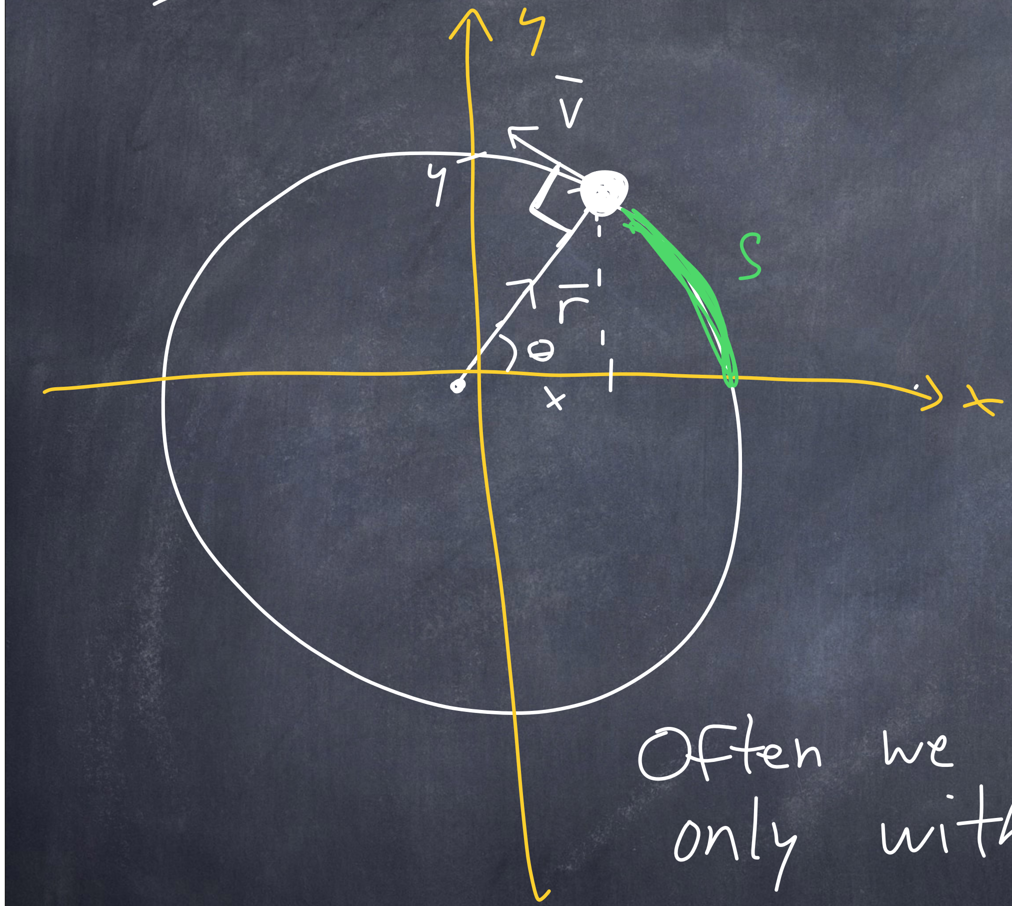


$\Delta S = r\,\Delta\Theta$

If $\Delta\Theta = 2\pi$,
then $\Delta S = 2\pi r$
(circumference)

Half a circle
$\Delta\Theta = 180^\circ = \pi$ radians $=$
$\Delta S = r\pi$

Circular motion (in 2D):



position $\bar{r}$, velocity $\bar{v}$

$\bar{r} \perp \bar{v}$

Distance traveled $\Rightarrow s = r\theta$

$$\underset{\text{speed}}{v} = |\bar{v}| = \frac{\text{circumference}}{\text{time to do a circle}}$$

$$v = \frac{2\pi r}{T}$$

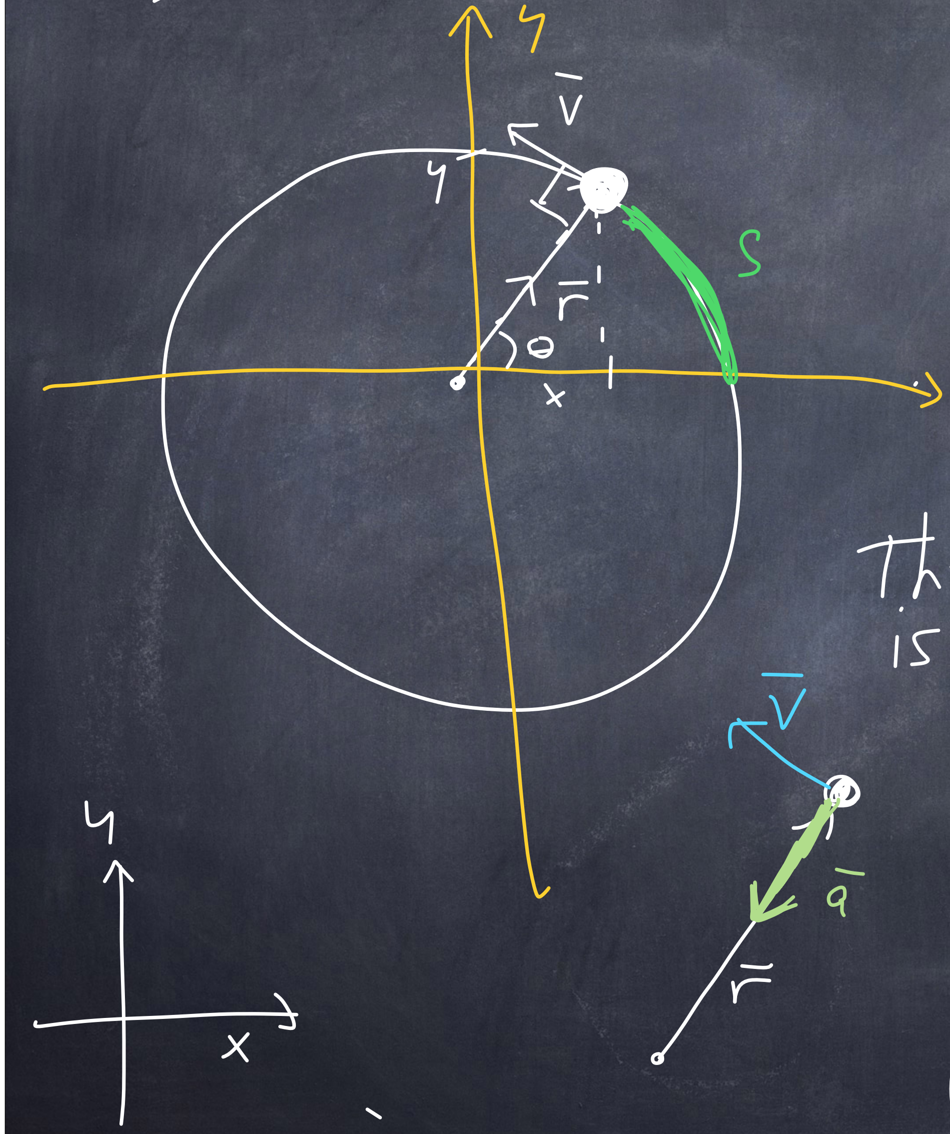
T: period of a cycle

Often we describe the position only with θ, since $|\bar{r}|$ is a constant

For the velocity, we often use $\frac{\Delta\theta}{\Delta t} = \omega$: angular velocity

$$\omega = \frac{v}{r} \quad \left[\frac{\frac{\cancel{m}}{s}}{\cancel{m}}\right] = \left[\frac{1}{s}\right] = \text{radians per second}$$

Circular motion (in 2D):



$a = \frac{v^2}{r}$ = centripetal acceleration

(Derivation of $a = \frac{v^2}{r}$ on the next page if you are interested)

The direction of $\bar{a}$ is toward the center of the circle.

$$\bar{a} = -\frac{v^2}{r}\hat{r}$$

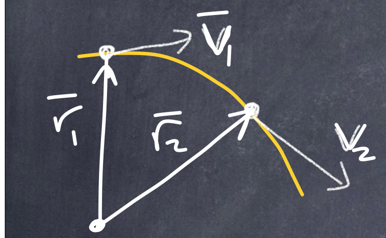
skip

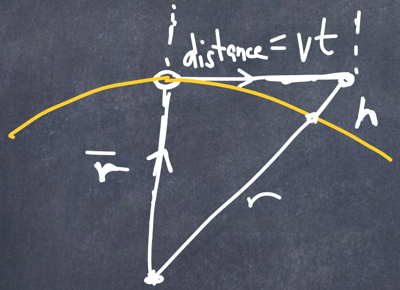
Motion in a circle, Ball on a string

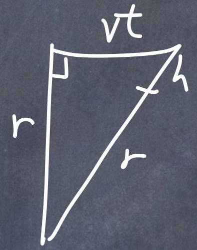
assumptions: The speed is the same vs. time

The velocity is changing (direction changes)

If the ball's velocity stayed the same, it would be at a height of r+h after a time t.







Pythagorean theorem:

$$(vt)^2 + r^2 = (r+h)^2$$

$$(vt)^2 + \cancel{r^2} = \cancel{r^2} + 2rh + h^2$$

For small time t, h is small

h is very small compared to r $(h \ll r)$

so $h^2 \ll rh$

Therefore, $(vt)^2 \cong 2rh + \cancelto{0}{h^2}$

so $h \approx \frac{1}{2}\left(\frac{v^2}{r}\right)t^2$

Compare this to $t = \frac{1}{2}at^2$, we see that $\boxed{a = \frac{v^2}{r}}$

This is the centripetal acceleration. Movement in circle.

A satellite is moving around the earth at a height of 100 km above the ground. How long does it take to go around the earth?

$$C_{earth} = 40{,}000 \text{ km}$$

$$g = 9.81 \frac{m}{s^2}$$

$$C = 2\pi r \Rightarrow r_{earth} = 6370 \text{ km}$$

$$r_{satellite} = 6470 \text{ km}$$

we know 2 things: $\left\{ \begin{array}{l} a = \frac{v^2}{r} \\ a \approx g \end{array} \right\}$ Therefore, $\frac{v^2}{r} = g \Rightarrow V = \sqrt{rg}$

$$V = \sqrt{(6470 \cancel{km}) \underbrace{\left(\frac{1000 m}{1 \cancel{km}}\right)}_{\text{This is just to get m instead of km}} \left(9.81 \frac{m}{s^2}\right)} = 7970 \frac{m}{s}$$

$$T = \frac{2\pi r}{V} = \frac{2\pi (6470 \text{ km})}{7.97 \frac{km}{s}} \cdot \left(\frac{1 \text{ minute}}{60 \text{ seconds}}\right) = 85 \text{ minutes}$$

$\Longrightarrow$ Space station takes 92 minutes

we are very close!

Experiments