

The Hidden Geometry of Particle Collisions

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Seminar in Theoretical Particle Physics, Zurich — October 19, 2021

The NSF AI Institute for Artificial Intelligence and Fundamental Interactions (IAIFI) *“eye-phi”*

Advance physics knowledge — from the smallest building blocks of nature to the largest structures in the universe — and galvanize AI research innovation



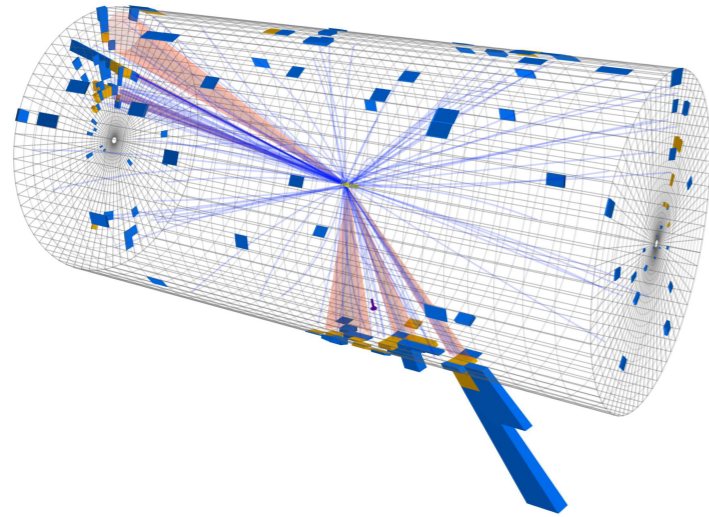
[<http://iaifi.org>, MIT News Announcement]

Optimal Transport for HEP?

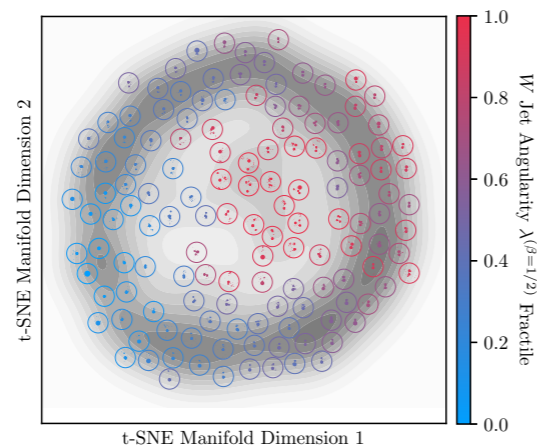


What does moving “earth” have to do with (theoretical) collider physics?

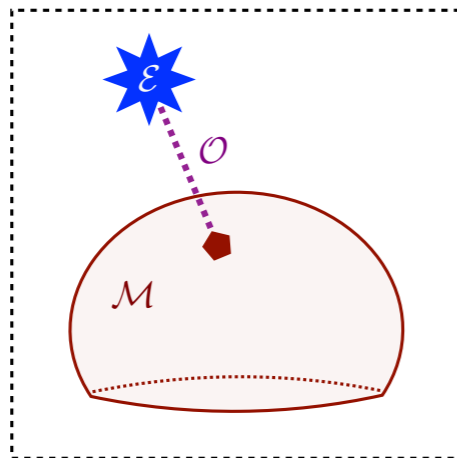
[image from [Wikipedia](#); see MITP Optimal Transport session by [Craig and Howard](#), June 2021]



From Manifest Geometry...



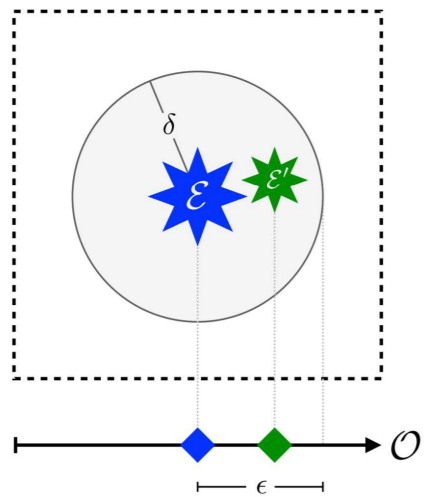
...to Emergent Geometry...



...to Hidden Geometry!

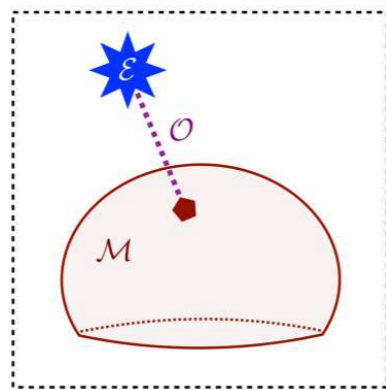
Six Decades of Collider Physics Translated into a New Geometric Language!

IRC Safety is smoothness in the space of events



Taming infinities

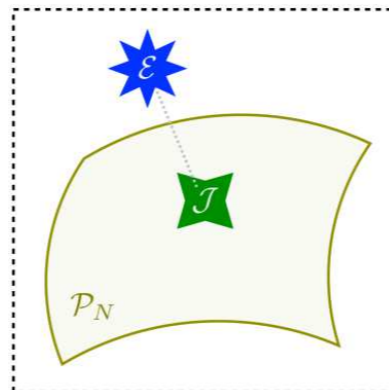
Event shapes are distances from events to manifolds.



$$O(\mathcal{E}) = \min_{\mathcal{E}' \in \mathcal{M}} \text{EMD}_{\beta, R}(\mathcal{E}, \mathcal{E}')$$

Event Shapes

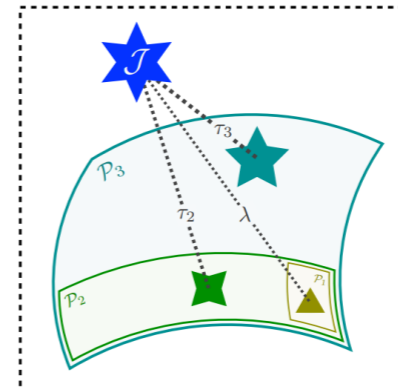
Jets are projections to few-particle manifolds.



$$J = \operatorname{argmin}_{\mathcal{E}' \in \mathcal{P}_N} \text{EMD}_{\beta, R}(\mathcal{E}, \mathcal{E}')$$

Jet Algorithms

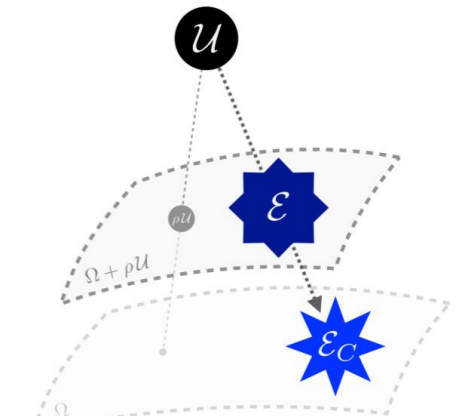
Substructure resolves emissions within the jet.



$$\tau(J) = \min_{\mathcal{E}' \in \mathcal{P}_N} \text{EMD}_{\beta}(\mathcal{J}, \mathcal{E}')$$

Jet Substructure

Pileup mitigation moves away from uniform radiation.



$$\mathcal{E}_C = \operatorname{argmin}_{\mathcal{E}'} \text{EMD}(\mathcal{E}, \mathcal{E}' + \rho \mathcal{U}).$$

Pileup



1962-1964

Infrared Safety
[Kinoshita, JMP 1962]
[Lee, Nauenberg, PR 1964]

1977

Thrust, Sphericity
[Farhi, PRL 1977]
[Georgi, Machacek, PRL 1977]

1993

k_T jet clustering
[Ellis, Soper, PRD 1993]
[Catani, Dokshitzer, Seymour, Webber, NPB 1993]

1997-1998

C/A jet clustering
[Wobisch, Wengler, 1998]
[Dokshitzer, Leder, Moretti, Webber, JHEP 1997]

2010-2015

N-(sub)jettiness, X Cone
[Stewart, Tackmann, Waalewijn, PRL 2010]
[Thaler, Van Tilburg, JHEP 2011]
[Stewart, Tackmann, Thaler, Vermilion, Wilkason, JHEP 2015]

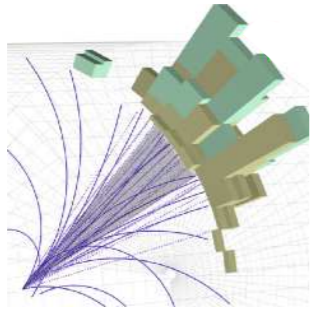
2014-2019

Constituent Subtraction
[Berta, Spousta, Miller, Leitner, JHEP 2014]
[Berta, Masetti, Miller, Spousta, JHEP 2019]

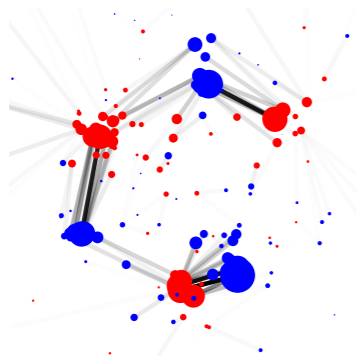
And many more!

[timeline from Eric Metodiev]

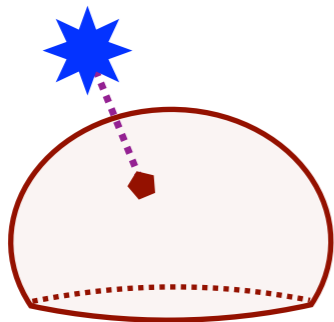
Outline



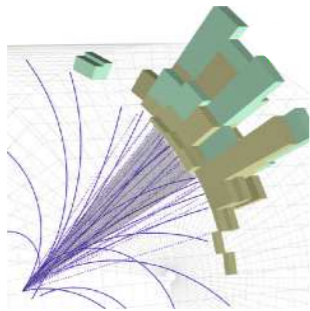
Going with the (Energy) Flow



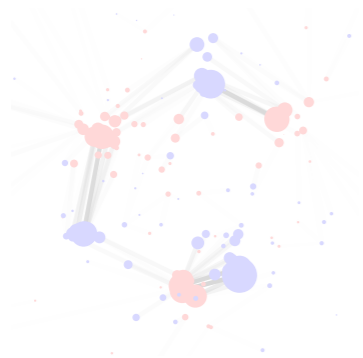
The Energy Mover's Distance



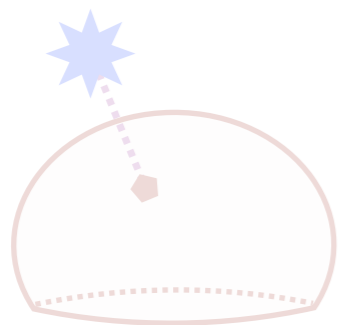
Revealing a Hidden Geometry



Going with the (Energy) Flow



The Energy Mover's Distance



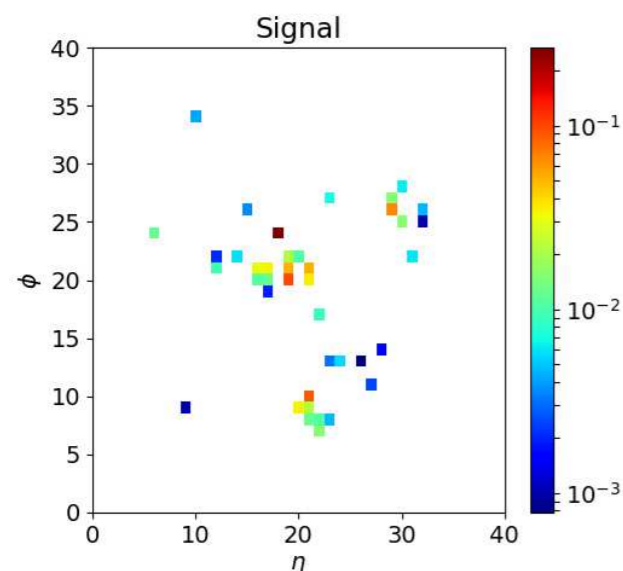
Revealing a Hidden Geometry

*Taking a step back to
supervised machine learning...*

Jet Representations

Pixelized Image

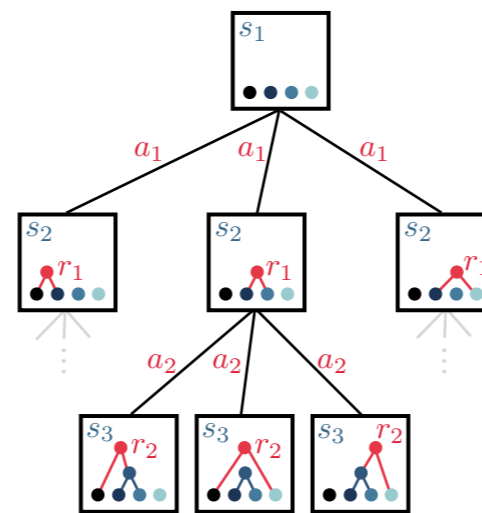
Calorimetry



[review in Kagan, [arXiv 2020](#)]

Hierarchical Tree

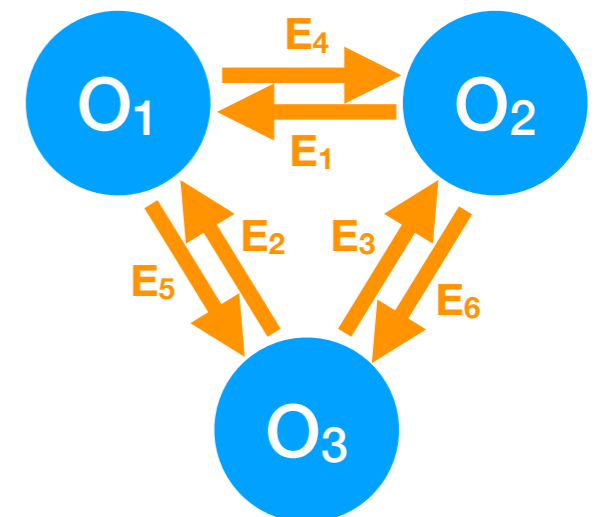
Binary Splittings



[e.g. Brehmer, Macaluso, Pappadopulo, Cranmer, [NeurIPS 2020](#)]

Graphs

Pairwise Interactions

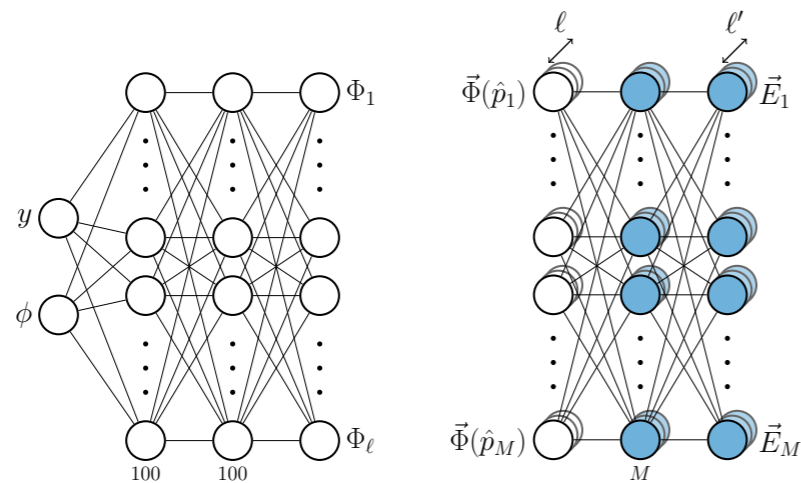


[e.g. Moreno, Cerri, Duarte, Newman, Nguyen, Periwal, Pierini, Serikova, Spiropulu, Vlimant, [EPJC 2020](#)]

*Imposes implicit **theoretical prior** (typically a good thing!)*
*Influences choice of **network architecture***

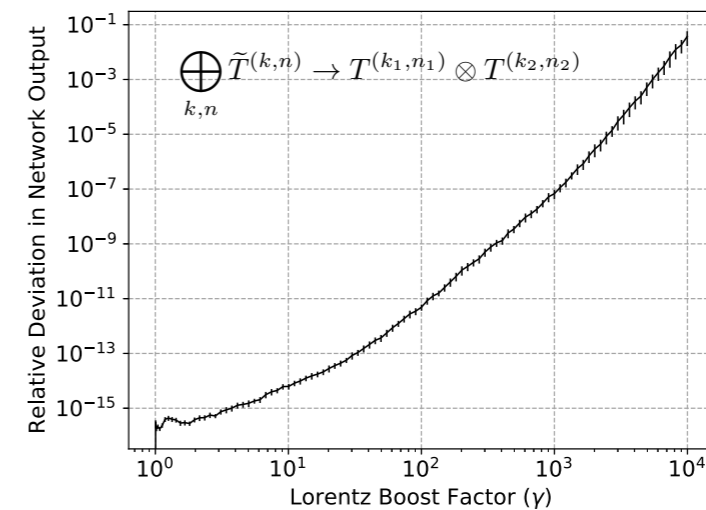
From Principles to Network Architectures

Permutation Equivariance



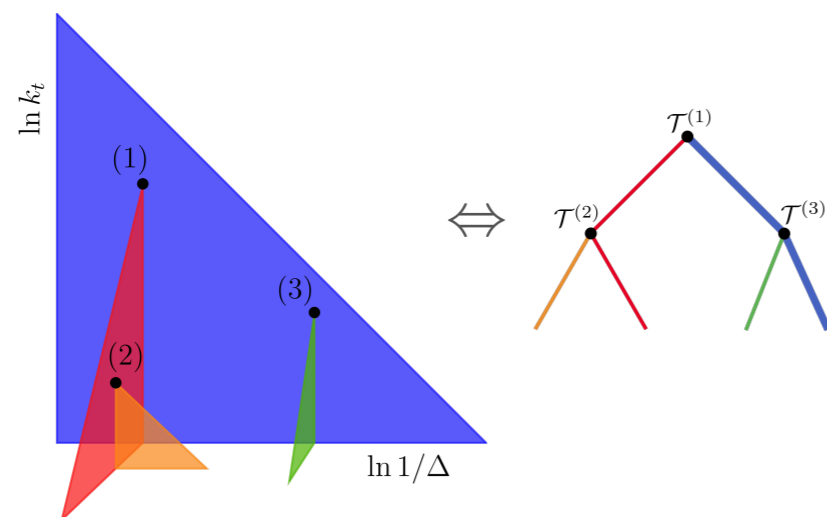
[Dolan, Ore, [PRD 2021](#)]

Lorentz Equivariance



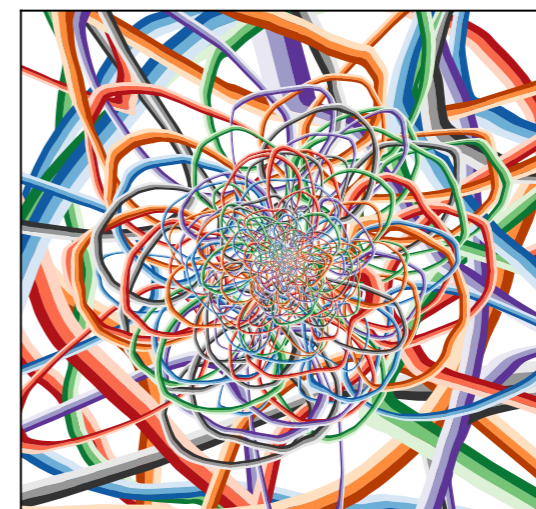
[Bogatskiy, Anderson, Offermann, Roussi, Miller, Kondor, [arXiv 2020](#)]

Lund Plane Emissions



[Dreyer, Qu, [JHEP 2021](#)]

Infrared and Collinear Safety

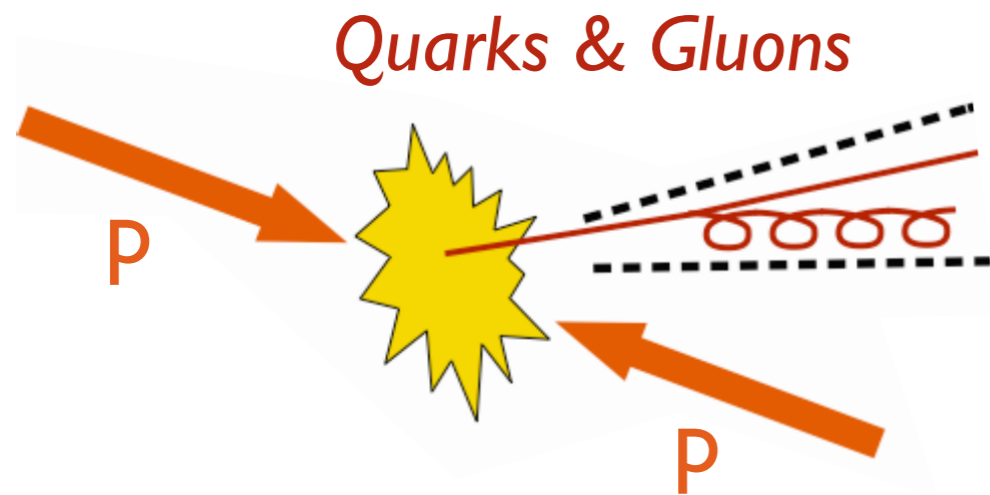


[Komiske, Metodiev, [JDT, JHEP 2019](#);
see also Konar, Ngairangbam, Spannowsky, [arXiv 2021](#)]

...

Energy Flow Representation

Emphasizes *infrared and collinear safety*

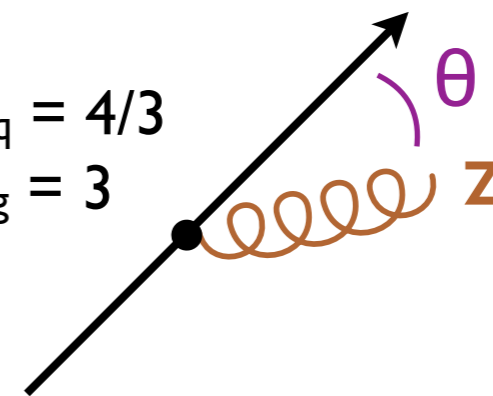


Altarelli-Parisi Splitting

Core prediction of **QCD**

$$C_q = 4/3$$

$$C_g = 3$$

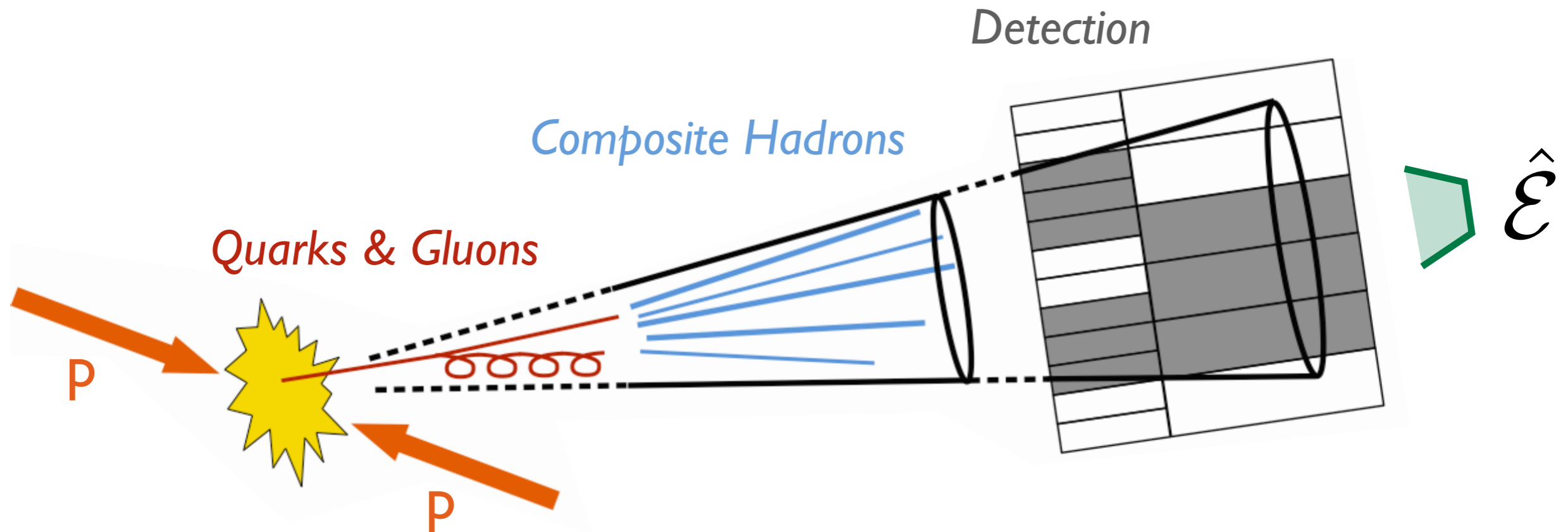


$$dP_{i \rightarrow ig} \simeq \frac{2\alpha_s}{\pi} C_i \underbrace{\frac{d\theta}{\theta}}_{\text{Collinear}} \underbrace{\frac{dz}{z}}_{\text{Soft}}$$

Energy Flow Representation

Emphasizes *infrared and collinear safety*

Theory



Energy Flow:

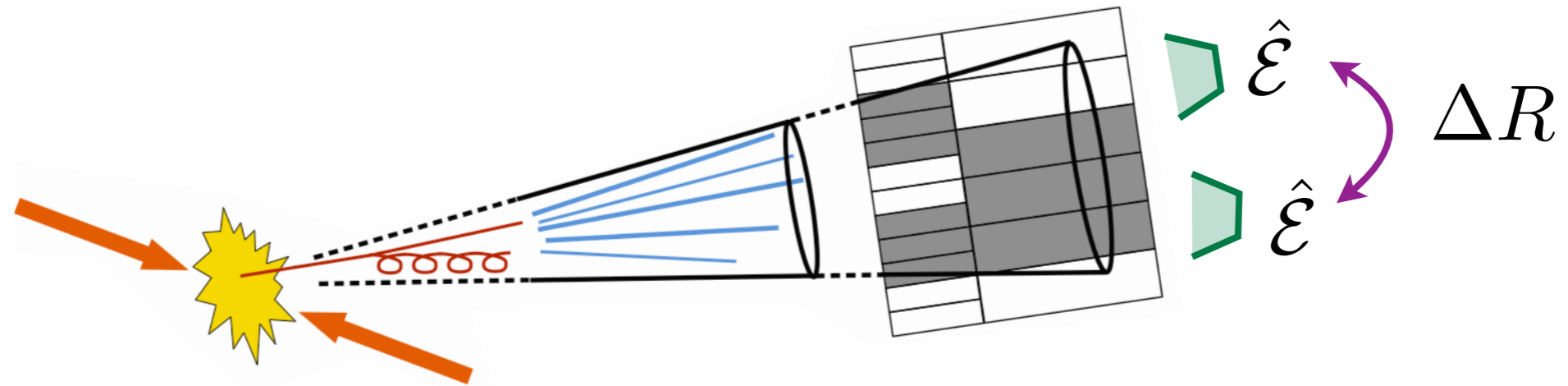
Robust to hadronization and detector effects
Well-defined for massless gauge theories

$$\hat{\mathcal{E}} \simeq \lim_{t \rightarrow \infty} \hat{n}_i T^{0i}(t, vt\hat{n})$$

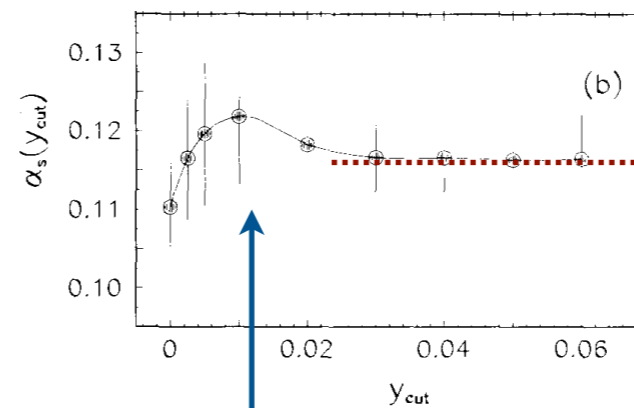
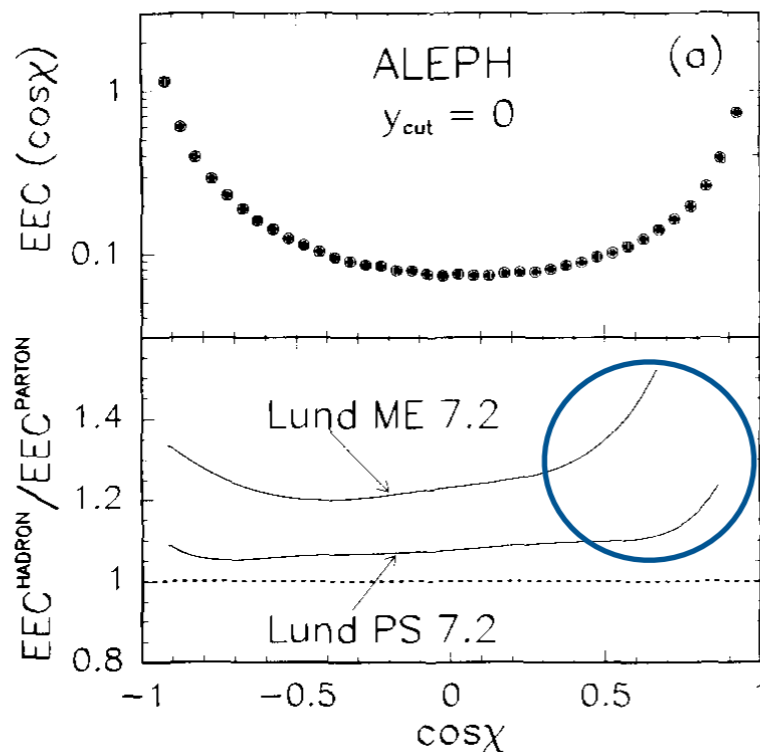
[see e.g. Sveshnikov, Tkachov, [PLB 1996](#); Hofman, Maldacena, [JHEP 2008](#); Mateu, Stewart, [JDT, PRD 2013](#); Belitsky, Hohenegger, Korchemsky, Sokatchev, Zhiboedov, [PRL 2014](#); Chen, Moul, Zhang, Zhu, [PRD 2020](#)]



Energy-Energy Correlators



A long history in probing collinear dynamics of QCD

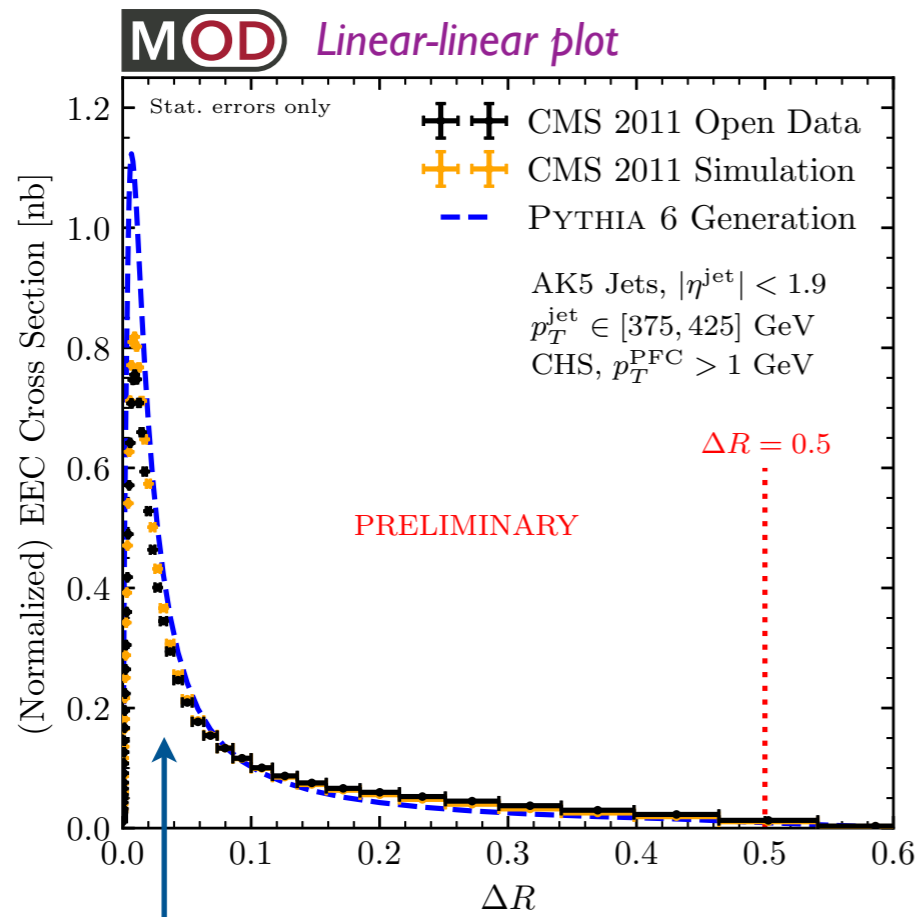


Extracting the strong coupling constant

Theoretical challenges with small angle (collinear) limit

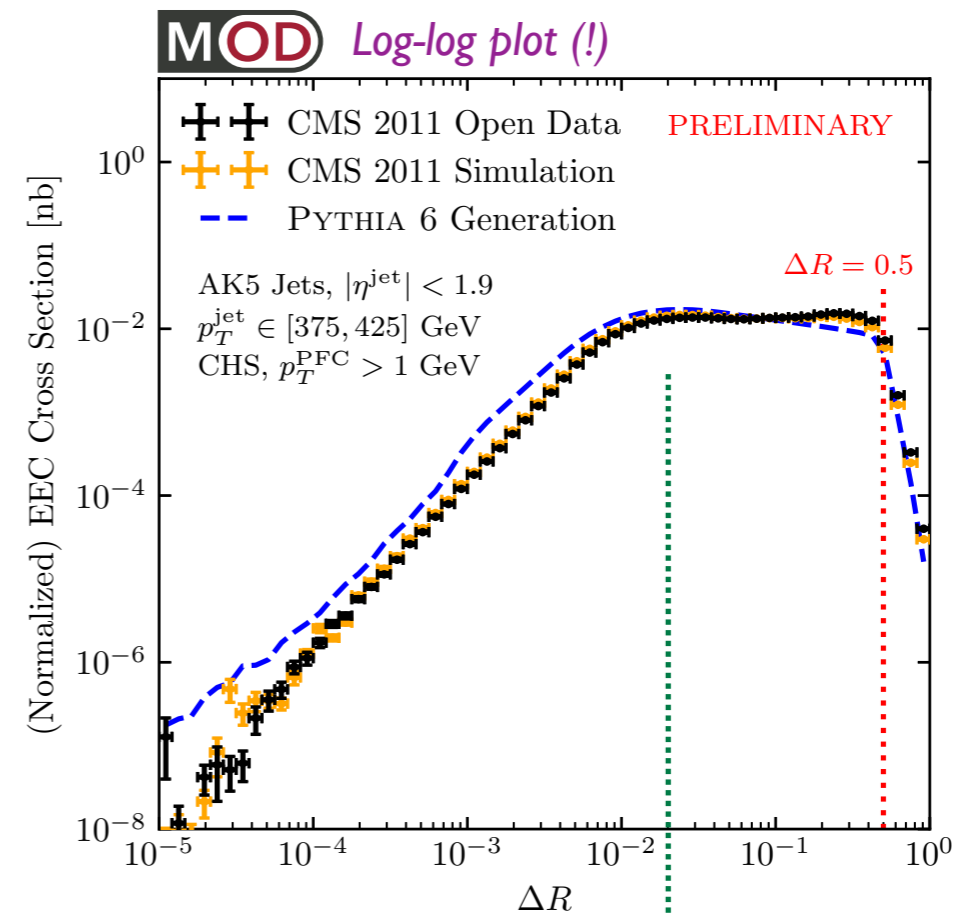
[Basham, Brown, Ellis, Love, *PRL* 1978; ALEPH, *PLB* 1991; see Chen, Mout, Zhang, Zhu, *PRD* 2020]

QCD Phase Transition in Jets?



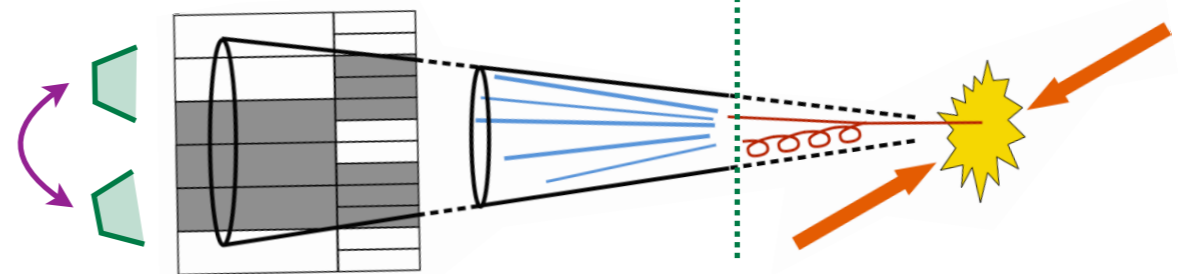
Are we learning something about small angle limit of QCD?

First Jet EEC Plot from the LHC (!)



Hadronic Phase

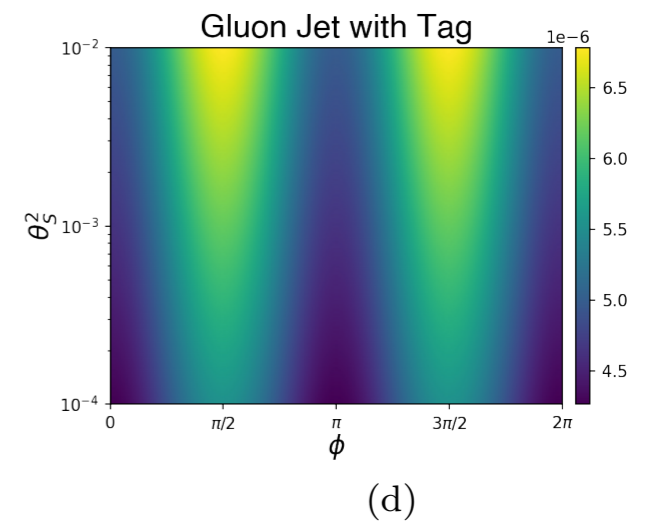
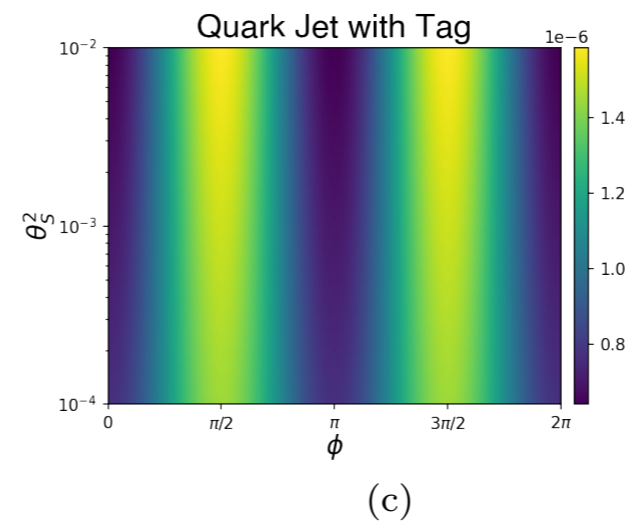
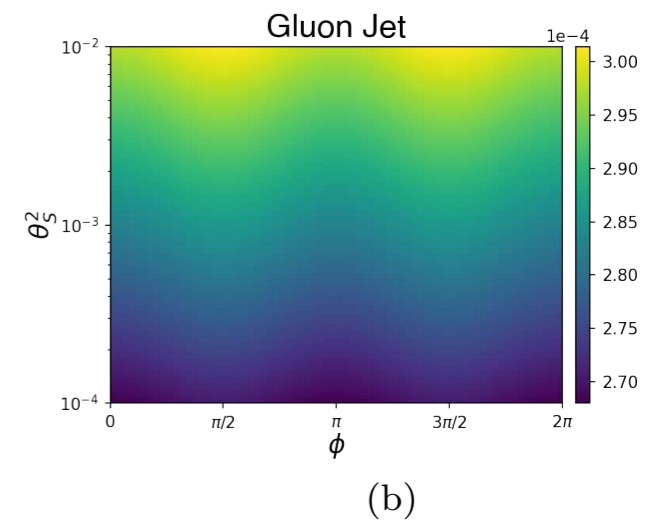
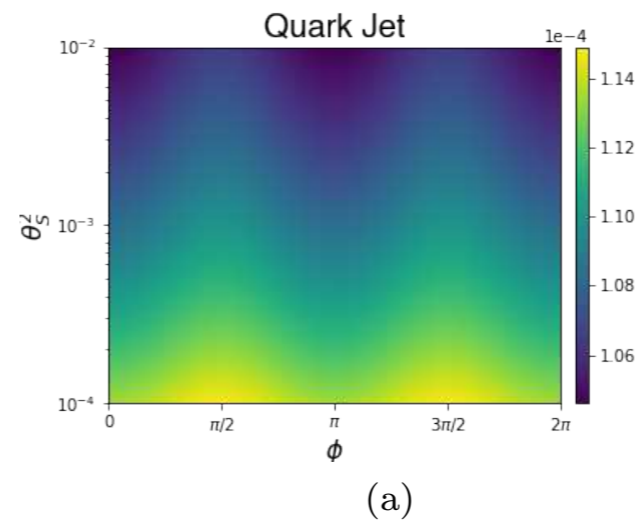
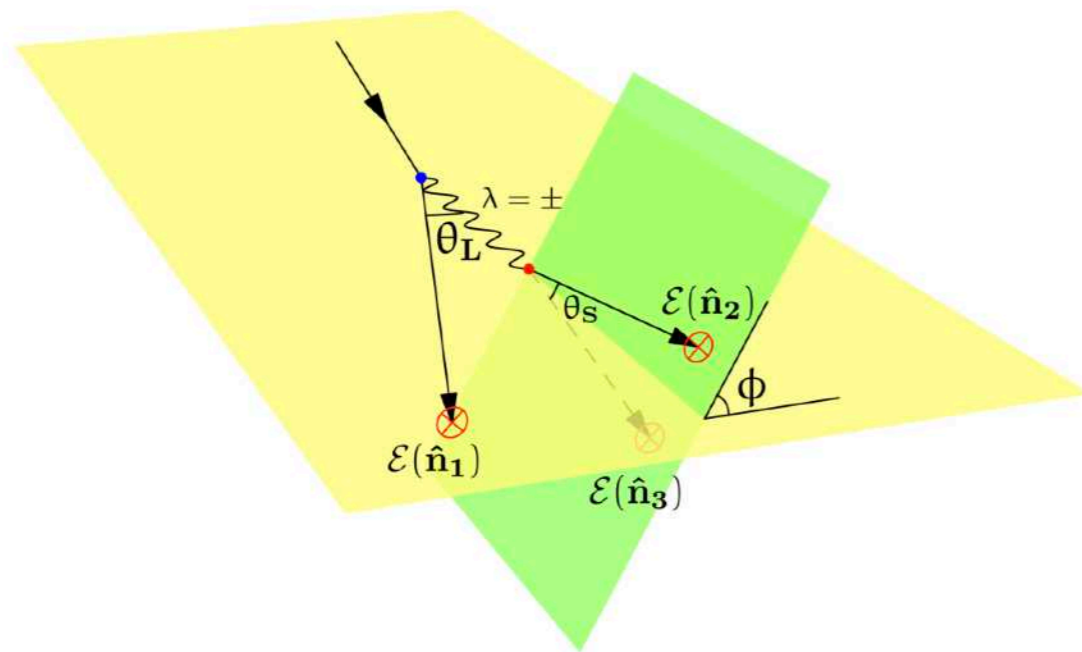
Partonic Phase



[Komiske, Mout, JDT, Zhu, in progress; see talks by Mout, BOOST 2019, BOOST 2020]



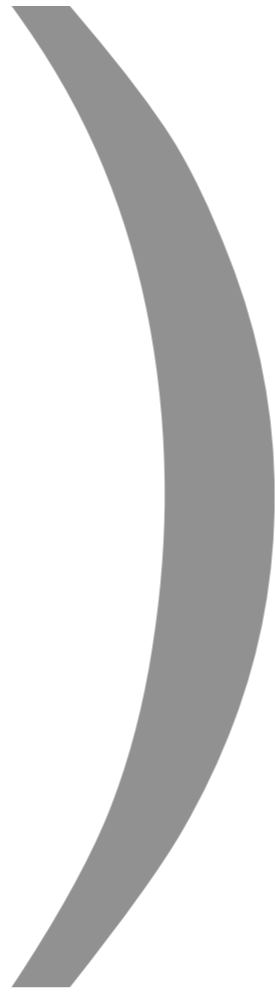
Fun with Three Point Correlators



(with help from b-tagging)

Extracting quantum interference effects of spinning gluons!

[Chen, Moul, Zhu, [PRL 2021](#)]

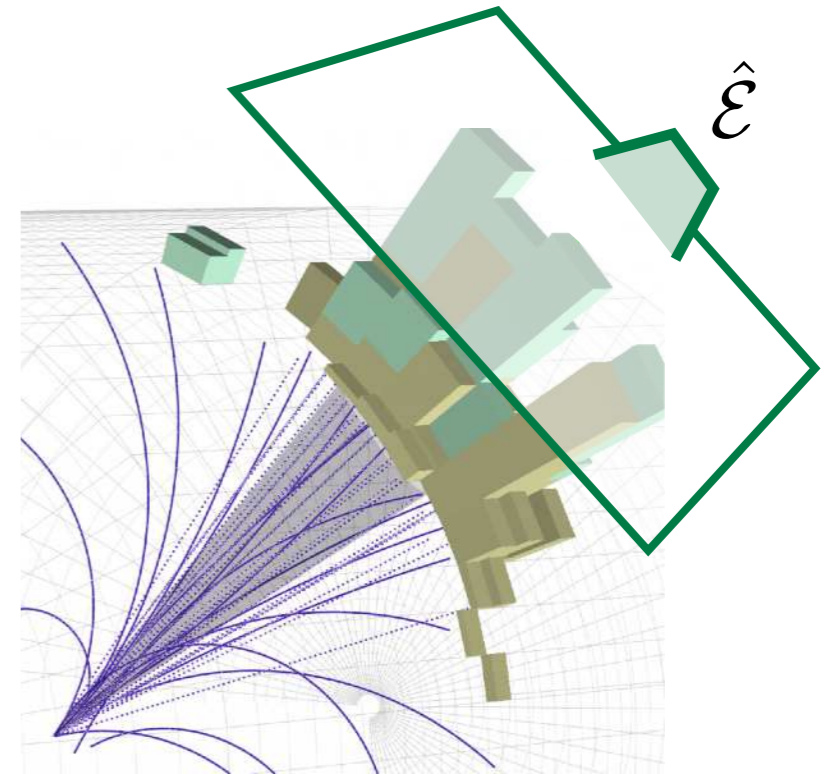


Jets as **Weighted Point Clouds**

- **Energy-Weighted Directions**

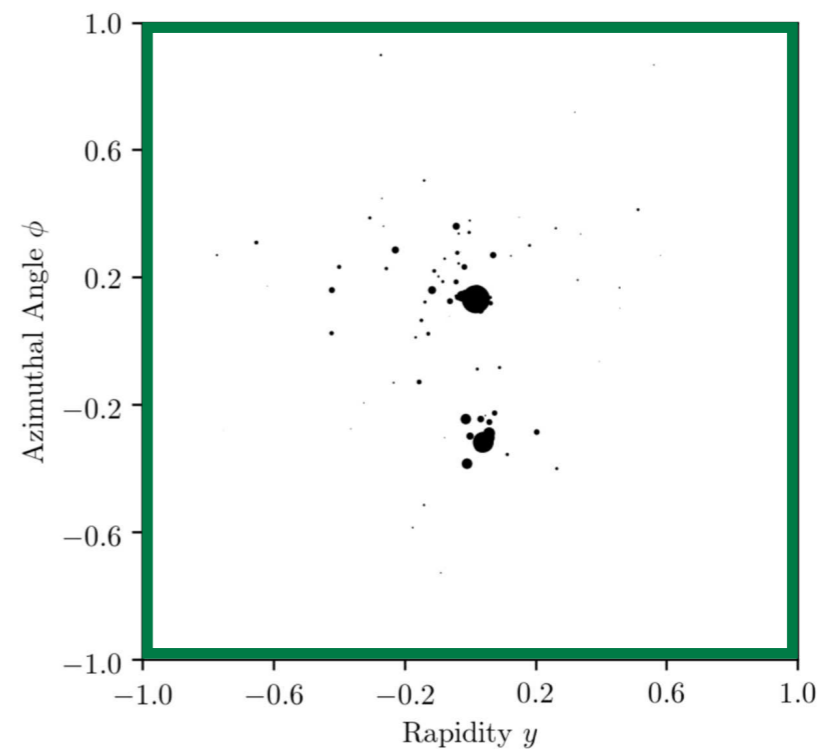
$$\vec{p} = \left\{ \underset{\substack{\uparrow \\ \text{Energy}}}{E}, \underbrace{\hat{n}_x, \hat{n}_y, \hat{n}_z}_{\text{Direction}} \right\}$$

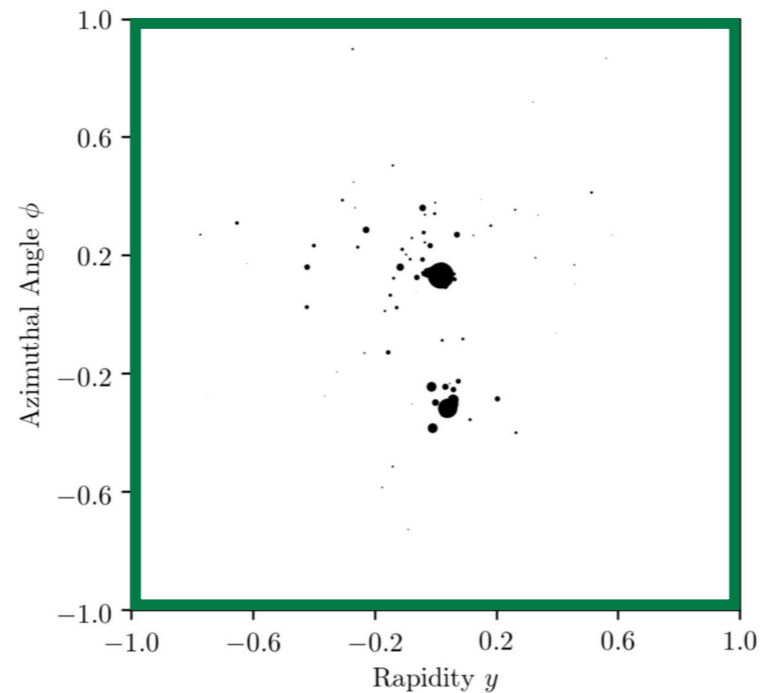
(suppressing “unsafe” charge/flavor information)



- Equivalently: **Energy Density**

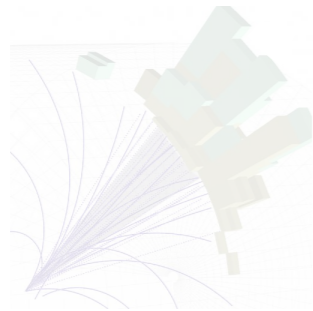
$$\rho(\hat{n}) = \sum_{i \in \mathcal{J}} \underset{\substack{\uparrow \\ \text{Energy}}}{E_i} \delta^{(2)}(\hat{n} - \underset{\substack{\uparrow \\ \text{Direction}}}{\hat{n}_i})$$



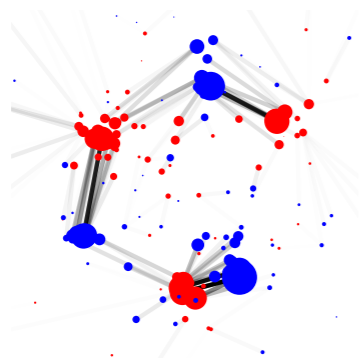


*When restricted to IRC safe information,
jets/events are naturally represented
as energy densities*

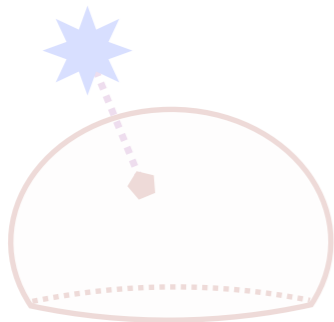
There is no obvious way to include unsafe information in this picture, since flow of charge/flavor is theoretically delicate, though interesting to consider



Going with the (Energy) Flow



The Energy Mover's Distance



Revealing a Hidden Geometry

If you ask your local computational geometry expert how to process densities...

The Earth Mover's Distance

Optimal Transport:

[Peleg, Werman, Rom, [IEEE 1989](#);
Rubner, Tomasi, Guibas, [ICCV 1998](#), [ICJV 2000](#);
Pele, Werman, [ECCV 2008](#); Pele Taskar, [GSI 2013](#)]

Minimum “work” (stuff x distance) to make one distribution look like another distribution



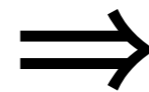
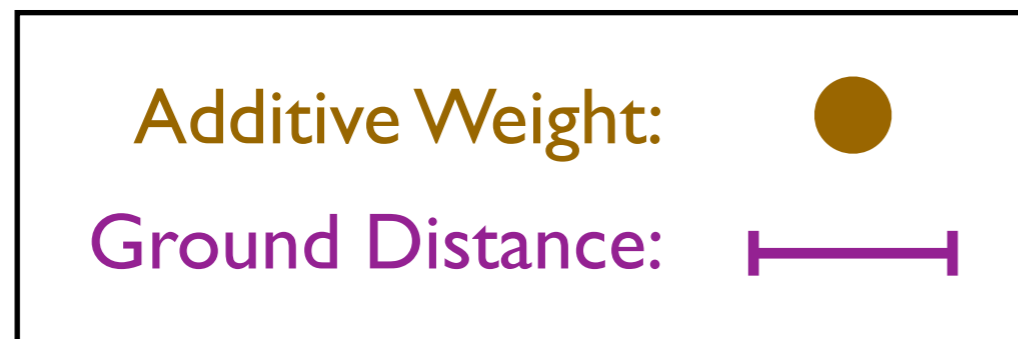
[h/t Niles-Weed, [ML4jets 2020](#); Monge, 1781; Kantorovich, 1939; Vaserštejn, 1969; [Wikipedia](#)]

The Earth Mover's Distance

Optimal Transport:

[Peleg, Werman, Rom, [IEEE 1989](#);
Rubner, Tomasi, Guibas, [ICCV 1998](#), [ICJV 2000](#);
Pele, Werman, [ECCV 2008](#); Pele Taskar, [GSI 2013](#)]

Minimum “work” (stuff x distance) to make
one distribution look like another distribution



Distance Between
Distributions



[h/t Niles-Weed, [ML4jets 2020](#); Monge, 1781; Kantorovich, 1939; Vaseršteĭn, 1969; [Wikipedia](#)]

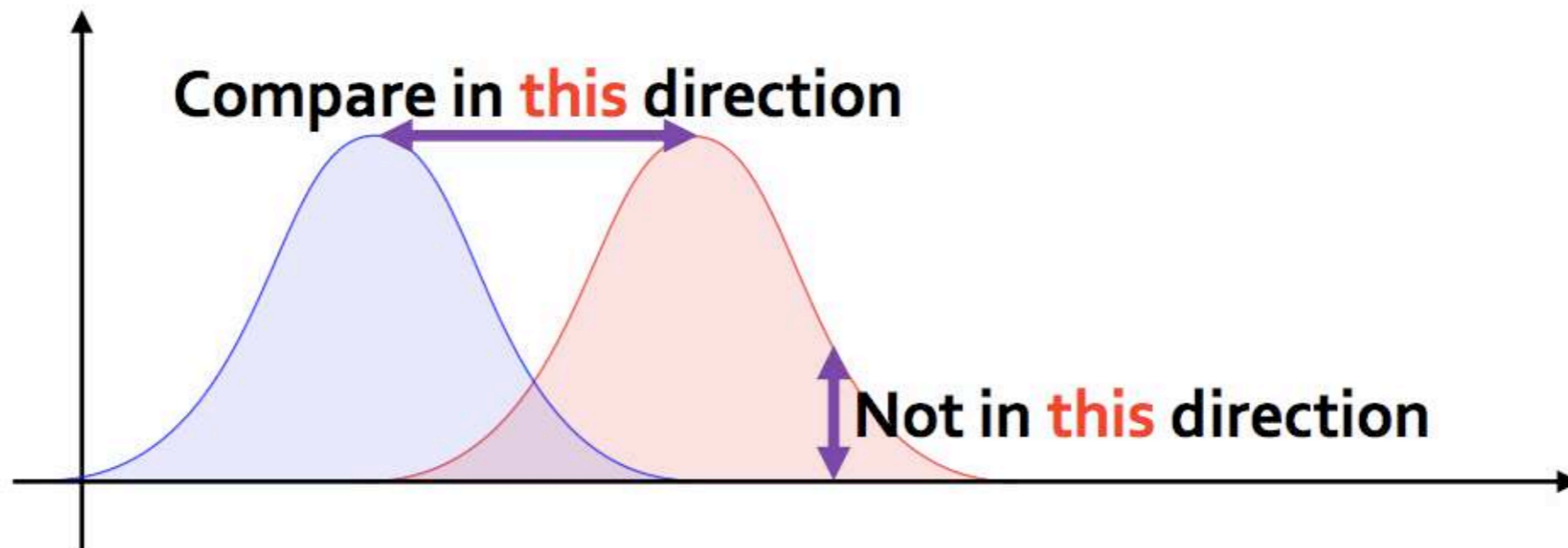
The Earth Mover's Distance

Optimal Transport:

[Peleg, Werman, Rom, [IEEE 1989](#);
Rubner, Tomasi, Guibas, [ICCV 1998](#), [ICJV 2000](#);
Pele, Werman, [ECCV 2008](#); Pele Taskar, [GSI 2013](#)]

Minimum “work” (stuff x distance) to make
one distribution look like another distribution

“Horizontal” comparison (EMD) yields better
dynamic range than “vertical” comparison (e.g. KL)

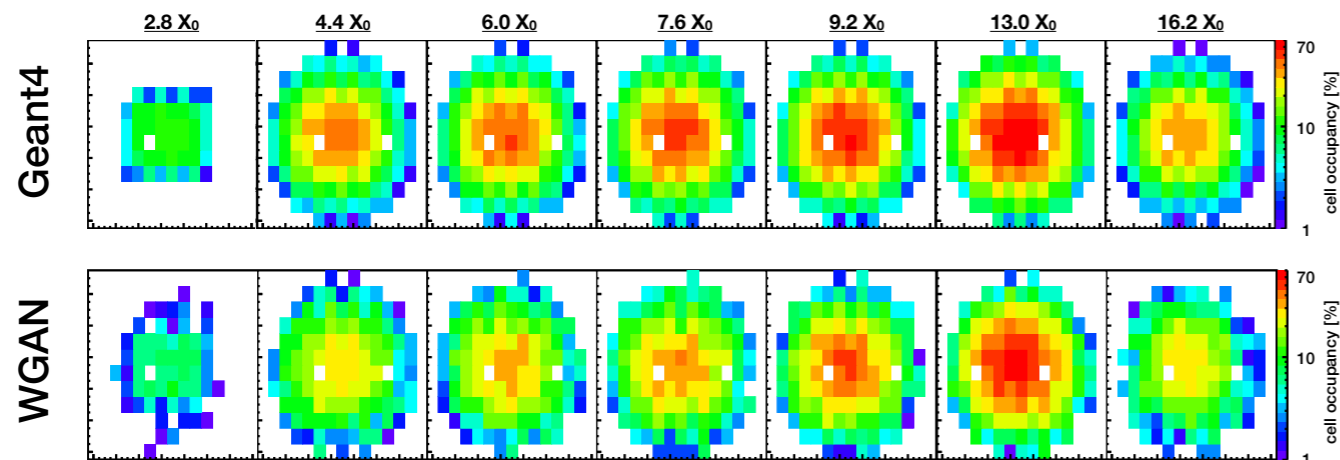


[figure from Kun, [Math n Programming](#)]

[h/t Niles-Weed, [ML4jets 2020](#); Monge, 1781; Kantorovich, 1939; Vaserštejn, 1969; [Wikipedia](#)]

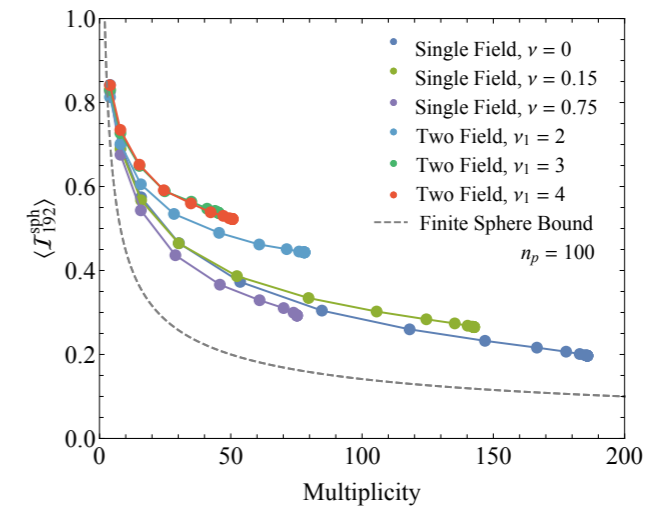
Wasserstein in HEP

Generative Modeling



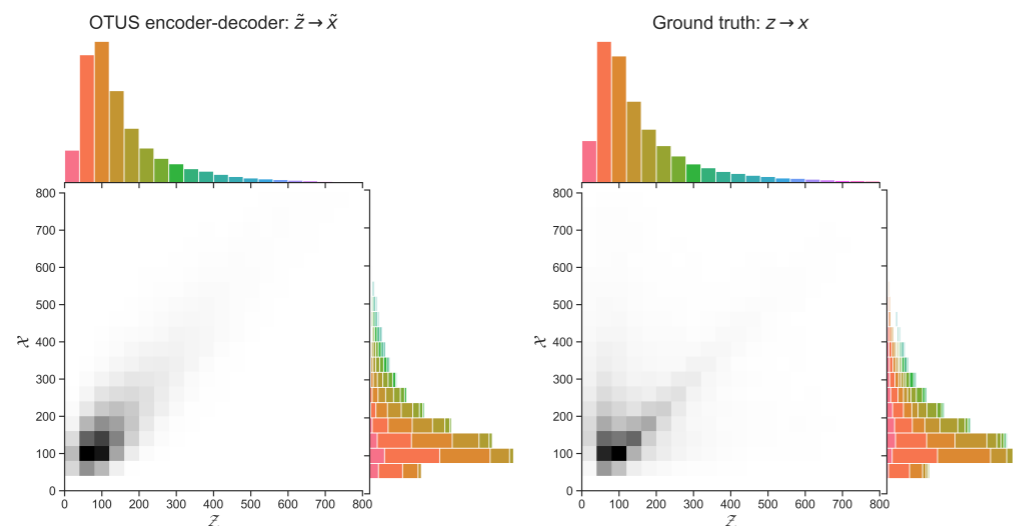
[Erdmann, Geiger, Glombitza, Schmidt, [CSBS 2018](#); Erdmann, Glombitza, Quast, [CSBS 2019](#); Chekalina, Orlova, Ratnikov, Ulyanov, Ustyuzhanin, Zakharov, [CHEP 2018](#)]

BSM Characterization



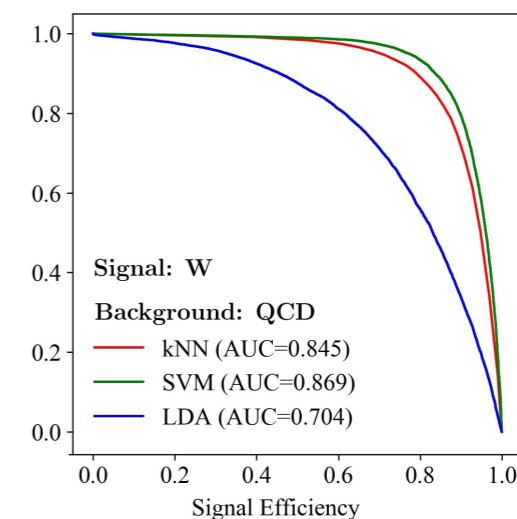
[Cesarotti, Reece, Strassler, [JHEP 2021](#), [arXiv 2020](#)]

Estimated Simulation/Unfolding



[Howard, Mandt, Whiteson, Yang, [arXiv 2021](#)]

Jet Classification

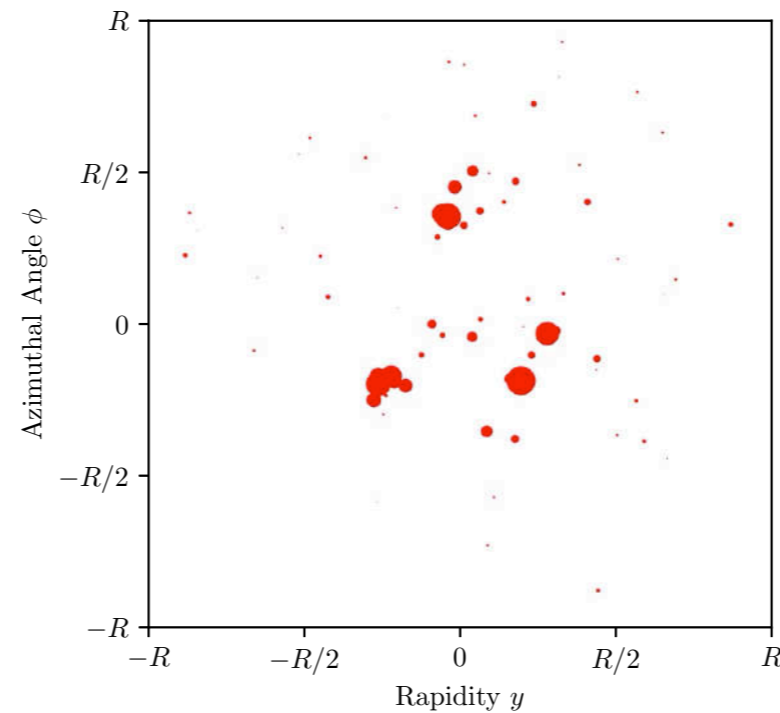


[Cai, Cheng, Craig, Craig, [PRD 2020](#)]

...

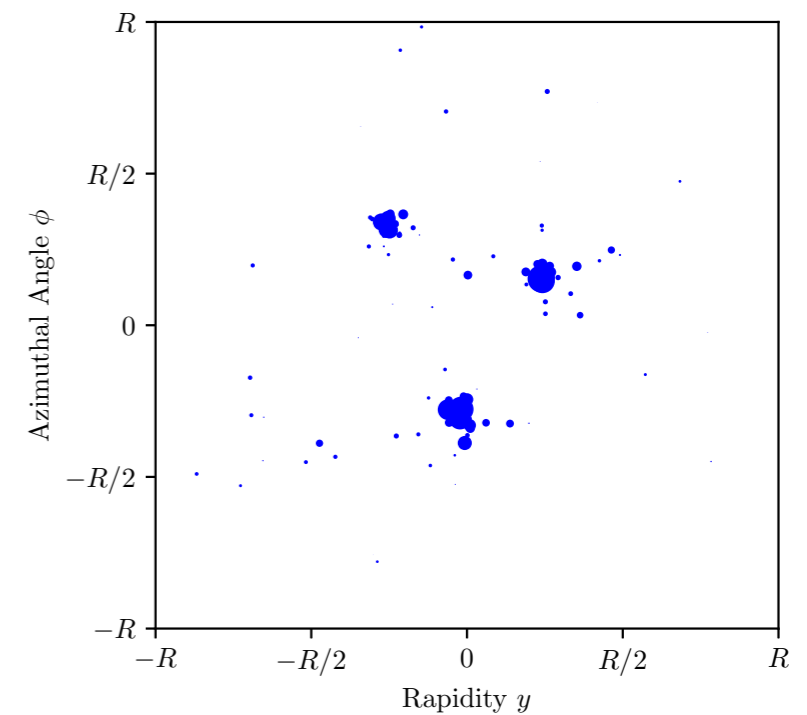
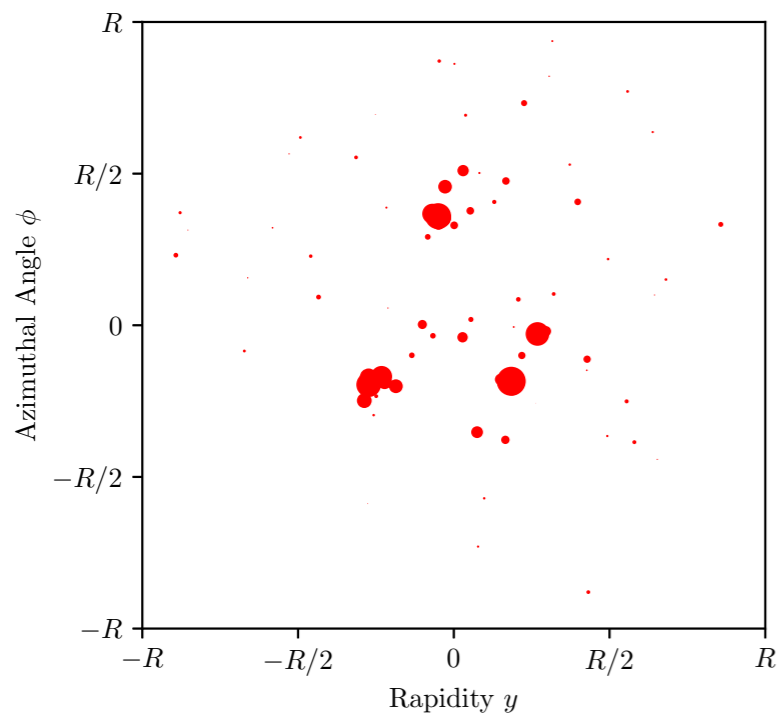
Similarity of Two Energy Flows?

$$\mathcal{E}(\hat{n}) = \sum_i E_i \delta(\hat{n} - \hat{n}_i)$$



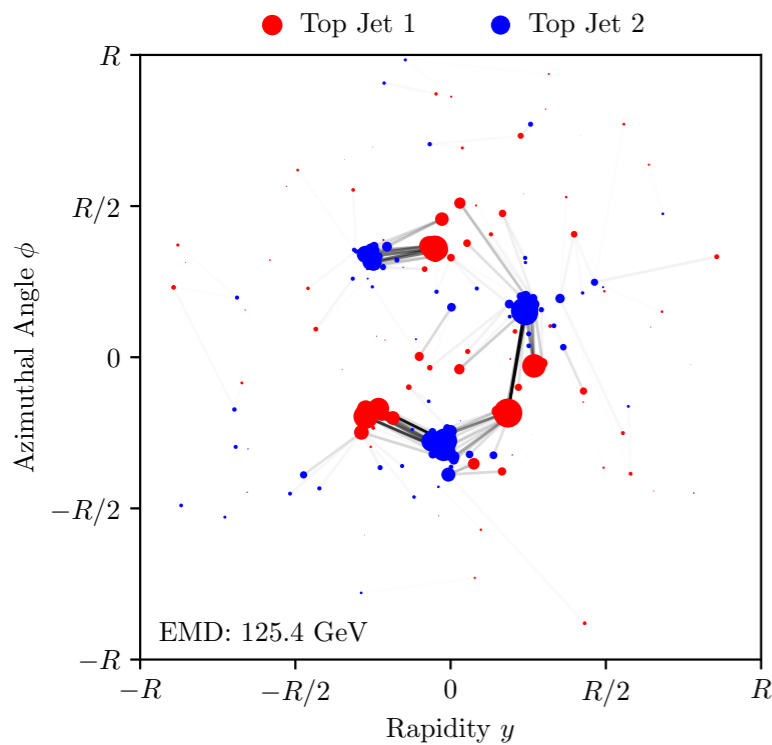
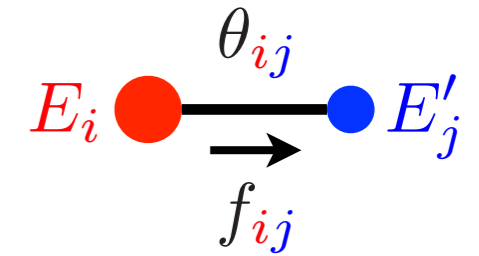
Optimal Transport:

Earth Mover's Distance
a.k.a. *1-Wasserstein metric*



[Komiske, Metodiev, JDT, [PRL 2019](#); code at Komiske, Metodiev, JDT, [energyflow.network](#)]

The Energy Mover's Distance



Optimal transport between energy flows...

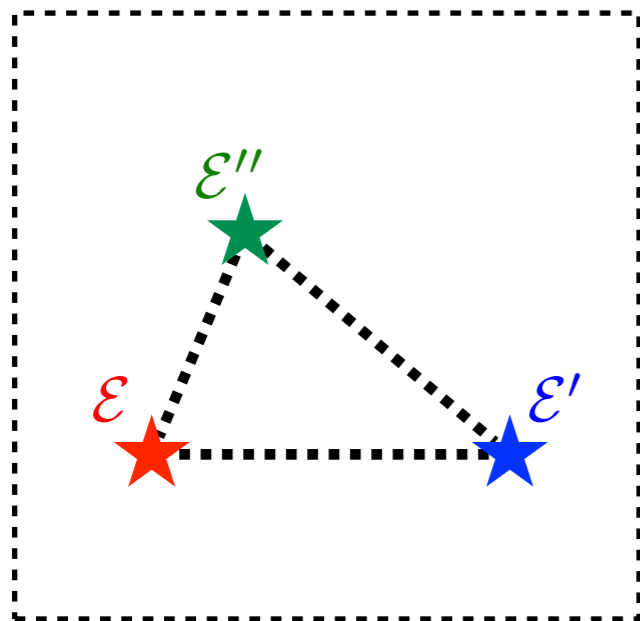
$$\text{EMD}(\mathcal{E}, \mathcal{E}') = \min_{\{f\}} \underbrace{\sum_i \sum_j f_{ij} \frac{\theta_{ij}}{R}}_{\text{Cost to move energy}} + \underbrace{\left| \sum_i E_i - \sum_j E'_j \right|}_{\text{Cost to create energy}}$$

↑
in GeV

...defines a metric on the space of events

$$0 \leq \text{EMD}(\mathcal{E}, \mathcal{E}') \leq \text{EMD}(\mathcal{E}, \mathcal{E}'') + \text{EMD}(\mathcal{E}', \mathcal{E}'')$$

(assuming $R \geq \theta_{\max}/2$, i.e. $R \geq$ jet radius for conical jets)



[Komiske, Metodiev, JDT, PRL 2019;

see also Pele, Werman, ECCV 2008; Pele, Taskar, GSI 2013;

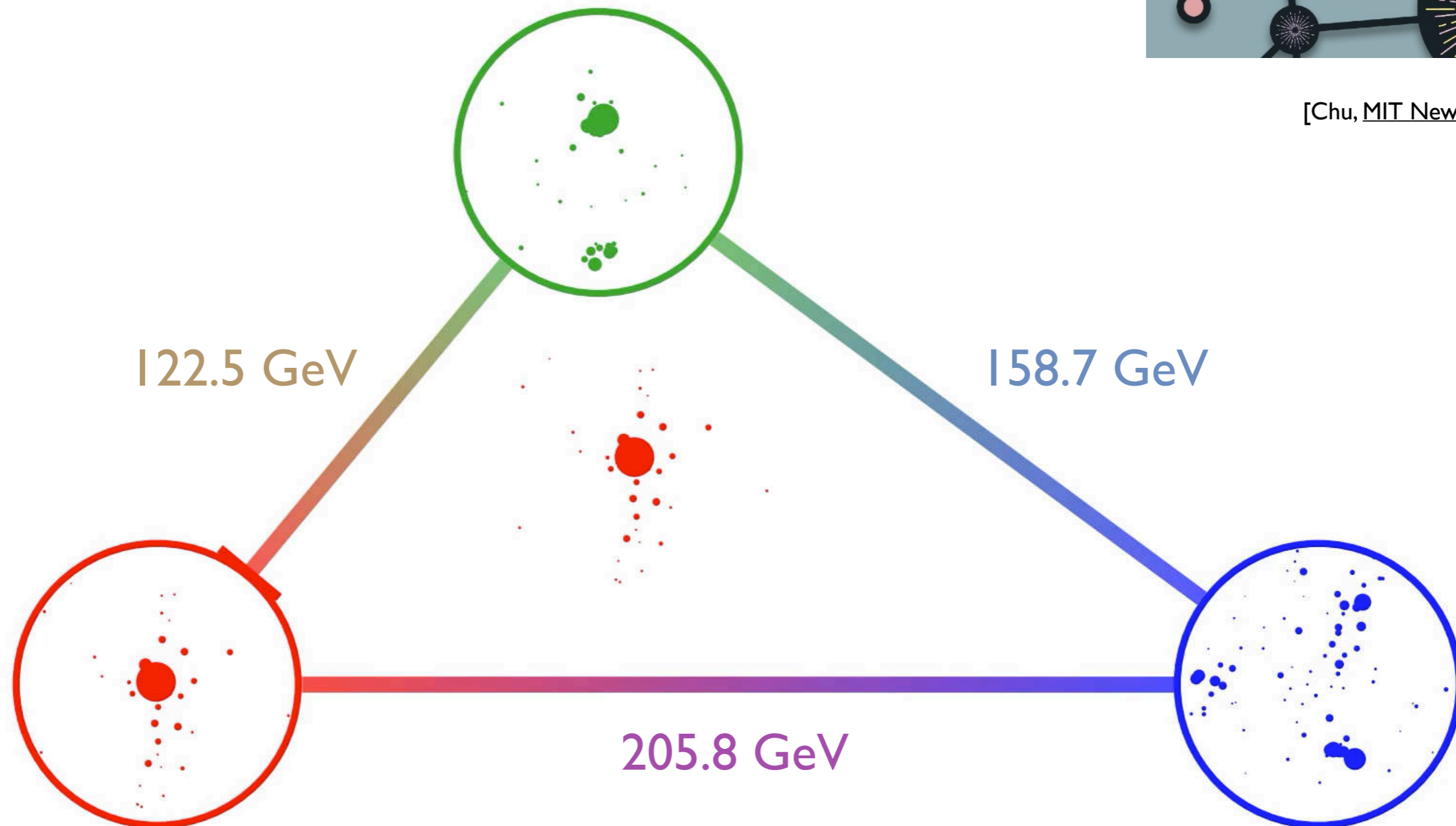
[see flavored variant in Crispim Romão, Castro, Milhano, Pedro, Vale, EPJC 2021]

[see computational speed up in Cai, Cheng, Craig, Craig, PRD 2020]

Similarity of Three Energy Flows?



[Chu, [MIT News July 2019](#)]



[Komiske, Metodiev, JDT, [PRL 2019](#); code at Komiske, Metodiev, JDT, [energyflow.network](#);
see alternative graph network approach in Mullin, Pacey, Parker, White, Williams, [JHEP 2021](#)]

Dimensionality of Space of Jets



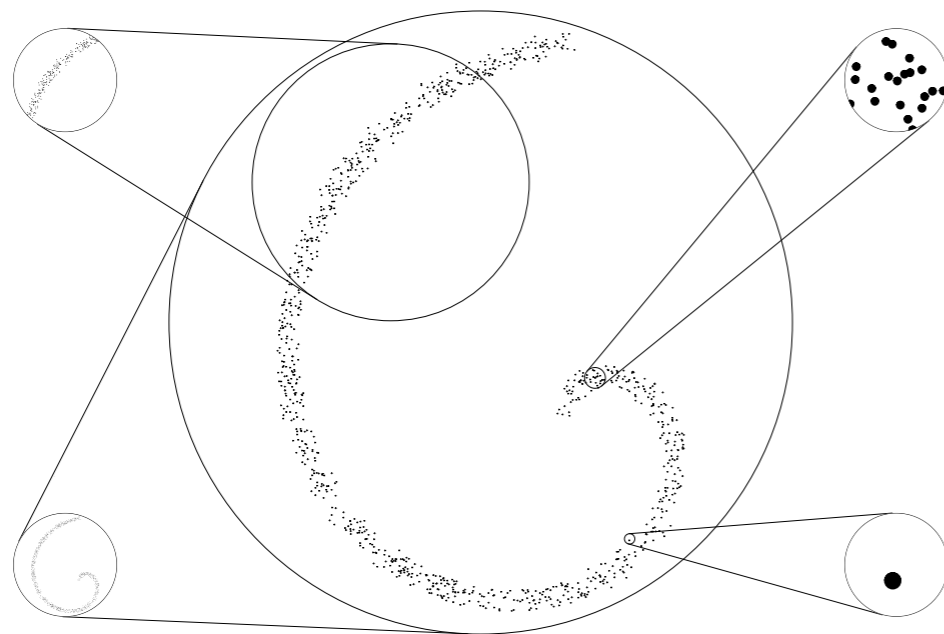
$$N_{\text{neighbors}}(r) \sim r^{\text{dim}}$$

$$\Rightarrow \text{dim}(r) \sim r \frac{\partial}{\partial r} \ln N_{\text{neighbors}}(r)$$

[Grassberger, Procaccia, PRL 1983; Kégl, NIPS 2002]

dim ≈ 1

dim ≈ 2

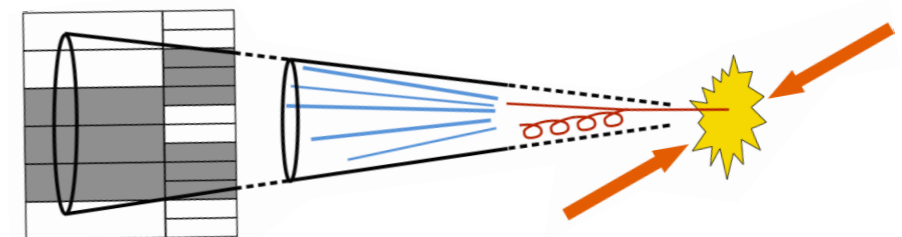
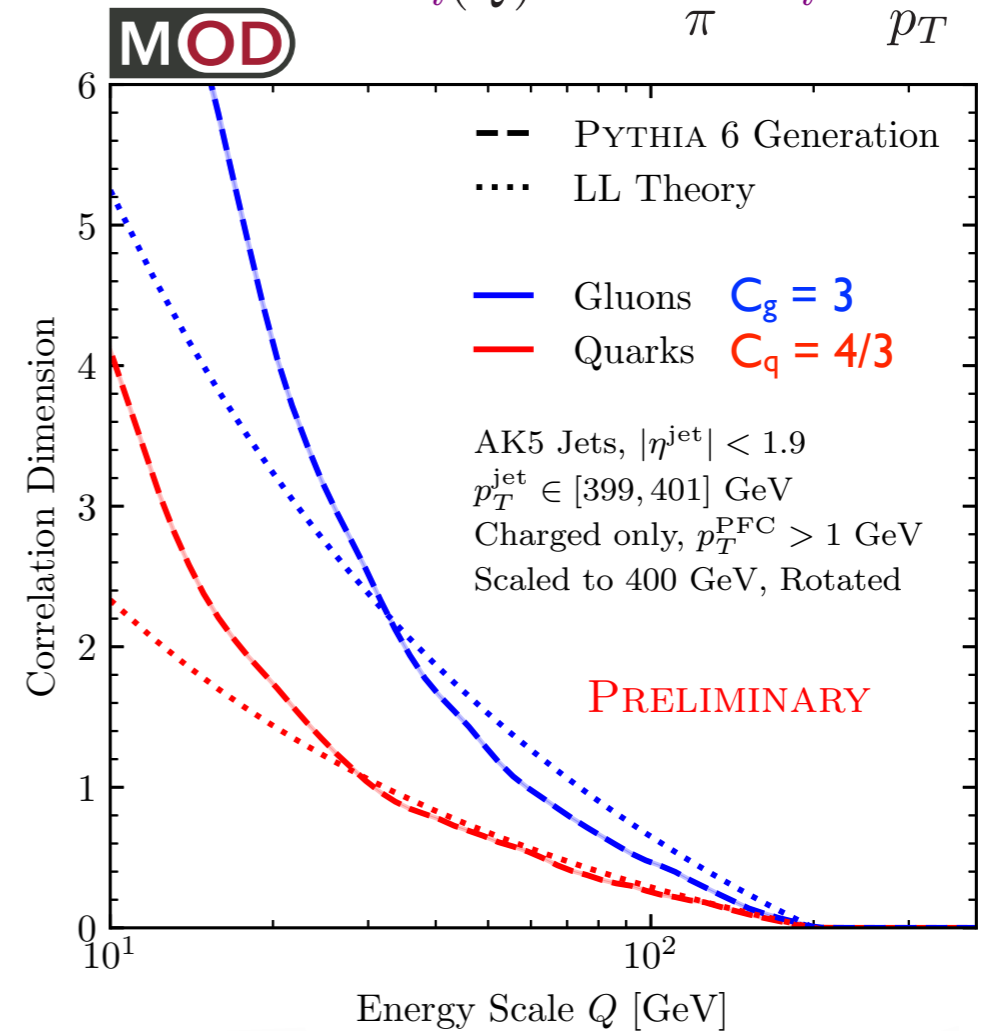


dim ≈ 2

dim ≈ 0

(eventually 0)

$$\text{dim}_i(Q) \simeq -\frac{8\alpha_s}{\pi} C_i \ln \frac{Q}{p_T}$$



Dimensionality of Space of Jets



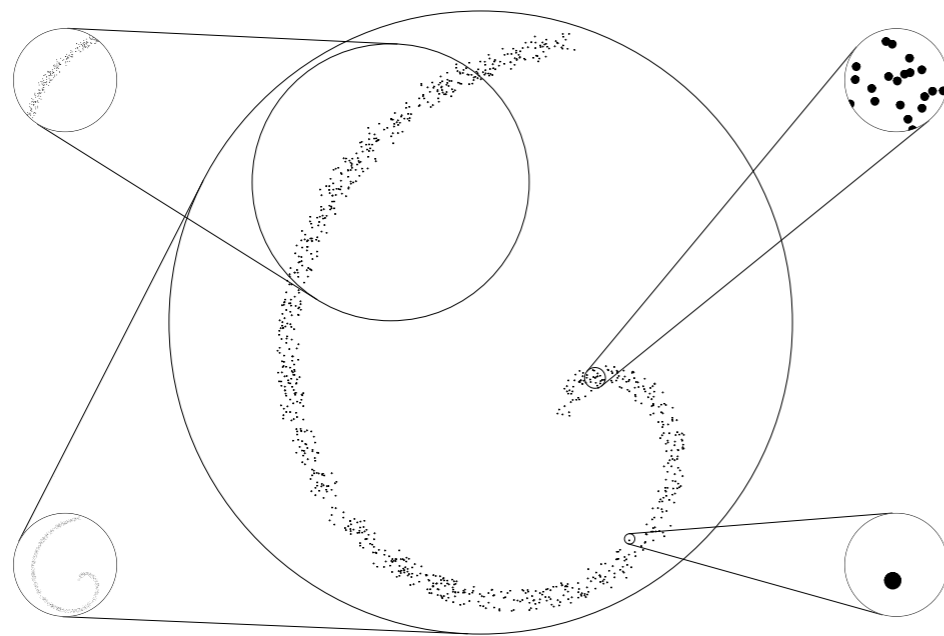
$$N_{\text{neighbors}}(r) \sim r^{\text{dim}}$$

$$\Rightarrow \text{dim}(r) \sim r \frac{\partial}{\partial r} \ln N_{\text{neighbors}}(r)$$

[Grassberger, Procaccia, PRL 1983; Kégl, NIPS 2002]

dim ≈ 1

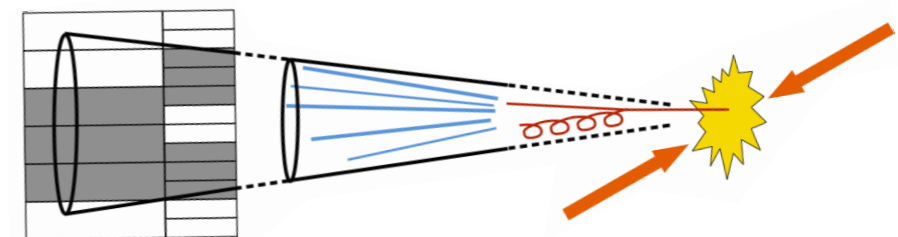
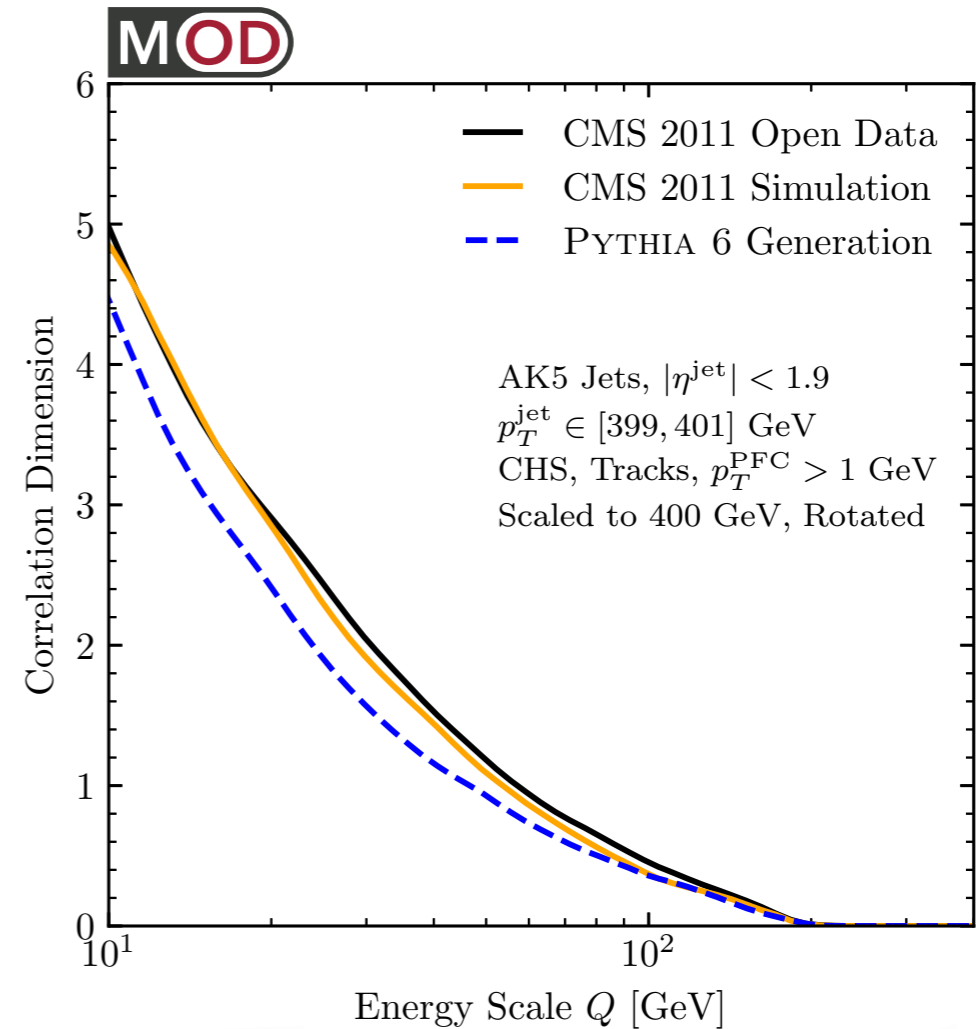
dim ≈ 2



dim ≈ 2

dim ≈ 0

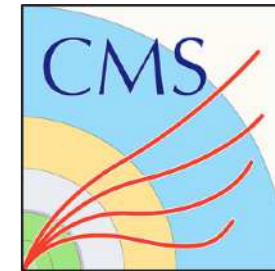
(eventually 0)



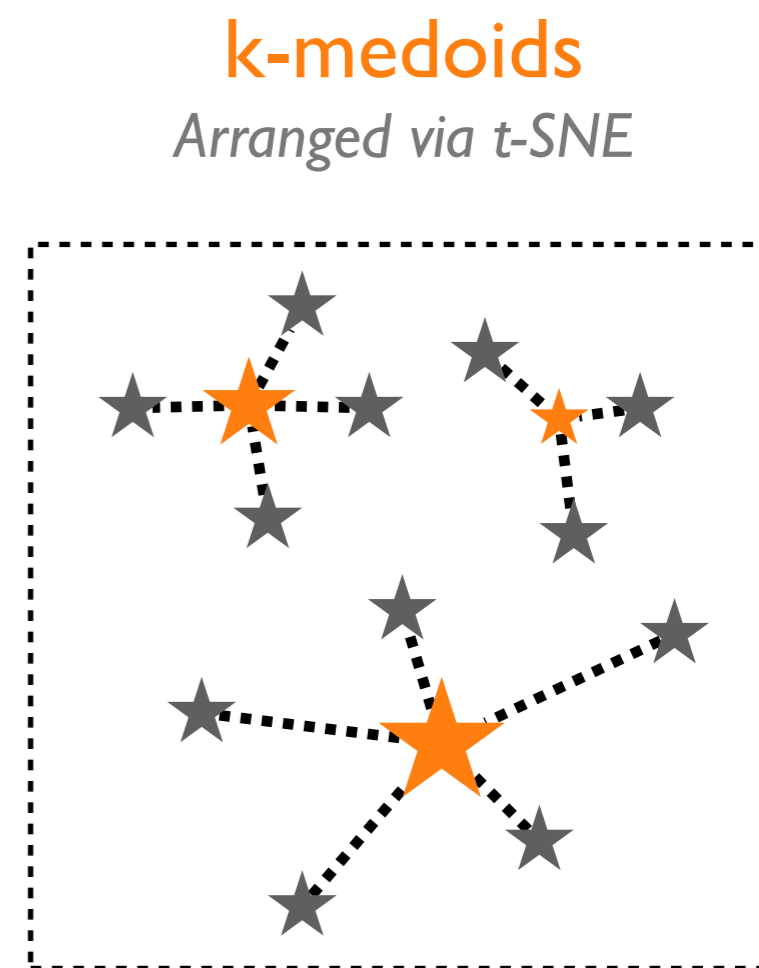
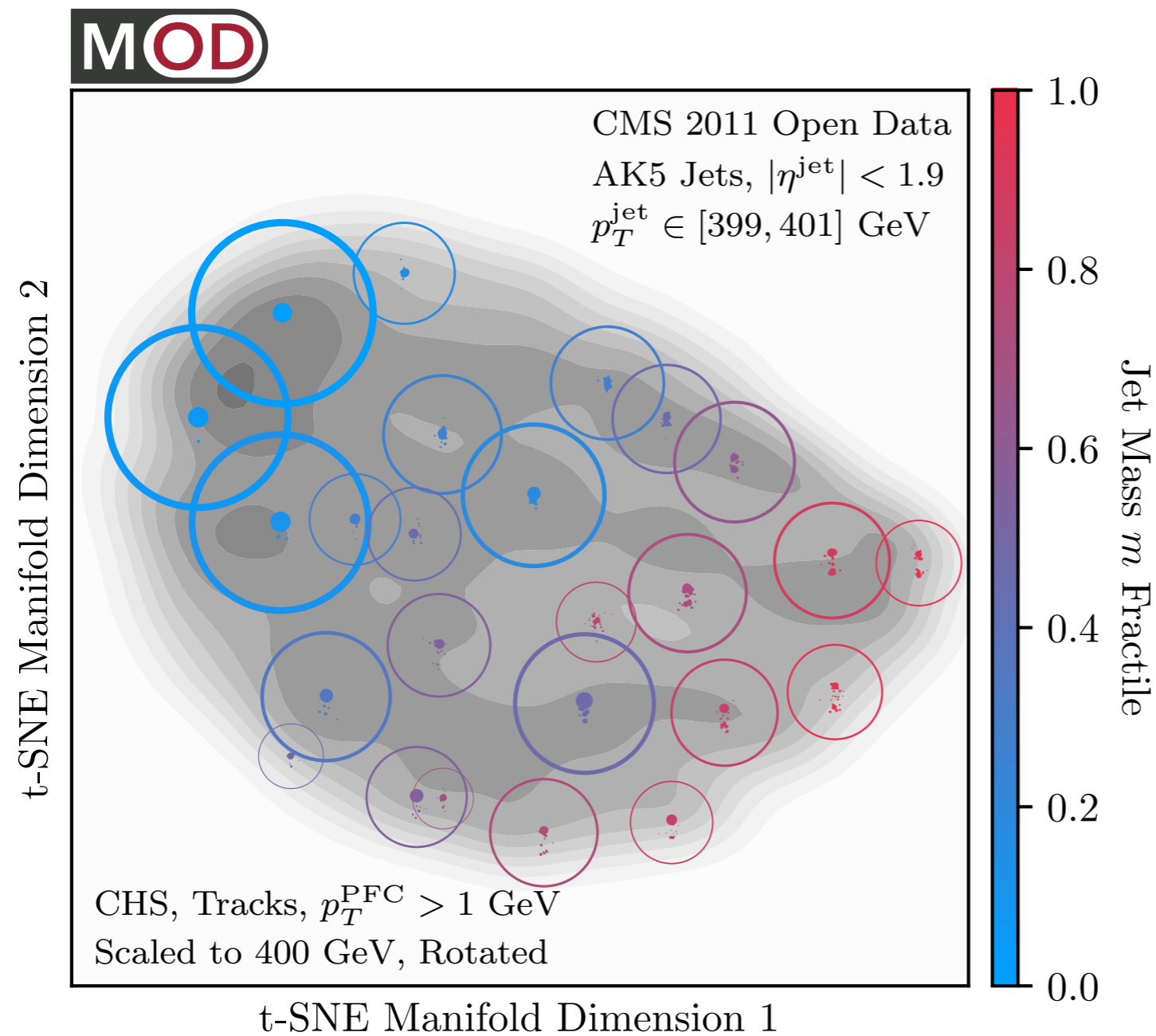
[Komiske, Mastandrea, Metodiev, Naik, JDT, PRD 2020; using CMS Open Data]



Most Representative Jets

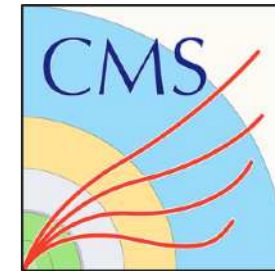


[<http://opendata.cern.ch/>]

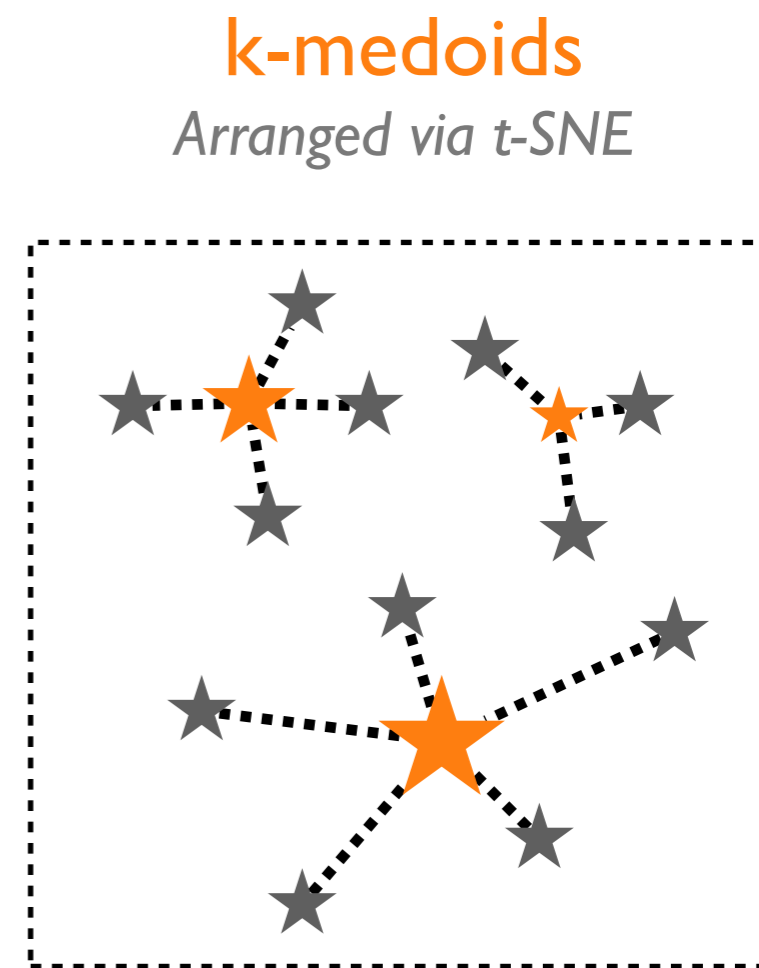
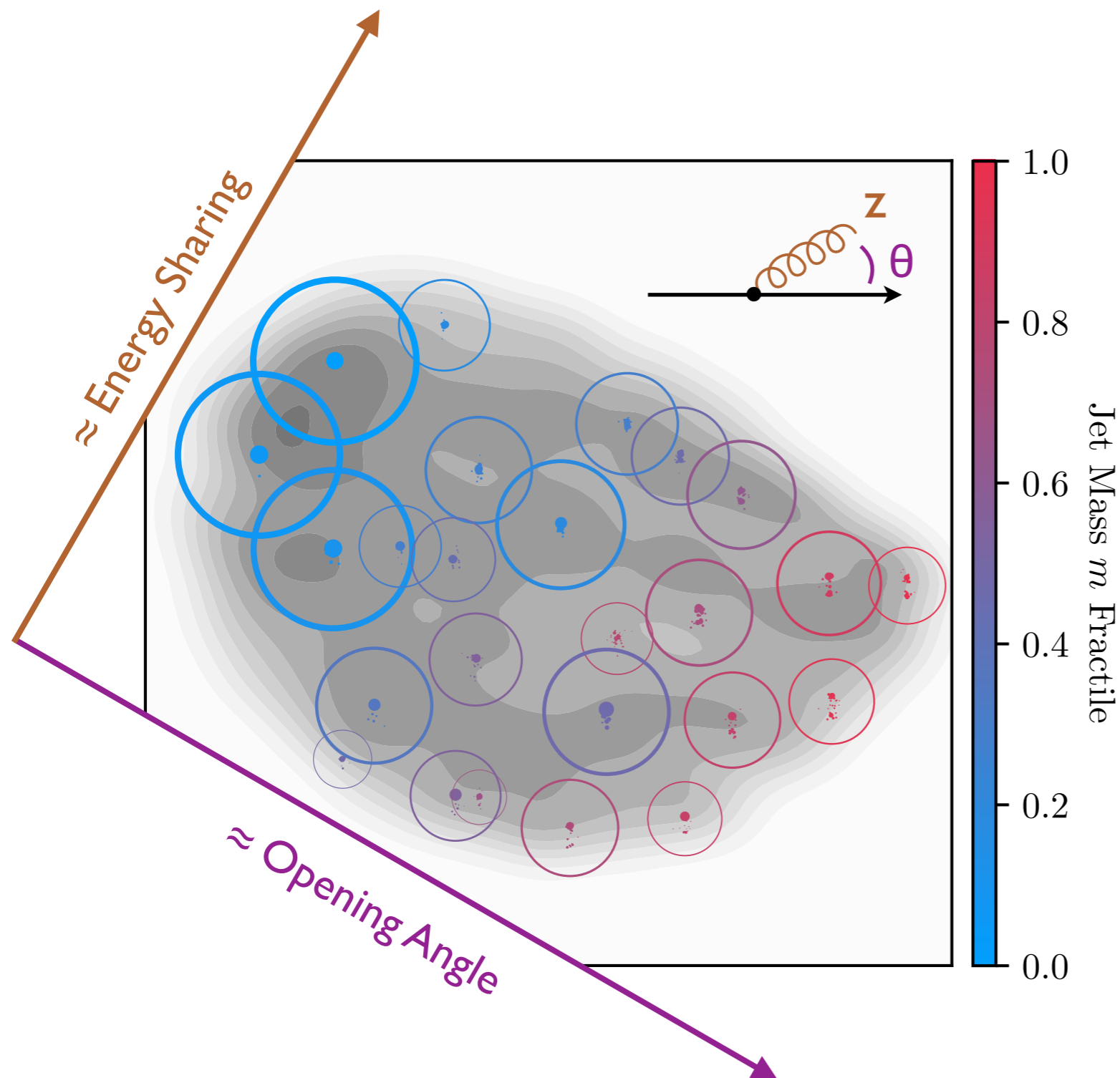


[Komiske, Mastandrea, Metodiev, Naik, JDT, PRD 2020; using van der Maaten, Hinton, JMLR 2008]

Most Representative Jets

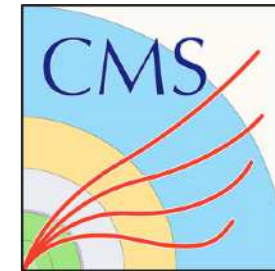


[<http://opendata.cern.ch/>]

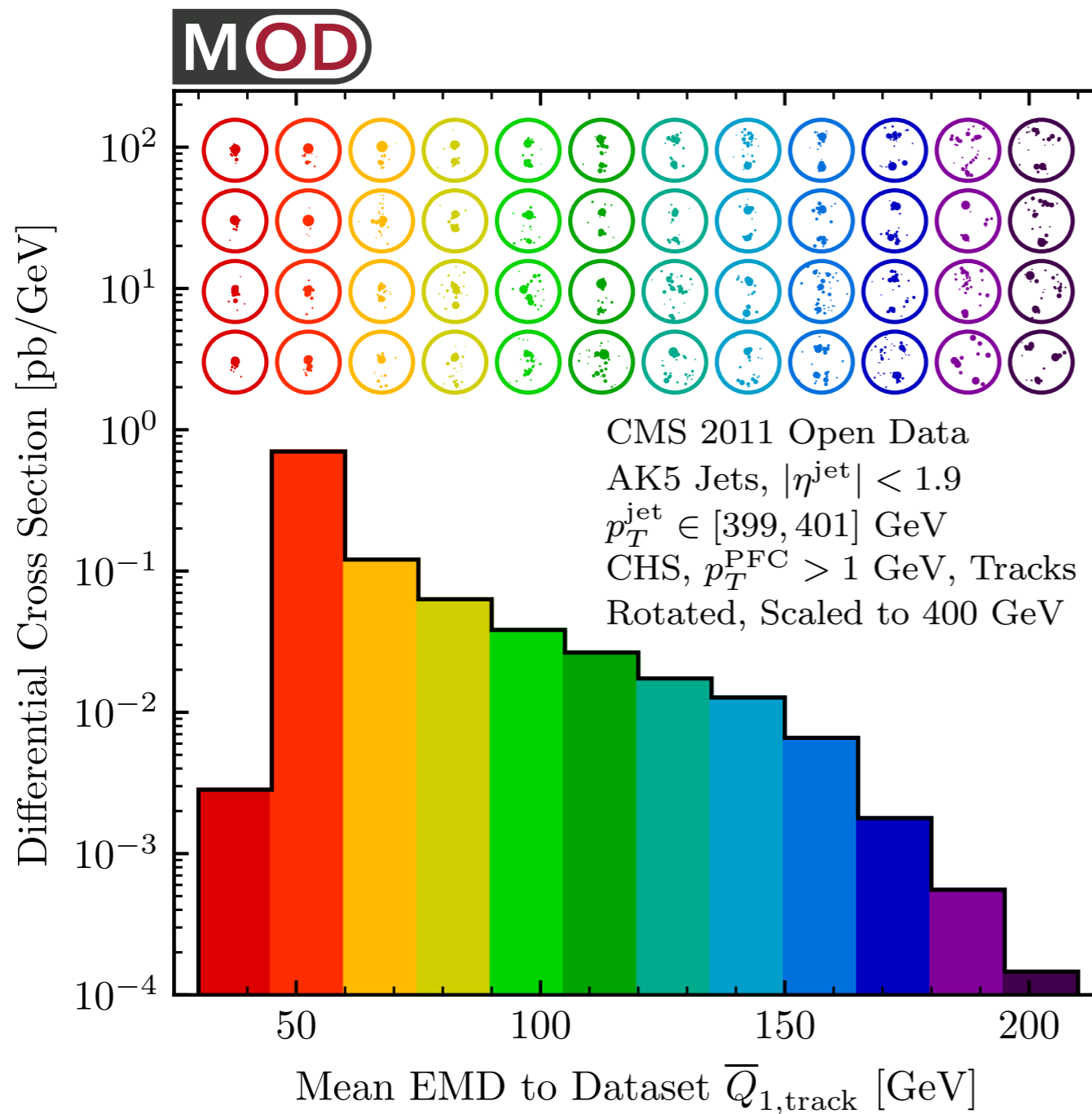


[Komiske, Mastandrea, Metodiev, Naik, JDT, PRD 2020; using van der Maaten, Hinton, JMLR 2008]

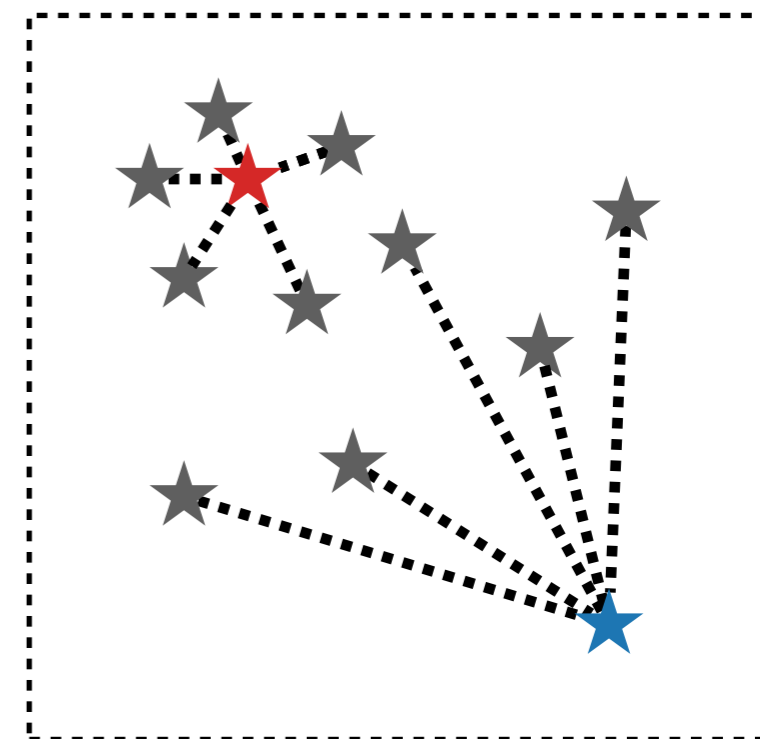
Least Representative Jets



[<http://opendata.cern.ch/>]



New Physics?
 Or tails of QCD?

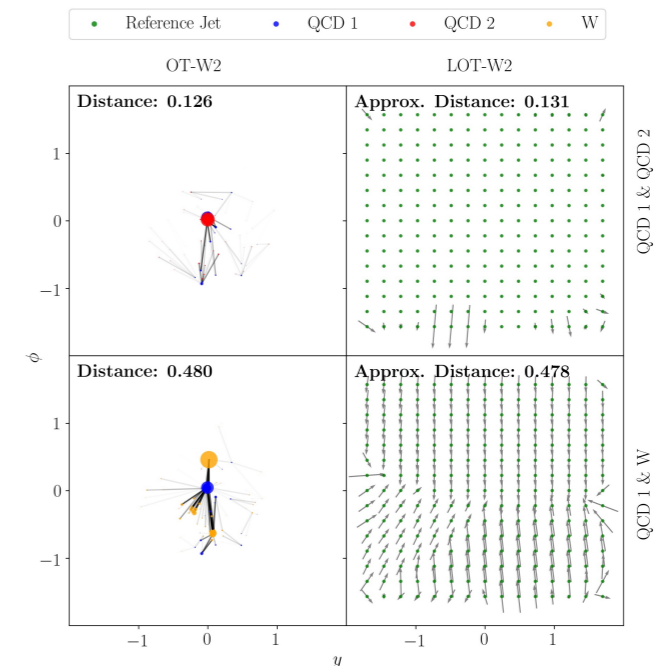


[Komiske, Mastandrea, Metodiev, Naik, JDT, PRD 2020]

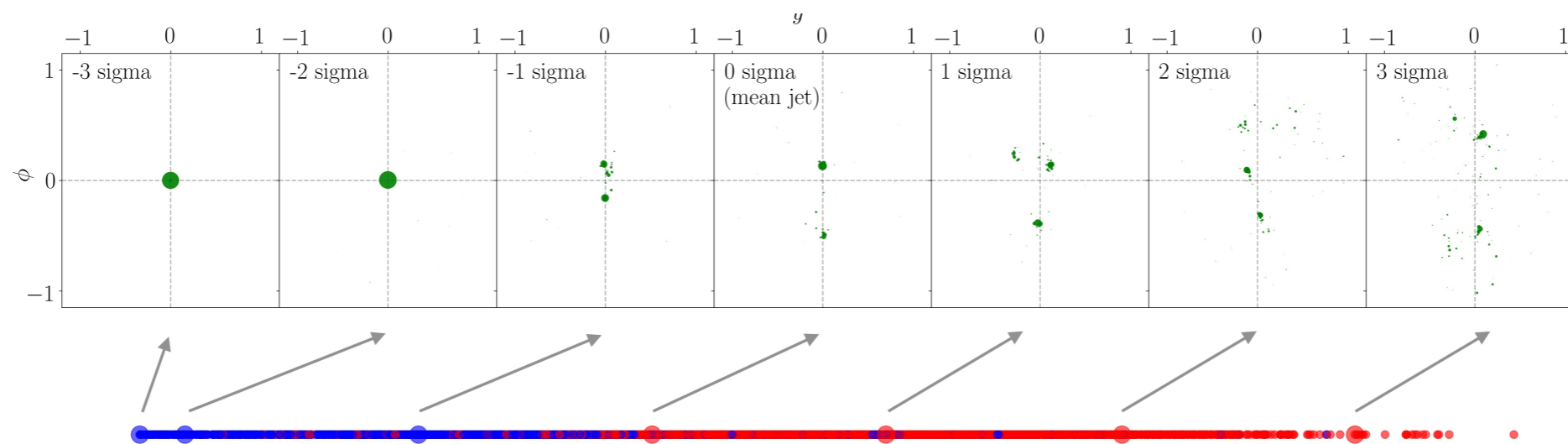
Linearized Optimal Transport

With the help of a **reference event**, transportation distances* can be **efficiently** mapped to **Euclidean distances**

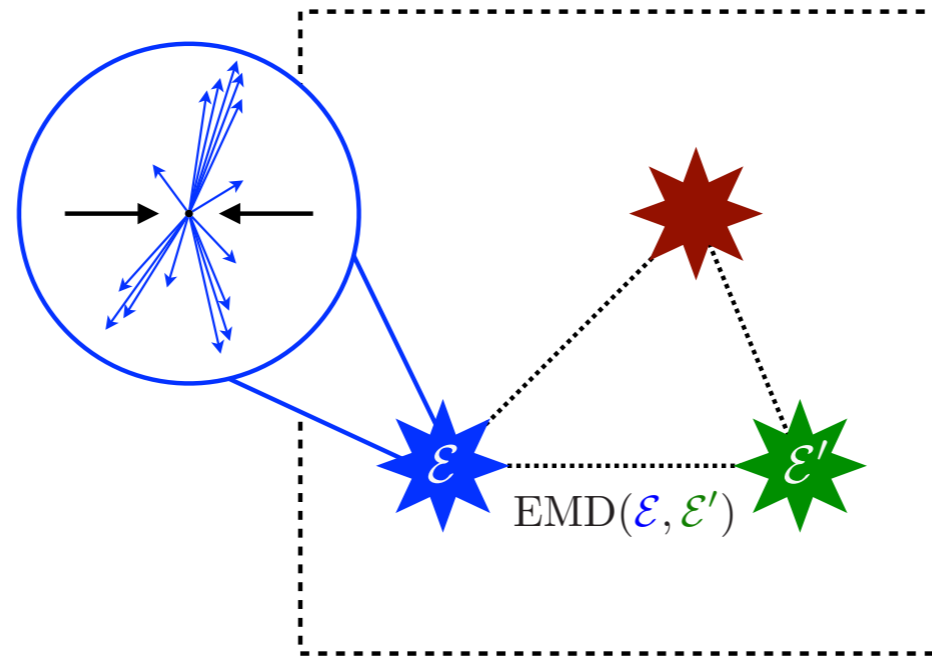
* assuming the 2-Wasserstein measure



Enables **coordinate-based techniques** like **Linear Discriminate Analysis**

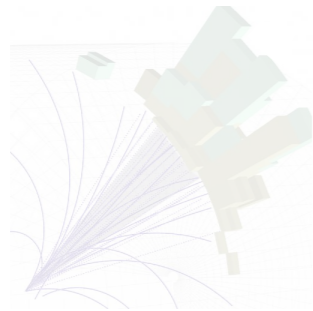


[Cai, Cheng, Craig, Craig, PRD 2020]

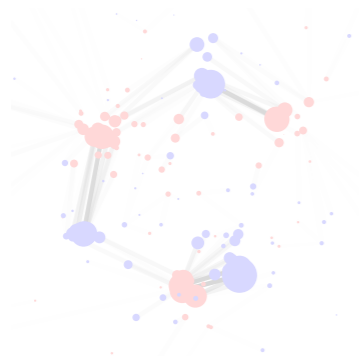


*Viewed through the data science lens,
the EMD unlocks a suite of
geometric analysis strategies*

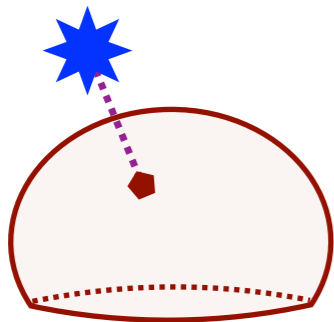
At this point, it should not be obvious why optimal transport distances should be particularly well-suited to collider applications



Going with the (Energy) Flow



The Energy Mover's Distance

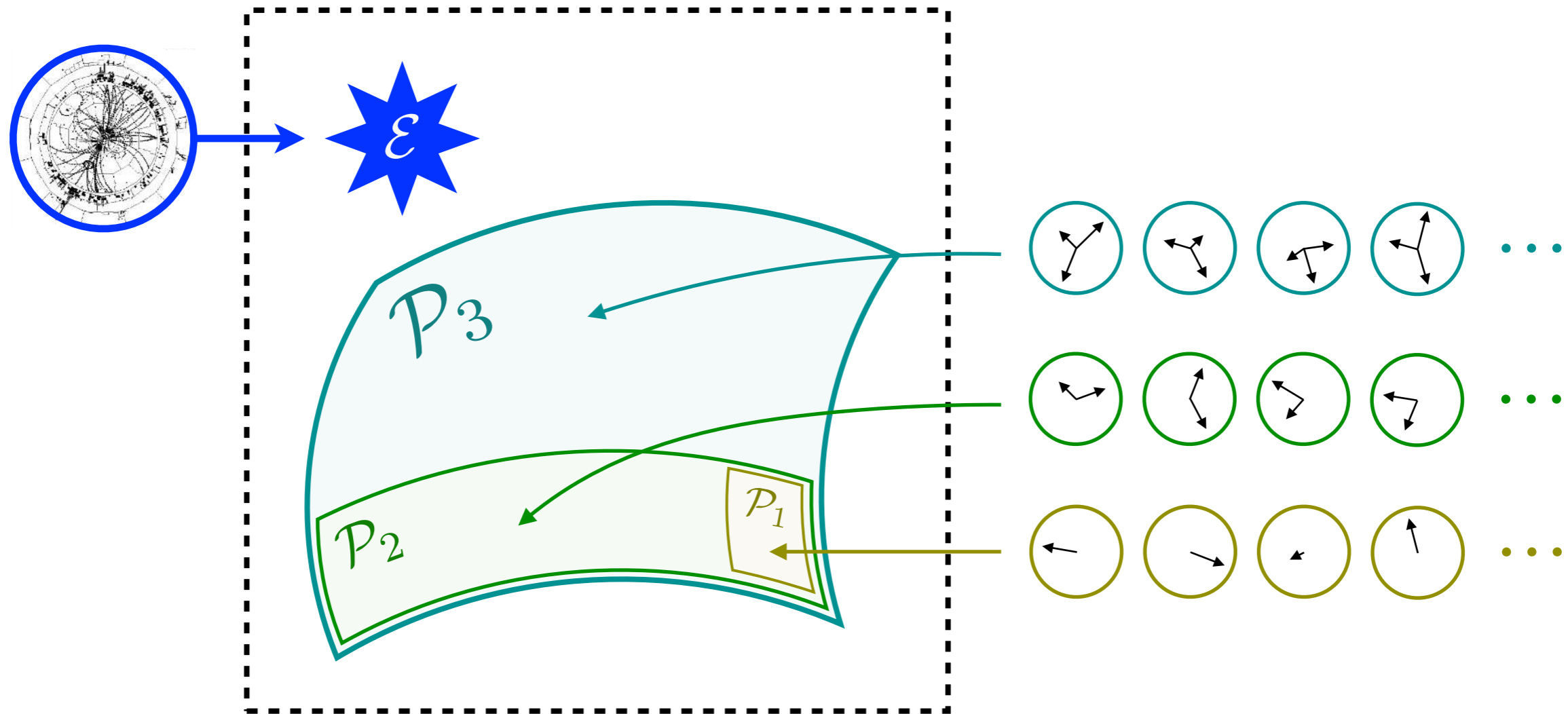


Revealing a Hidden Geometry

*Given a metric space, the first geometric object
you might think to construct is...*

Introducing N-particle Manifolds

\mathcal{P}_N = set of all N-particle configurations



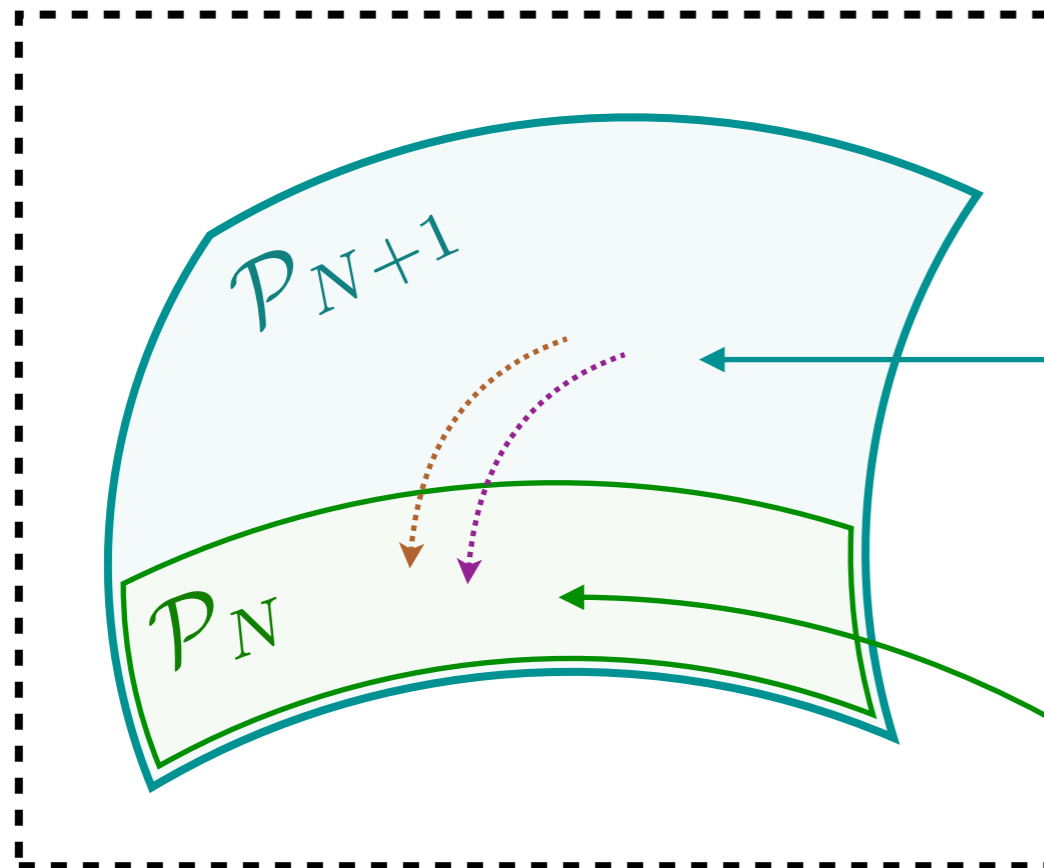
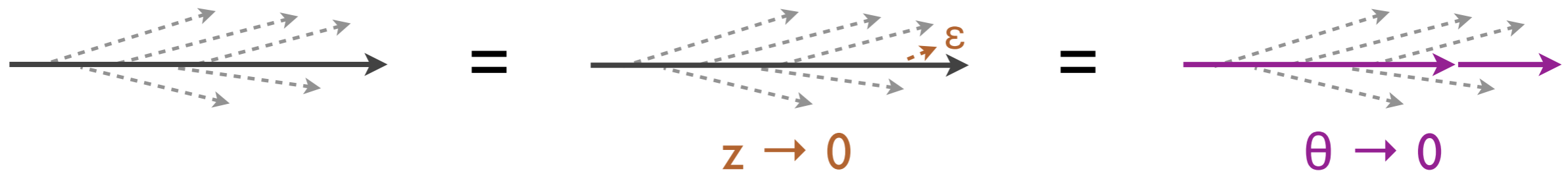
$\mathcal{P}_N \supset \mathcal{P}_{N-1} \supset \dots \supset \mathcal{P}_2 \supset \mathcal{P}_1$ by **soft/collinear** limits

[see related discussion in Larkoski, Melia, [PRD 2020](#)]

When are Two Events **the Same**?

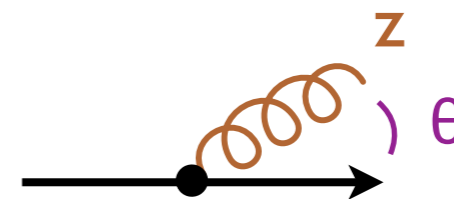
$$\mathcal{E}(\hat{n}) = \sum_i E_i \delta(\hat{n} - \hat{n}_i)$$

Energy Flow unchanged by infinitesimal **soft/collinear** emissions



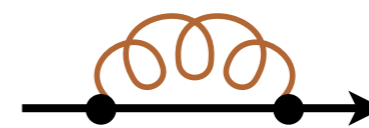
Infrared divergences “live” together!

Real:



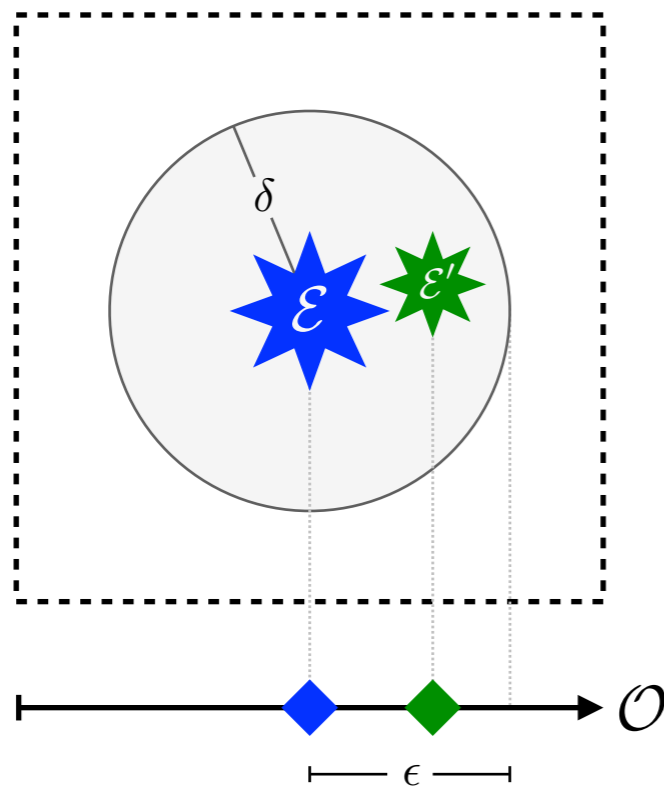
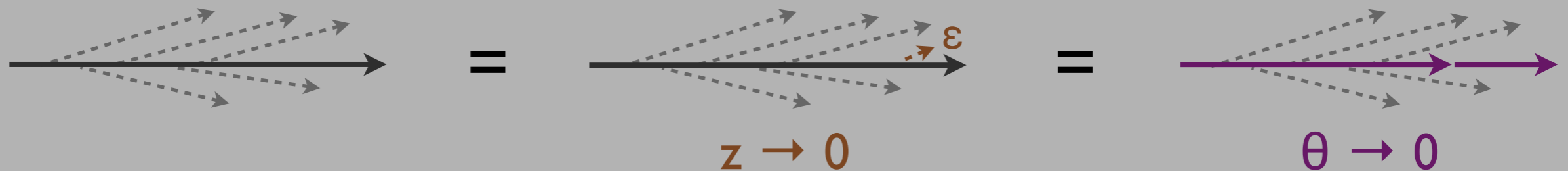
$$dP_{i \rightarrow ig} \simeq \frac{2\alpha_s}{\pi} C_i \frac{dz}{z} \frac{d\theta}{\theta}$$

Virtual:



When are Two Events the Same?

Energy Flow unchanged by infinitesimal *soft/collinear* emissions



Infrared & Collinear Safety

\approx calculable in perturbative quantum field theory

iS^* ← (see backup for subtleties)

Continuity in EMD Space

[Komiske, Metodiev, JDT, JHEP 2020]

[Sterman, Weinberg, PRL 1977; Sterman, PRD 1979]

[see also Banfi, Salam, Zanderighi, JHEP 2005; Larkoski, Marzani, JDT, PRD 2015]

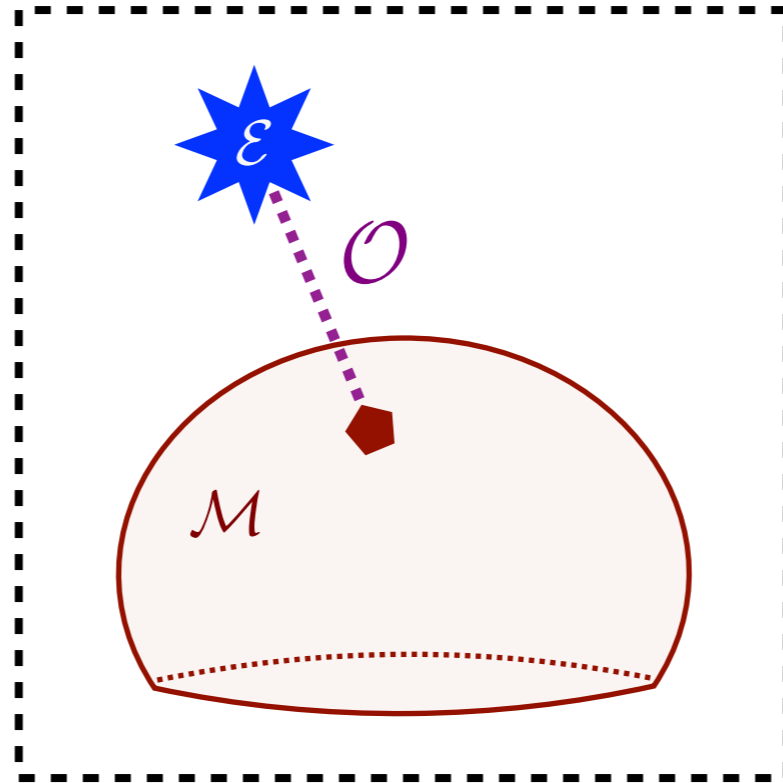
The EMD seems to define the “natural” geometry for massless gauge theories

Open question: Can you define $|\mathcal{M}_{AB \rightarrow 12 \dots n}|^2$ directly in this space?

What does it mean to “integrate” in this space? How do you account for charge/flavor?

Manifolds for Observables

One Event



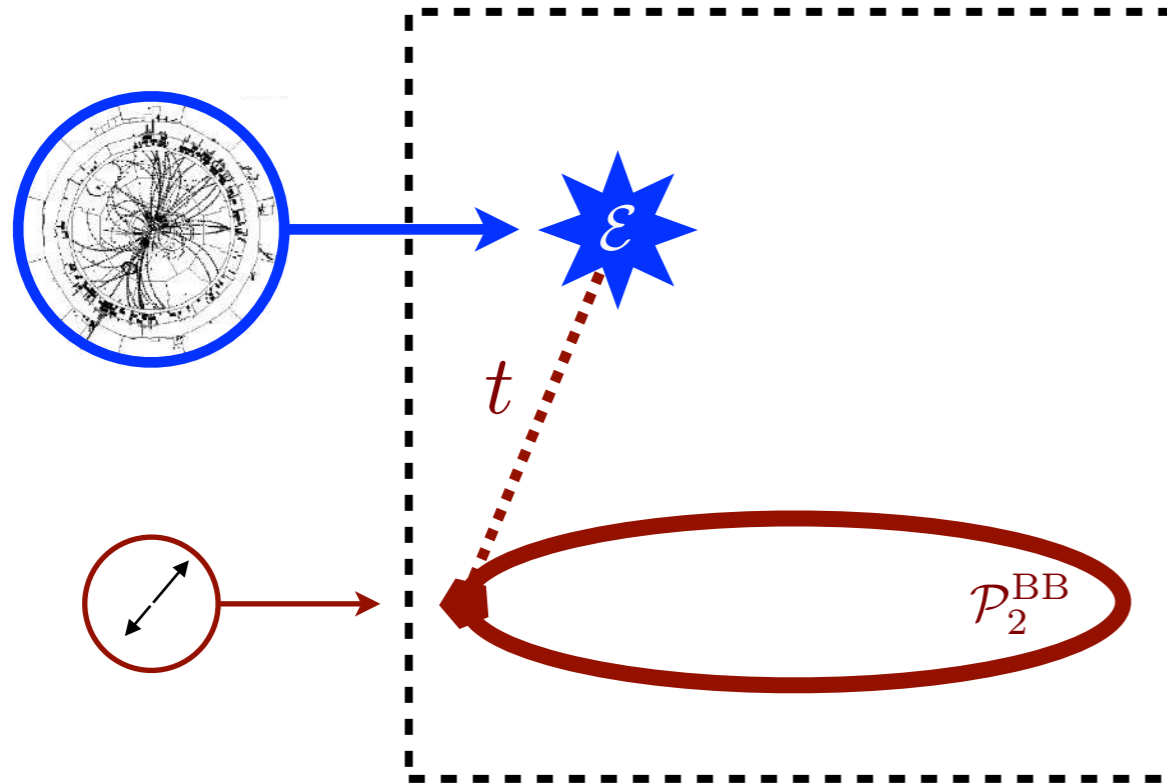
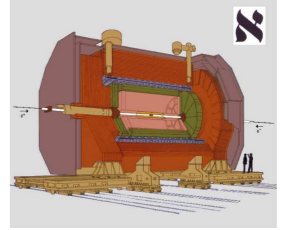
Set of Events

Distance of Closest Approach \Rightarrow Observable

$$\mathcal{O}(\mathcal{E}) = \min_{\mathcal{E}' \in \mathcal{M}} \text{EMD}(\mathcal{E}, \mathcal{E}')$$

E.g. Thrust

How dijet-like is an event?

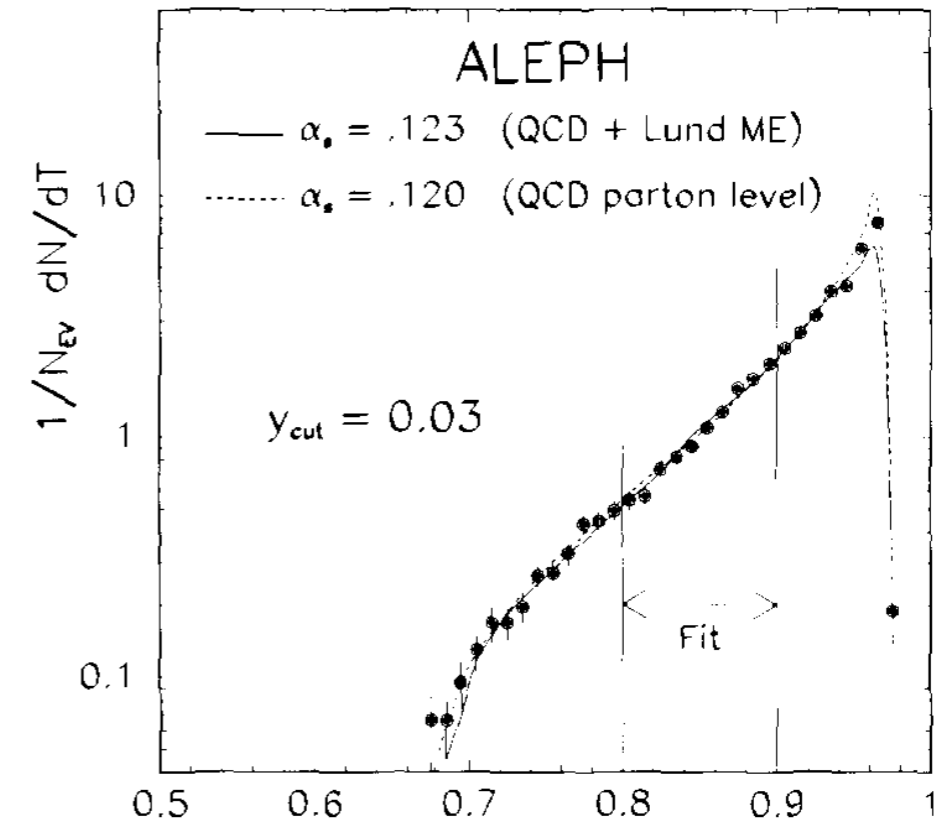


All Back-to-Back Two Particle Configurations

$$\mathcal{P}_2^{\text{BB}} = \left\{ \text{diagram 1} \quad \text{diagram 2} \quad \text{diagram 3} \quad \text{diagram 4} \quad \dots \right\}$$

(using $\beta=2$ EMD variant)

$$t(\mathcal{E}) = \min_{\mathcal{E}' \in \mathcal{P}_2^{\text{BB}}} \text{EMD}_2(\mathcal{E}, \mathcal{E}')$$



$$1 - \frac{t}{2E_{\text{CM}}}$$

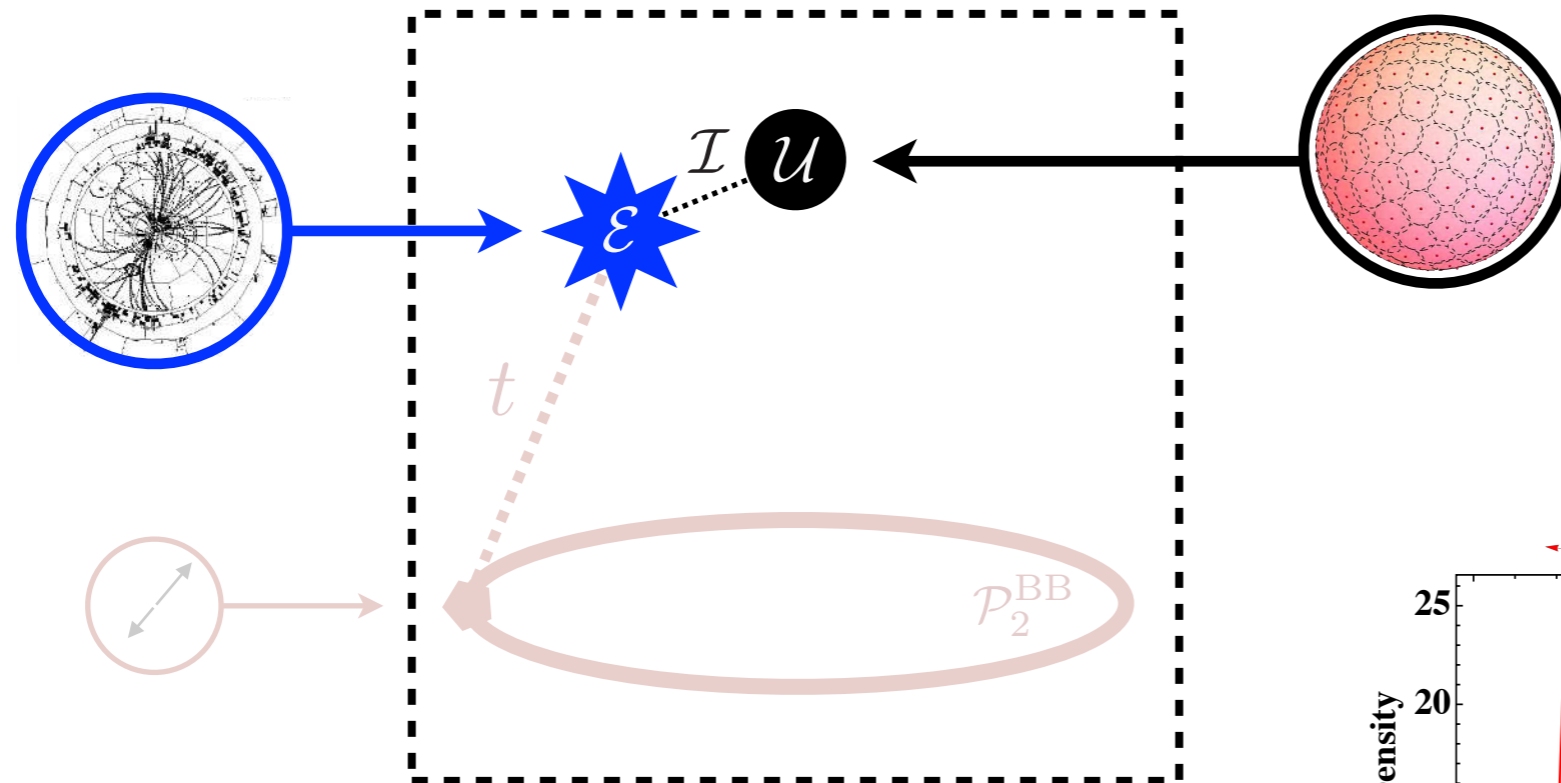
$$\text{cf. } T(\mathcal{E}) = \max_{\hat{n}} \frac{\sum_i |\vec{p}_i \cdot \hat{n}|}{\sum_j |\vec{p}_j|}$$

[Komiske, Metodiev, JDT, JHEP 2020]

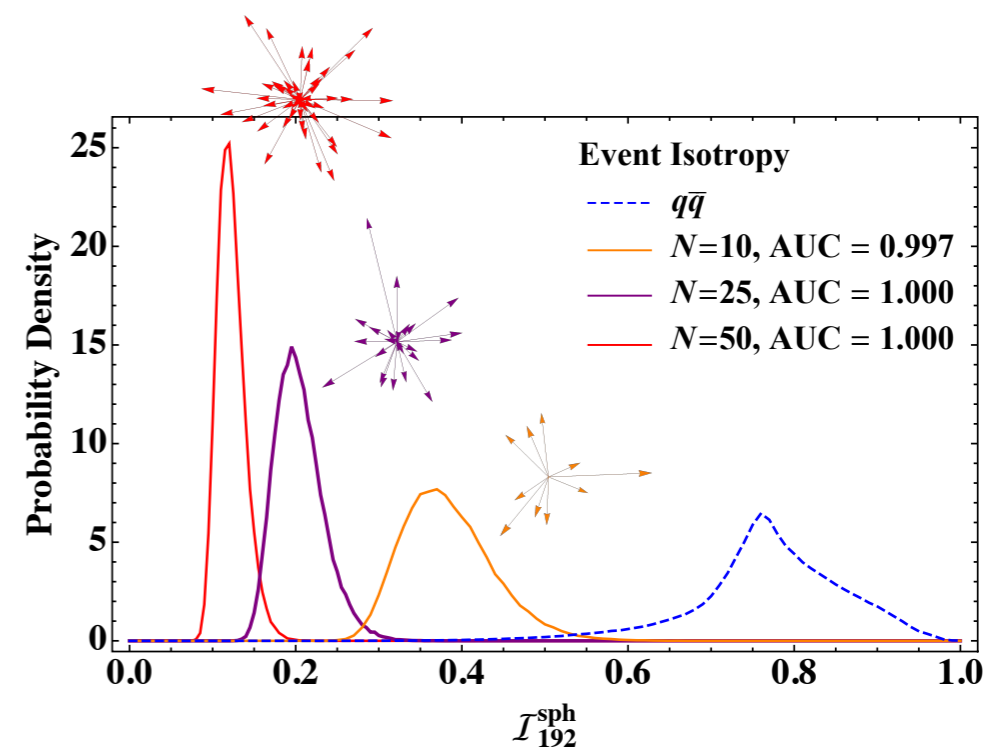
[Brandt, Peyrou, Sosnowski, Wroblewski, PL 1964; Farhi, PRL 1977; ALEPH, PLB 1991]

New! Event Isotropy

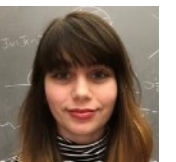
How isotropic is an event?



$$\mathcal{I}(\mathcal{E}) = \text{EMD}(\mathcal{E}, \mathcal{U})$$

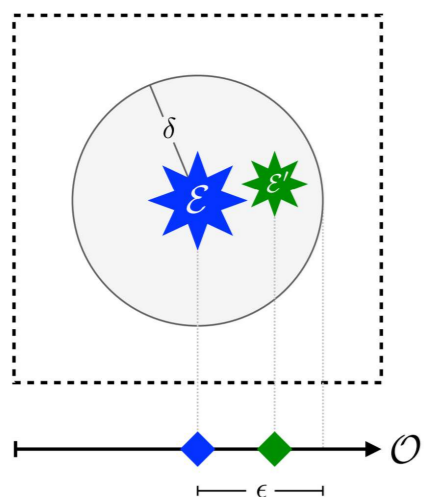


[Cesarotti, JDT, JHEP 2020;
see also Cesarotti, Reece, Strassler, JHEP 2021]



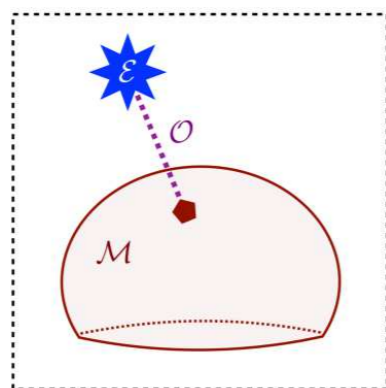
Six Decades of Collider Physics Translated into a New Geometric Language!

IRC Safety is smoothness in the space of events



Taming infinities

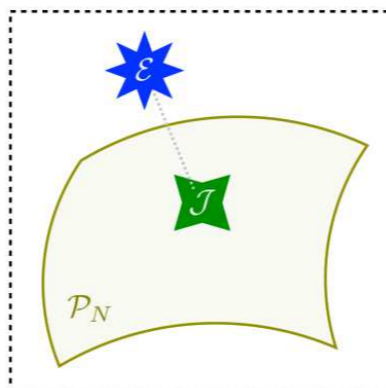
Event shapes are distances from events to manifolds.



$$O(\mathcal{E}) = \min_{\mathcal{E}' \in \mathcal{M}} \text{EMD}_{\beta,R}(\mathcal{E}, \mathcal{E}')$$

Event Shapes

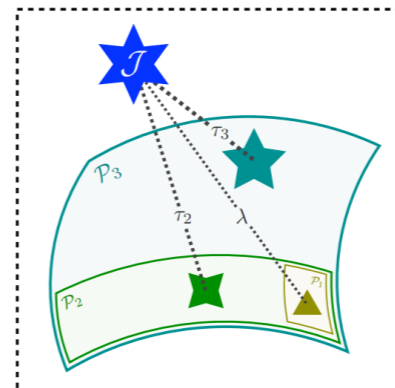
Jets are projections to few-particle manifolds.



$$J = \operatorname{argmin}_{\mathcal{E}' \in \mathcal{P}_N} \text{EMD}_{\beta,R}(\mathcal{E}, \mathcal{E}')$$

Jet Algorithms

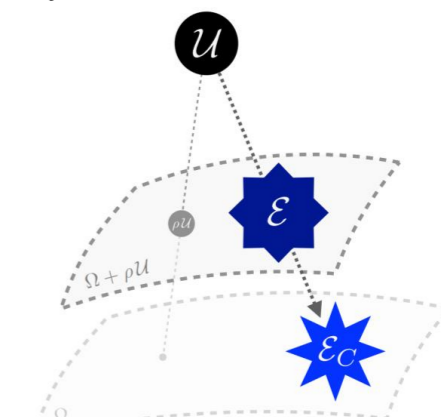
Substructure resolves emissions within the jet.



$$\tau(J) = \min_{\mathcal{E}' \in \mathcal{P}_N} \text{EMD}_{\beta}(\mathcal{J}, \mathcal{E}')$$

Jet Substructure

Pileup mitigation moves away from uniform radiation.



$$\mathcal{E}_C = \operatorname{argmin}_{\mathcal{E}'} \text{EMD}(\mathcal{E}, \mathcal{E}' + \rho \mathcal{U}).$$

Pileup

1960

1962-1964

Infrared Safety

[Kinoshita, JMP 1962]
[Lee, Nauenberg, PR 1964]

1977

Thrust, Sphericity

[Farhi, PRL 1977]
[Georgi, Machacek, PRL 1977]

1993

k_T jet clustering

[Ellis, Soper, PRD 1993]
[Catani, Dokshitzer, Seymour, Webber, NPB 1993]

1997-1998

C/A jet clustering

[Wobisch, Wengler, 1998]
[Dokshitzer, Leder, Moretti, Webber, JHEP 1997]

2010-2015

N-(sub)jettiness, X Cone

[Stewart, Tackmann, Waalewijn, PRL 2010]
[Thaler, Van Tilburg, JHEP 2011]
[Stewart, Tackmann, Thaler, Vermilion, Wilkason, JHEP 2015]

2014-2019

Constituent Subtraction

[Berta, Spousta, Miller, Leitner, JHEP 2014]
[Berta, Masetti, Miller, Spousta, JHEP 2019]

2020

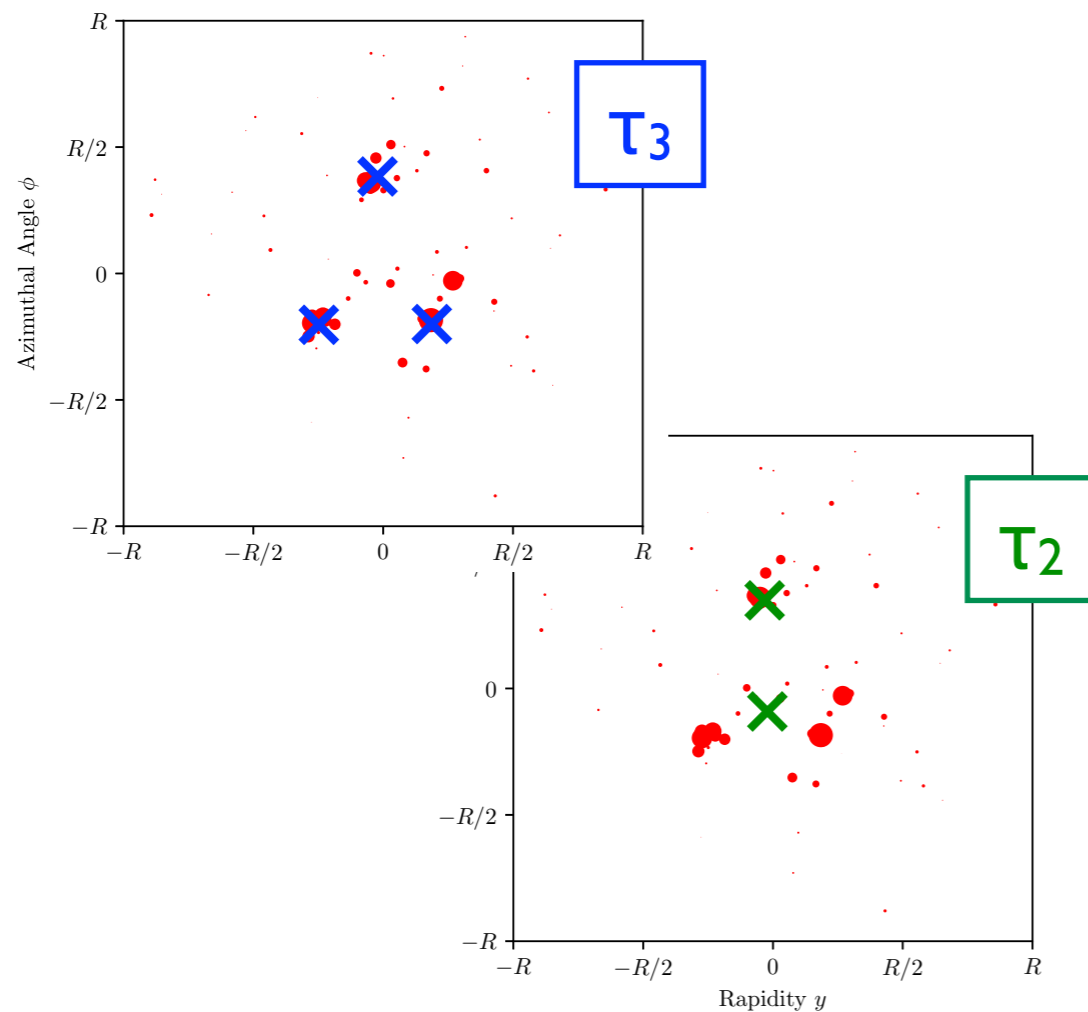
And many more!

[timeline from Eric Metodiev]

N-subjettiness

Ubiquitous jet substructure observable used for almost a decade...

$$\tau_N(\mathcal{J}) = \min_{N \text{ axes}} \sum_i E_i \min \{ \theta_{1,i}, \theta_{2,i}, \dots, \theta_{N,i} \}$$



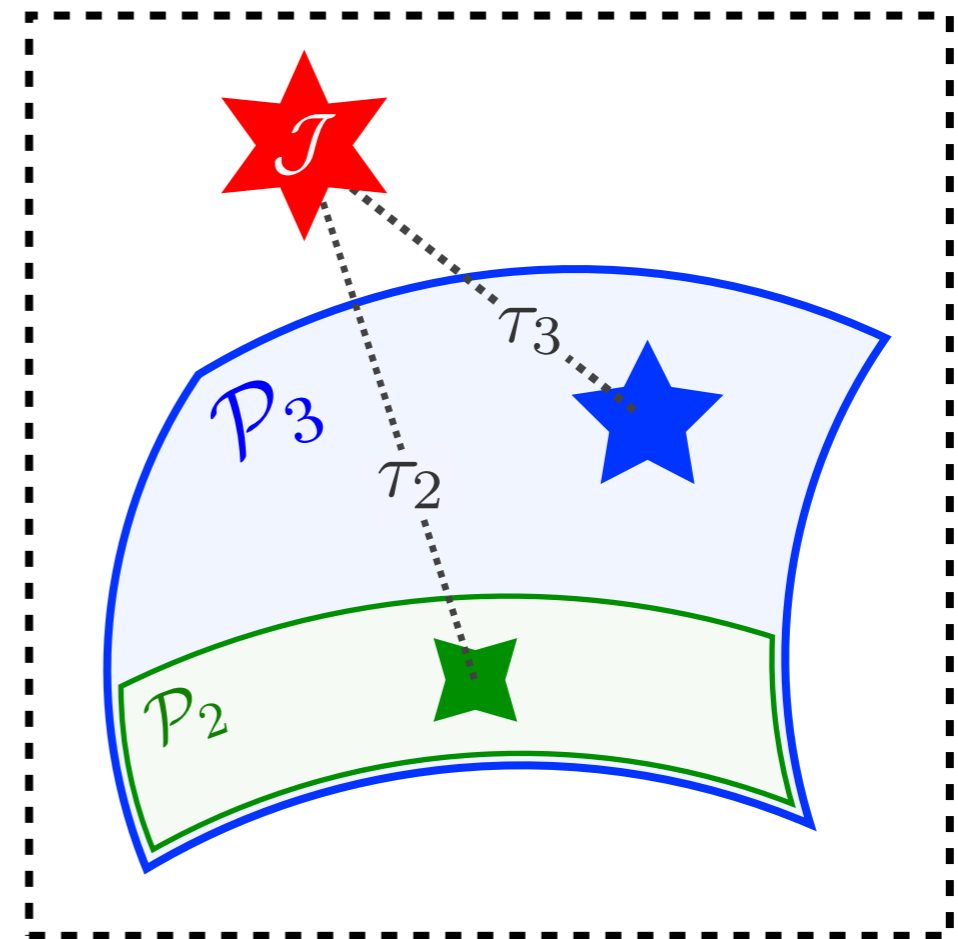
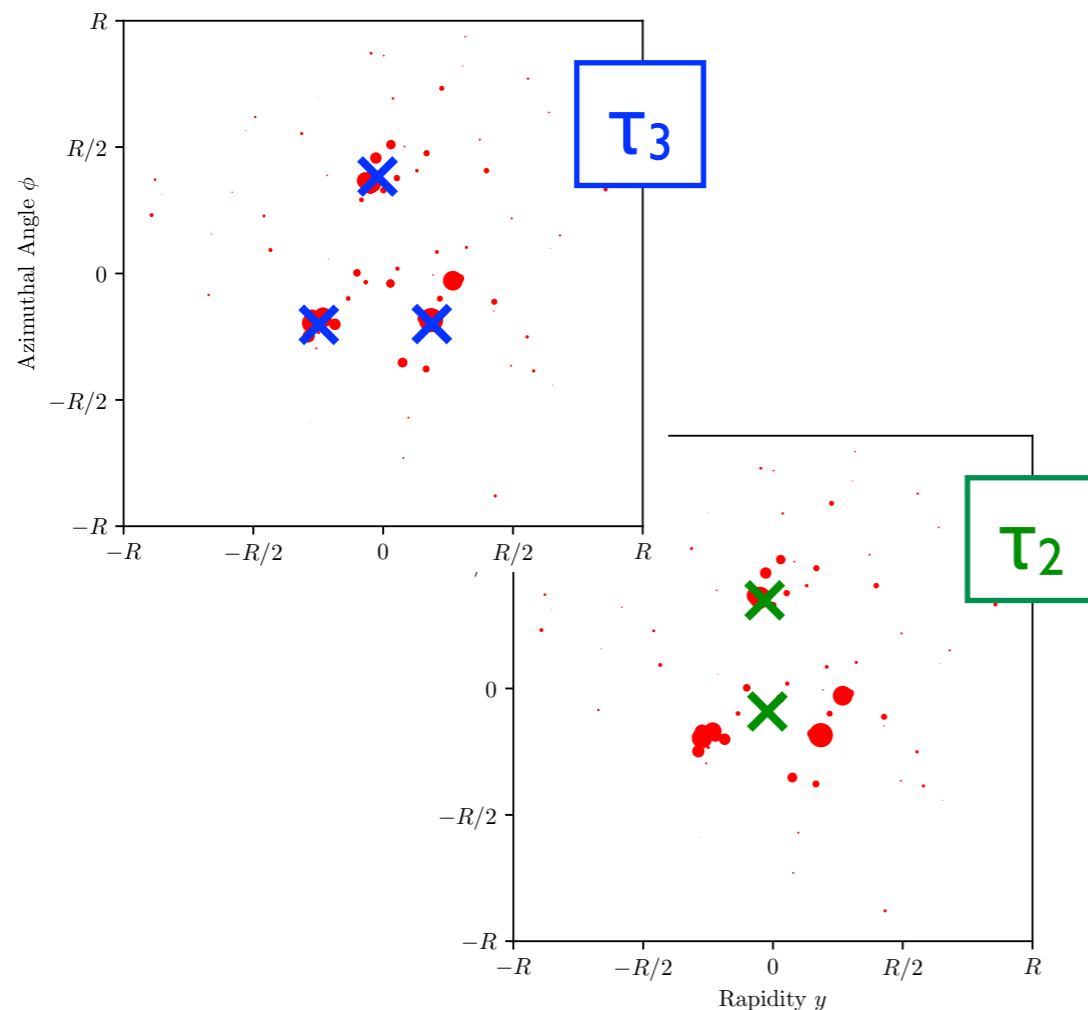
[JDT, Van Tilburg, [JHEP 2011](#), [JHEP 2012](#);
based on Brandt, Dahmen, [ZPC 1979](#); Stewart, Tackmann, Waalewijn, [PRL 2010](#)]



N-subjettiness = Point to Manifold EMD

...is secretly an optimal transport problem

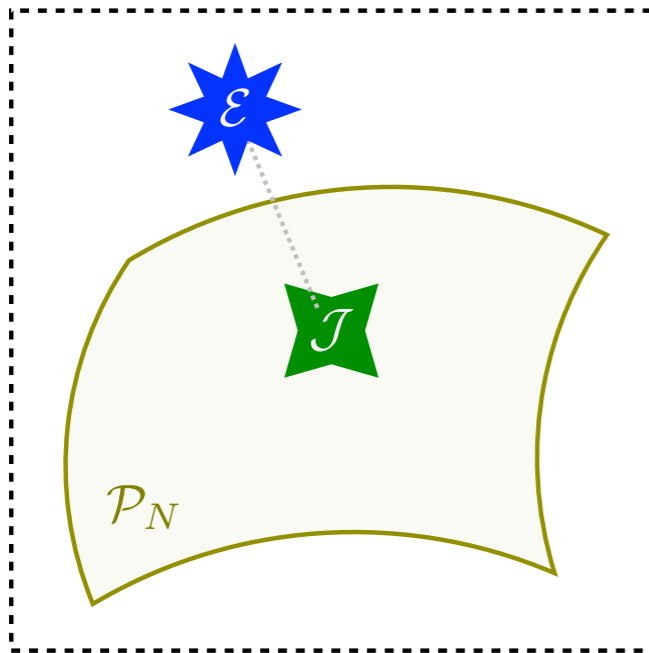
$$\tau_N(\mathcal{J}) = \min_{\mathcal{J}' \in \mathcal{P}_N} \text{EMD}(\mathcal{J}, \mathcal{J}')$$



[JDT, Van Tilburg, JHEP 2011, JHEP 2012;
rephrased in the language of Komiske, Metodiev, JDT, PRL 2019]



More Fun with N-particle Manifolds



N-jettiness

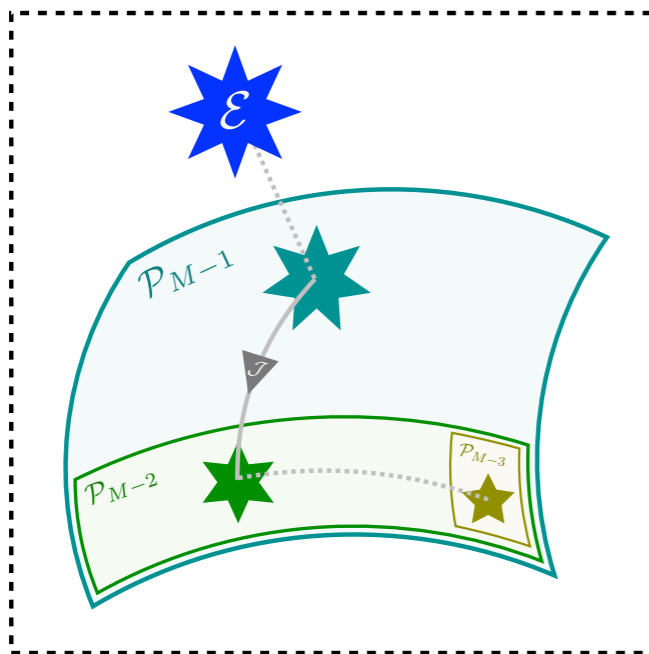
Distance of closest approach to N-particle manifold

[Brandt, Dahmen, ZPC 1979; Stewart, Tackmann, Waalewijn, PRL 2010]

Exclusive Cone Jet Finding

Point of closest approach on N-particle manifold

[Stewart, Tackmann, JDT, Vermilion, Wilkason, JHEP 2015]



Sequential Jet Recombination

Iteratively stepping between various N-particle manifolds

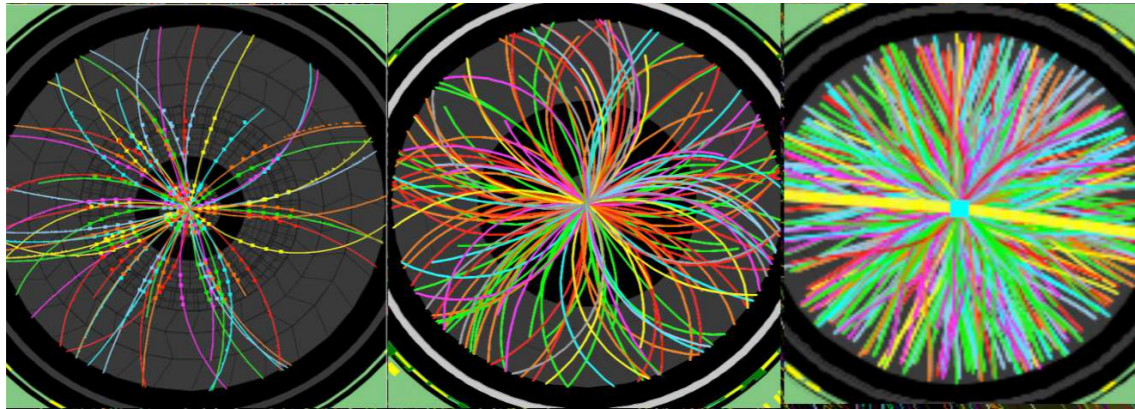
[Catani, Dokshitzer, Seymour, Webber, NPB 1993; Ellis, Soper, PRD 1993]

[Dokshitzer, Leder, Moretti, Webber, JHEP 1997; Wobisch, Wengler, arXiv 1999]

[Butterworth, Couchman, Cox, Waugh, CPC 2003; Larkoski, Neill, JDT, JHEP 2014]

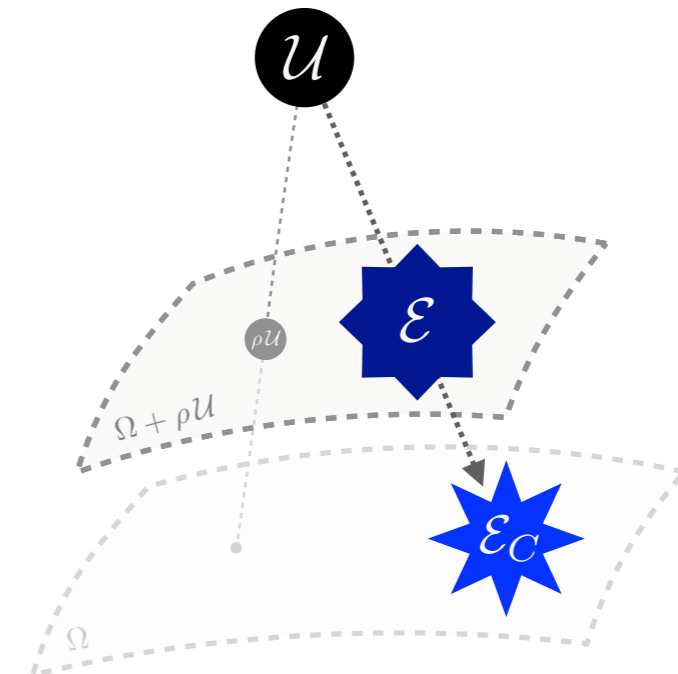
[Komiske, Metodiev, JDT, JHEP 2020]

Pileup Mitigation



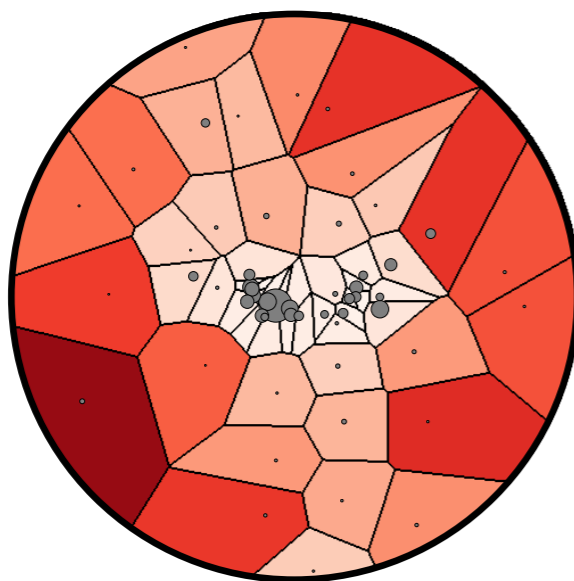
[see review in Soyez, PR 2019]

Uniform event contamination from overlapping proton-proton collisions



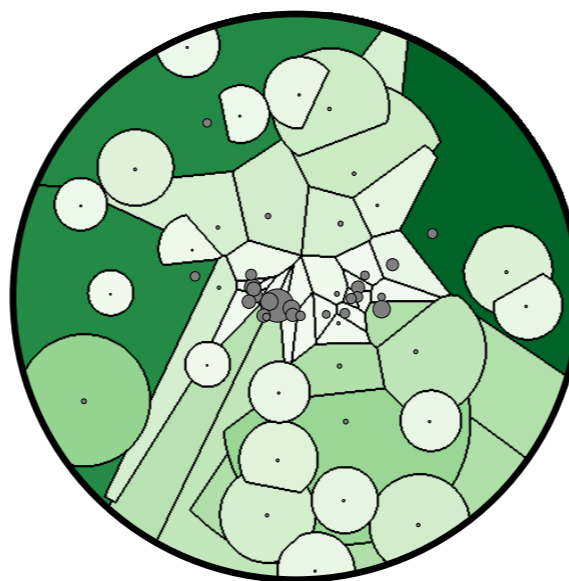
Pileup Mitigation:
“Move away” from uniform event

Voronoi



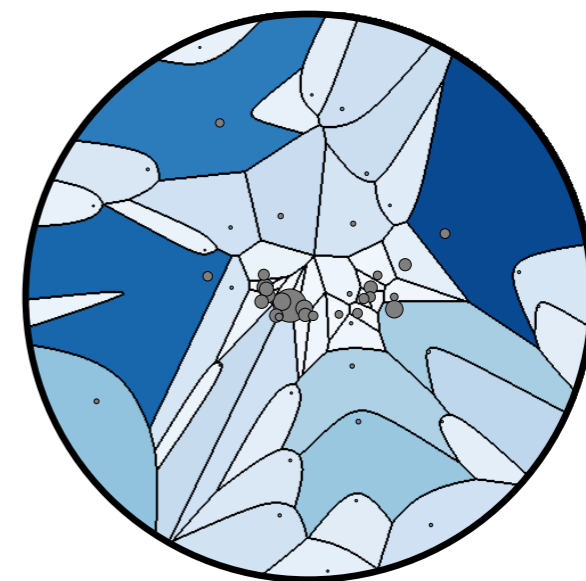
[Cacciari, Salam, Soyez, JHEP 2008]

Constituent Subtraction



[Berta, Spousta, Miller, Leitner, JHEP 2014]

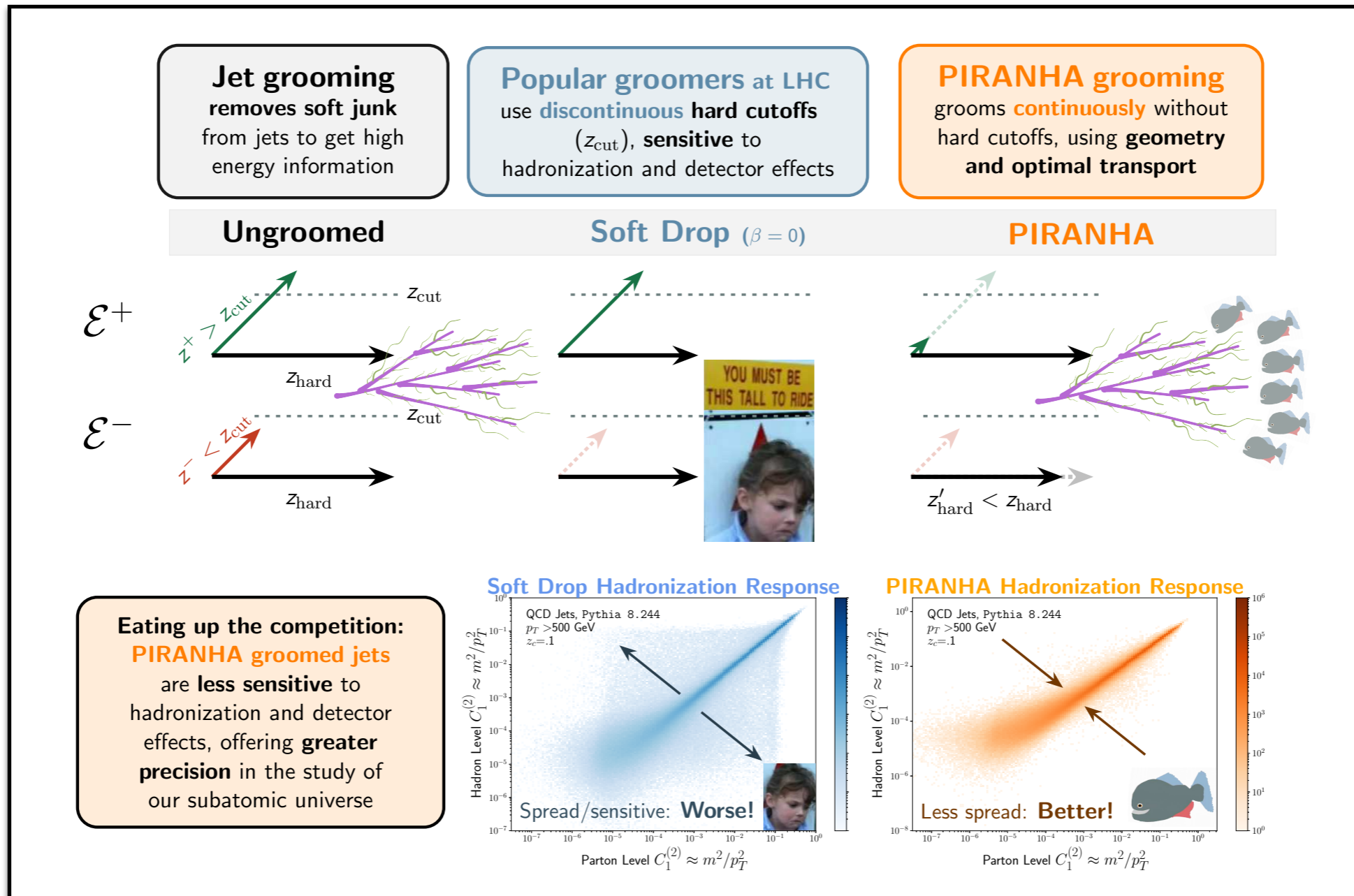
Apollonius



[Komiske, Metodiev, JDT, JHEP 2020]

Pileup and Infrared Radiation AnNiHiAtion

Recursive Safe Subtraction: tree-based approx. to optimal transport grooming



[Slides from Sam Alipour-fard]
[Alipour-fard, Komiske, Metodiev, JDT, in progress]

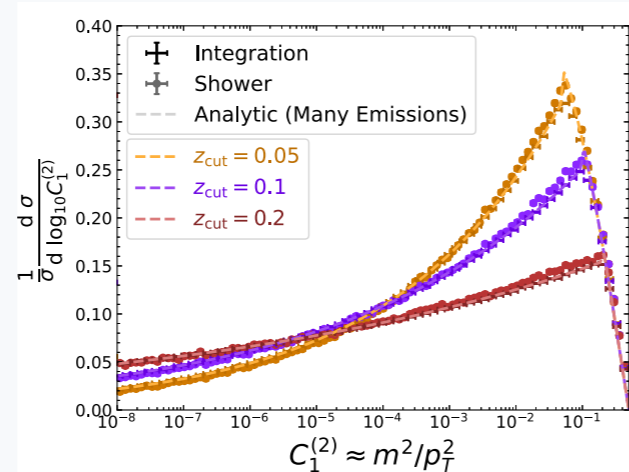


Pileup and Infrared Radiation AnNiHiAtion

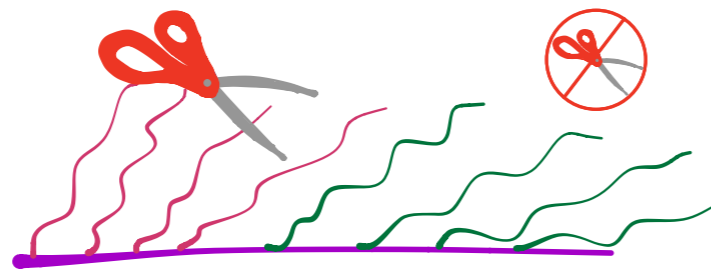
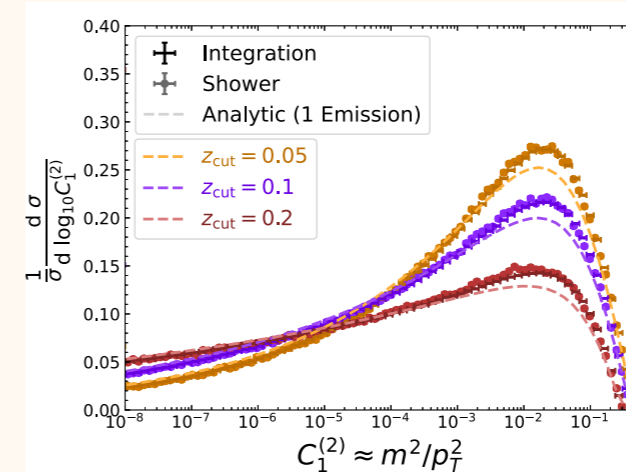
Recursive Safe Subtraction: tree-based approx. to optimal transport grooming

Fixed coupling, **multiple emission** calculations:

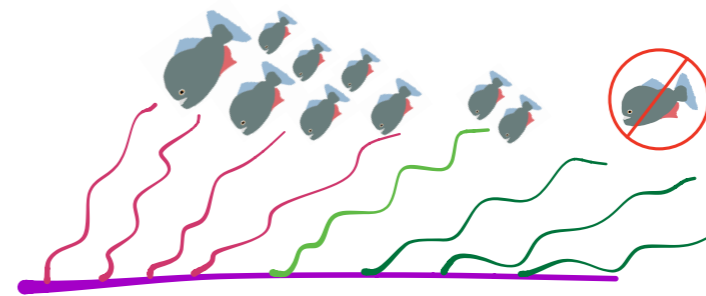
Soft Drop/mMDT



PIRANHA-RSS ($f = 1$)



Sharp cutoff \rightarrow kink

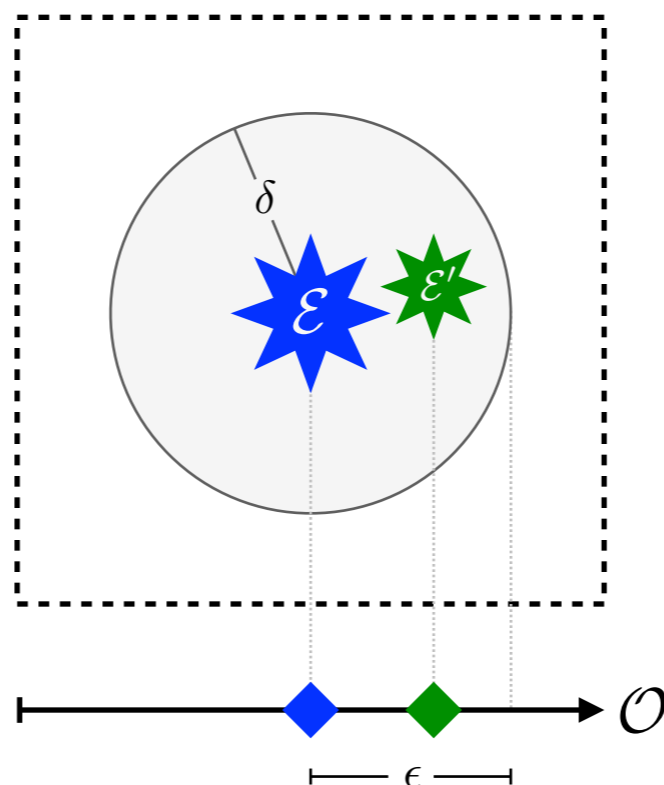


No sharp cutoff \rightarrow smooth

[Slides from Sam Alipour-fard]

[Alipour-fard, Komiske, Metodiev, JDT, in progress]

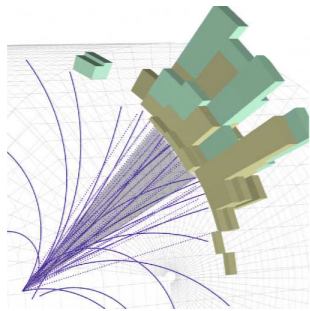




We are just beginning to leverage the *conceptual richness* of optimal transport for high-energy physics application

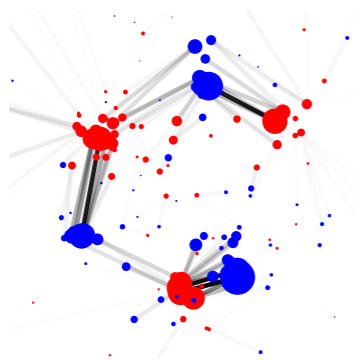
Ask me how far down this rabbit hole goes!

Summary



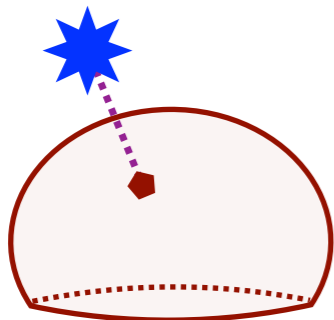
Going with the (Energy) Flow

Restricting our attention to IRC safe information is a theoretically motivated data analysis strategy



The Energy Mover's Distance

Optimal transport allows us to triangulate the space of collider events and define an emergent geometry



Revealing a Hidden Geometry

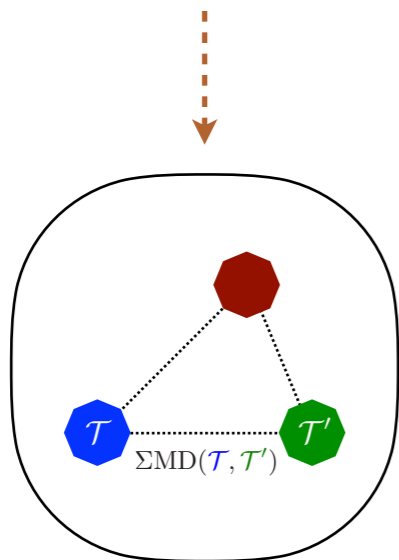
We can gain new perspectives on concepts/techniques in QFT and collider physics from the last half century

How far down does this rabbit hole go?

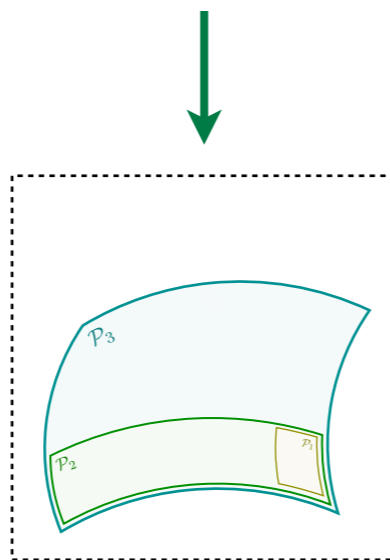
Master Formula for Collider Physics

$$\sigma_{\text{obs}} \simeq \frac{1}{2E_{\text{CM}}^2} \sum_{n=2}^{\infty} \int d\Phi_n |\mathcal{M}_{AB \rightarrow 12\dots n}|^2 f_{\text{obs}}(\Phi_n)$$

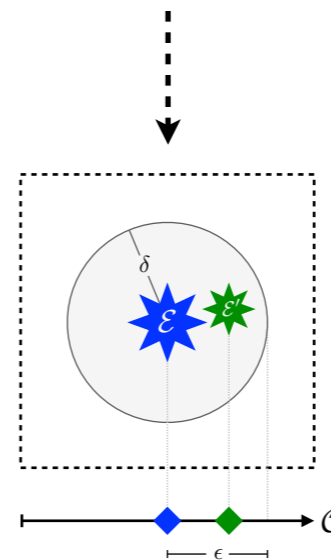
cross section



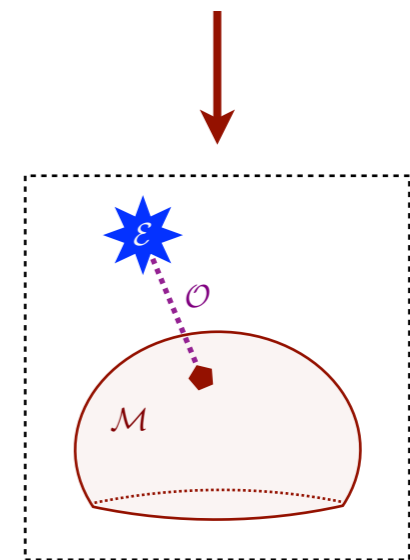
phase space



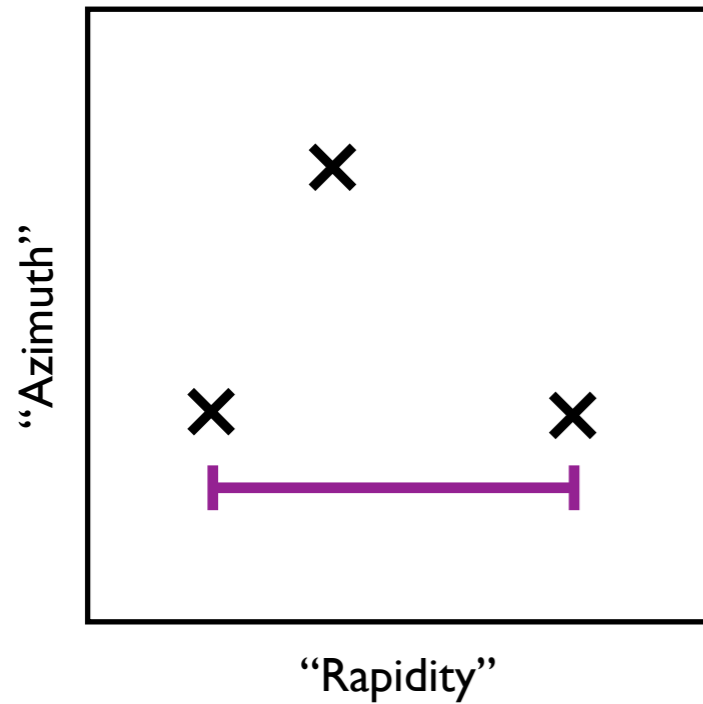
amplitude



observable



Direction Space



x = Direction

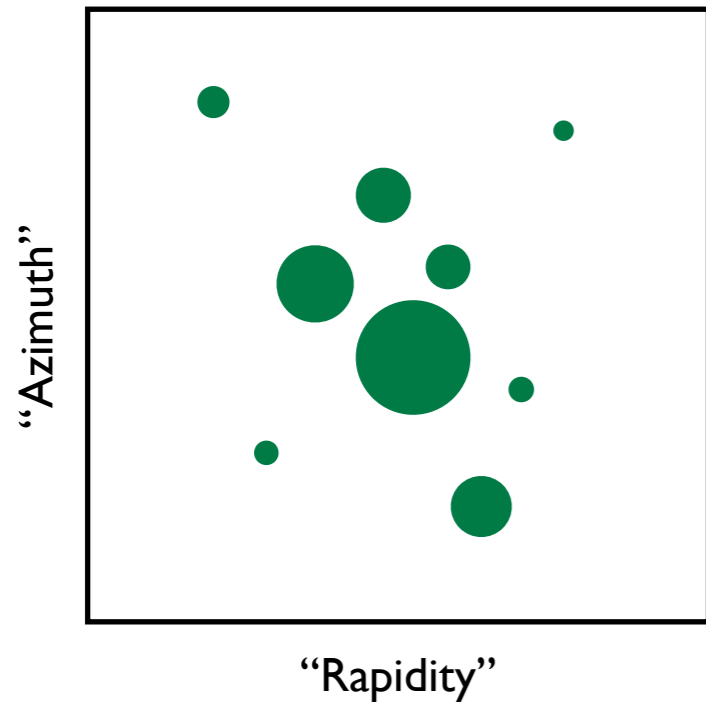
— = Angular Distance

$$n_i^\mu = \frac{p_i^\mu}{E_i} = (1, \hat{n})^\mu$$

$$\theta_{ij} = \sqrt{2n_i^\mu n_{j\mu}}$$

(for massless particles)

Direction Space Distribution



● = Weighted Direction

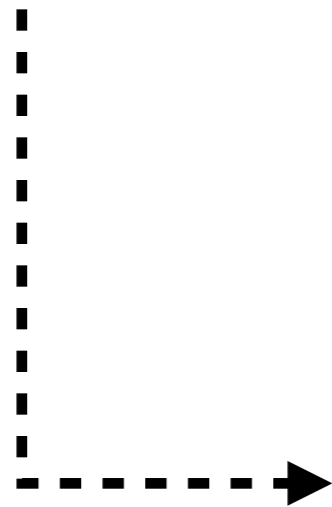
— = Angular Distance

$$n_i^\mu = \frac{p_i^\mu}{E_i} = (1, \hat{n})^\mu$$

$$w_i = E_i$$

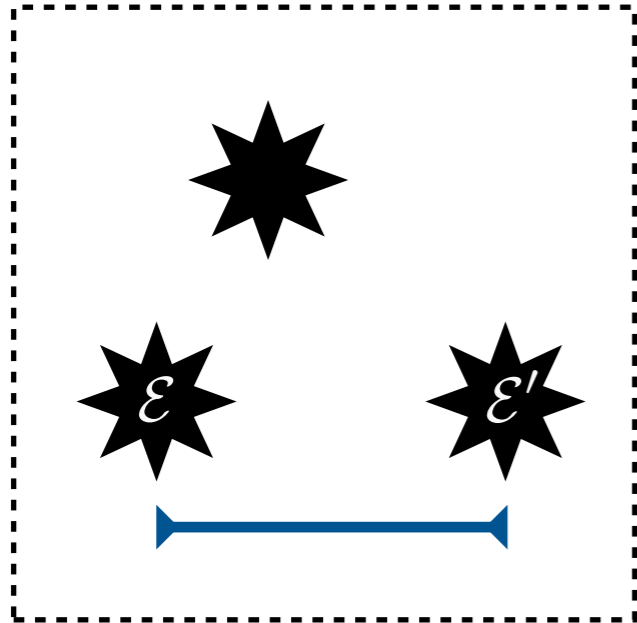
$$\theta_{ij} = \sqrt{2n_i^\mu n_{j\mu}}$$

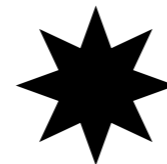
(for massless particles)



★ \mathcal{E} = Event

Event Space



 = Event

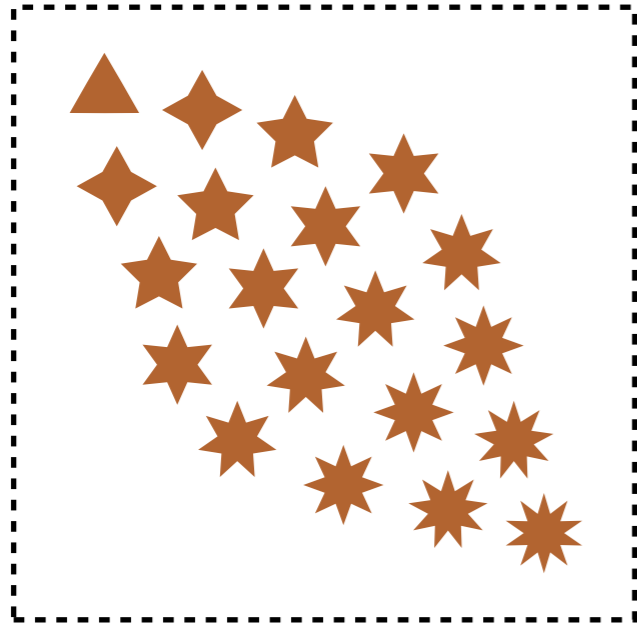
 = EMD
Energy
Mover's Distance


$$\mathcal{E}(\hat{n}) = \sum_i E_i \delta(\hat{n} - \hat{n}_i)$$

$$\text{EMD}(\mathcal{E}, \mathcal{E}') = \min_{\{f\}} \sum_i \sum_j f_{ij} \theta_{ij}$$

(for equal total energy)

Event Space Distribution



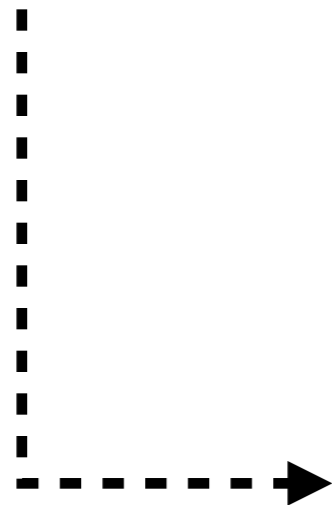
 = **Weighted Event**


$$\mathcal{E}(\hat{n}) = \sum_i E_i \delta(\hat{n} - \hat{n}_i)$$

$$w_a = \sigma_a$$

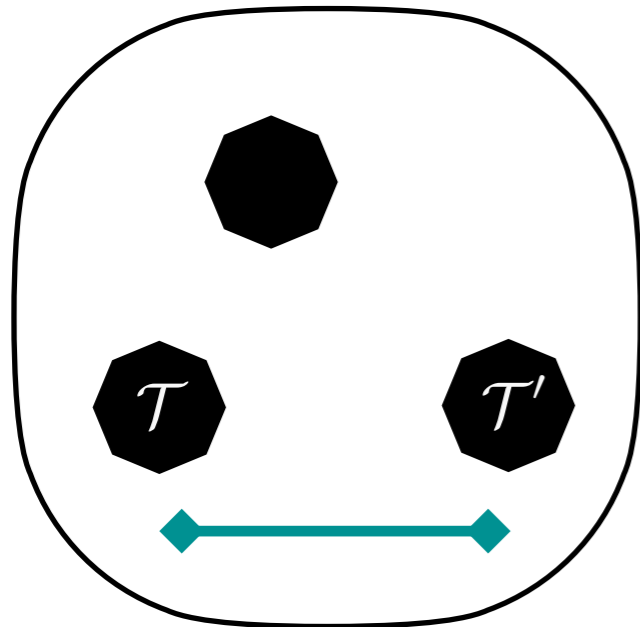
 = **EMD**
 Energy
 Mover's Distance


$$\text{EMD}(\mathcal{E}, \mathcal{E}') = \min_{\{f\}} \sum_i \sum_j f_{ij} \theta_{ij}$$
 (for equal total energy)




 = **Theory**

Theory Space



 = Theory

 = ΣMD
Cross-Section
Mover's Distance

$$\mathcal{T}(\mathcal{E}) = \sum_a \sigma_a \delta(\mathcal{E} - \mathcal{E}_a)$$

$$\Sigma\text{MD}(\mathcal{T}, \mathcal{T}') = \min_{\{\mathcal{F}\}} \sum_a \sum_b \mathcal{F}_{ab} \text{EMD}(\mathcal{E}_a, \mathcal{E}'_b)$$

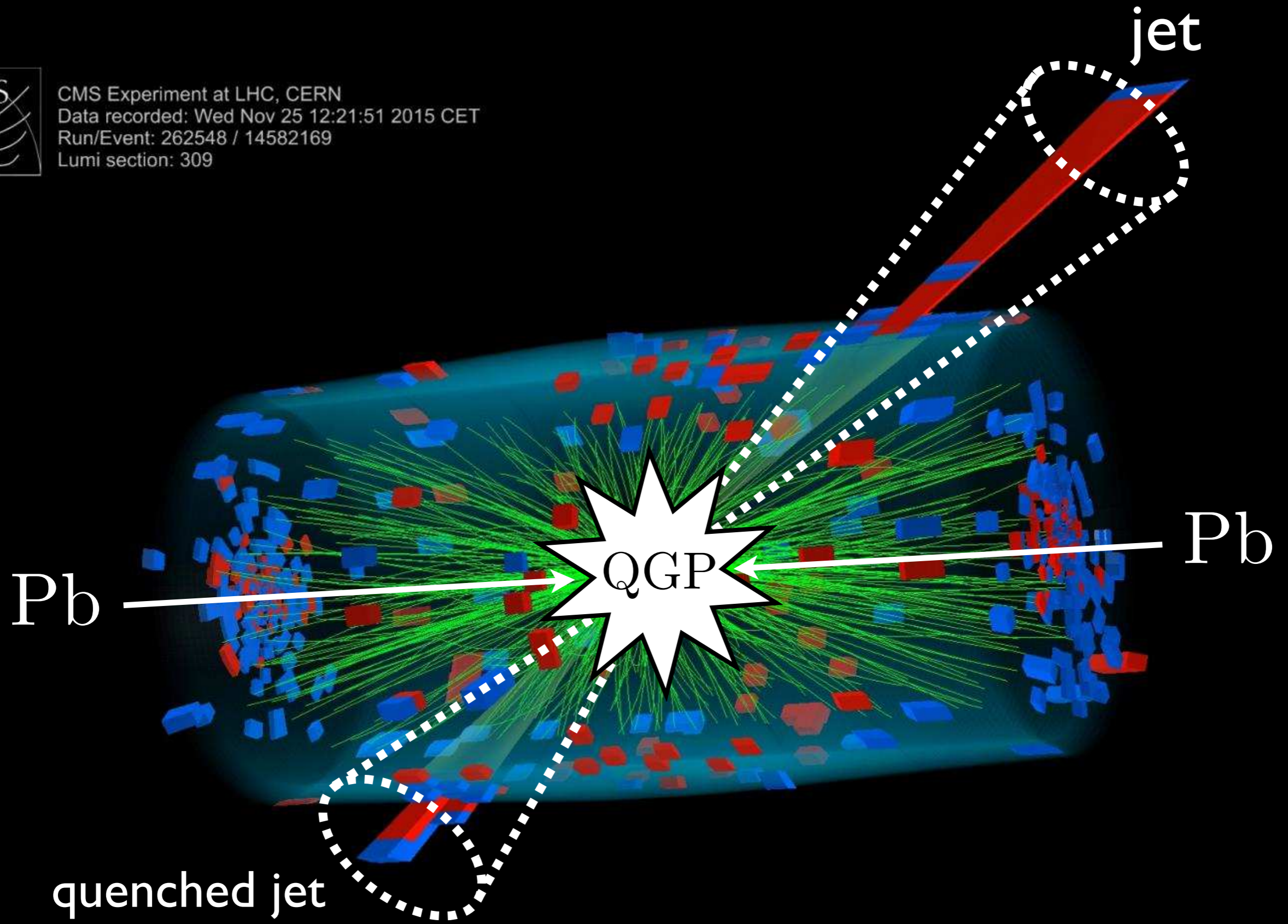
(for equal total xsec)

A distance between theories!

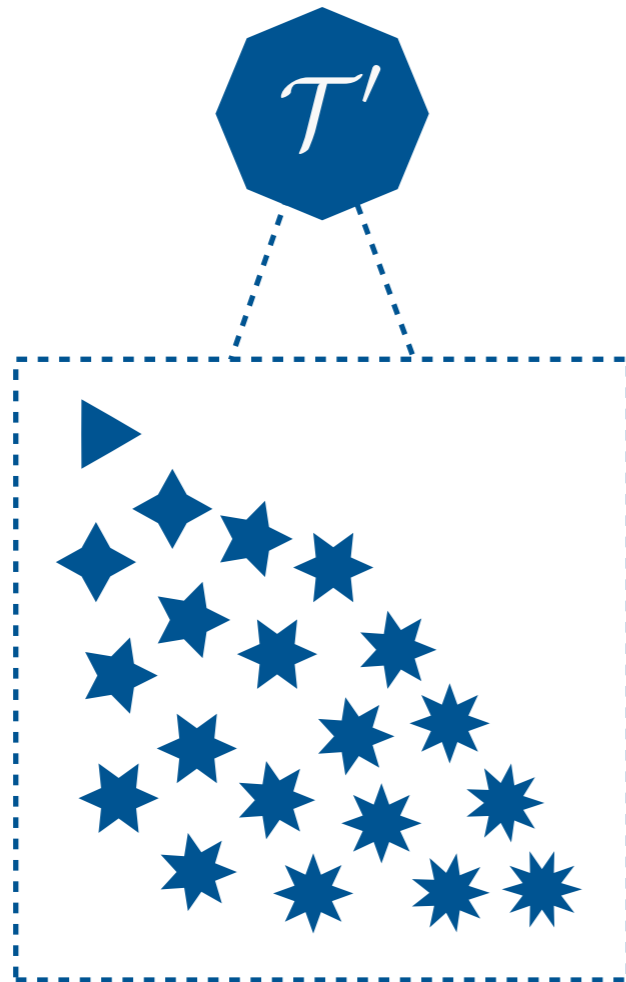
(e.g. EMD : N-jettiness :: ΣMD : k-eventiness)



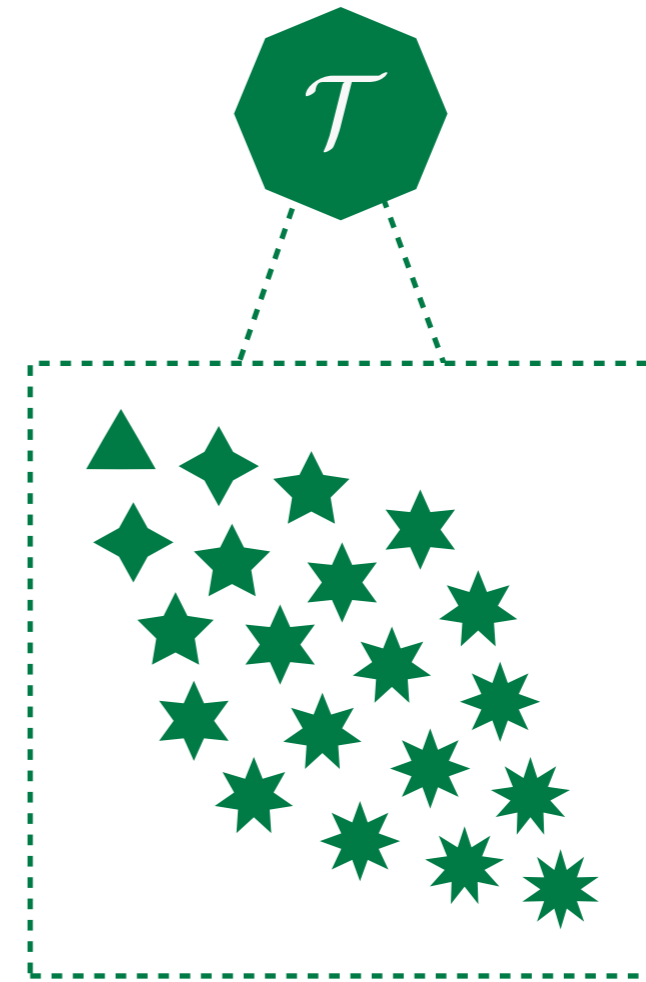
CMS Experiment at LHC, CERN
Data recorded: Wed Nov 25 12:21:51 2015 CET
Run/Event: 262548 / 14582169
Lumi section: 309



Theory Prime: In-Medium QCD



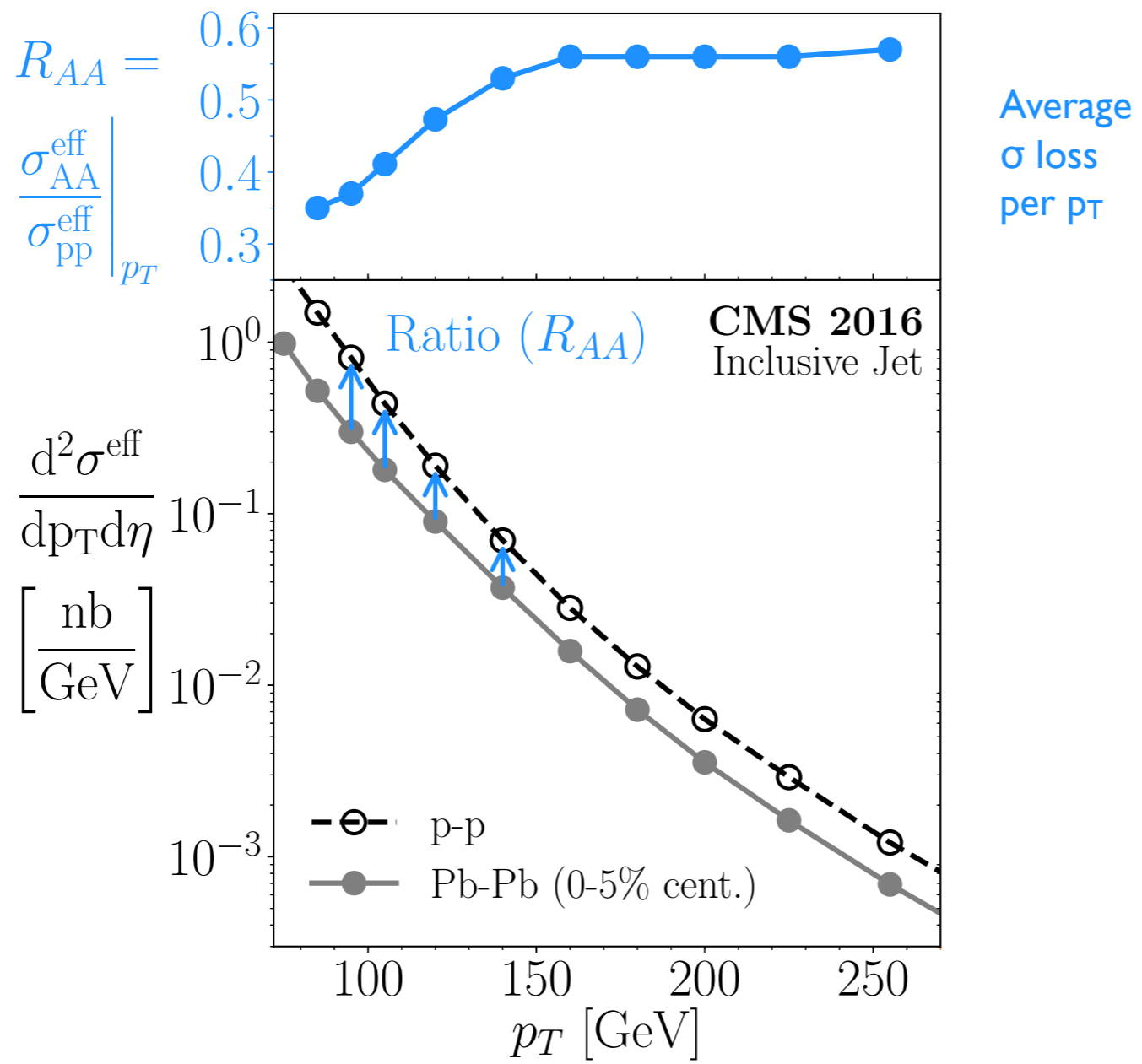
Theory: Vacuum QCD



Σ MMD

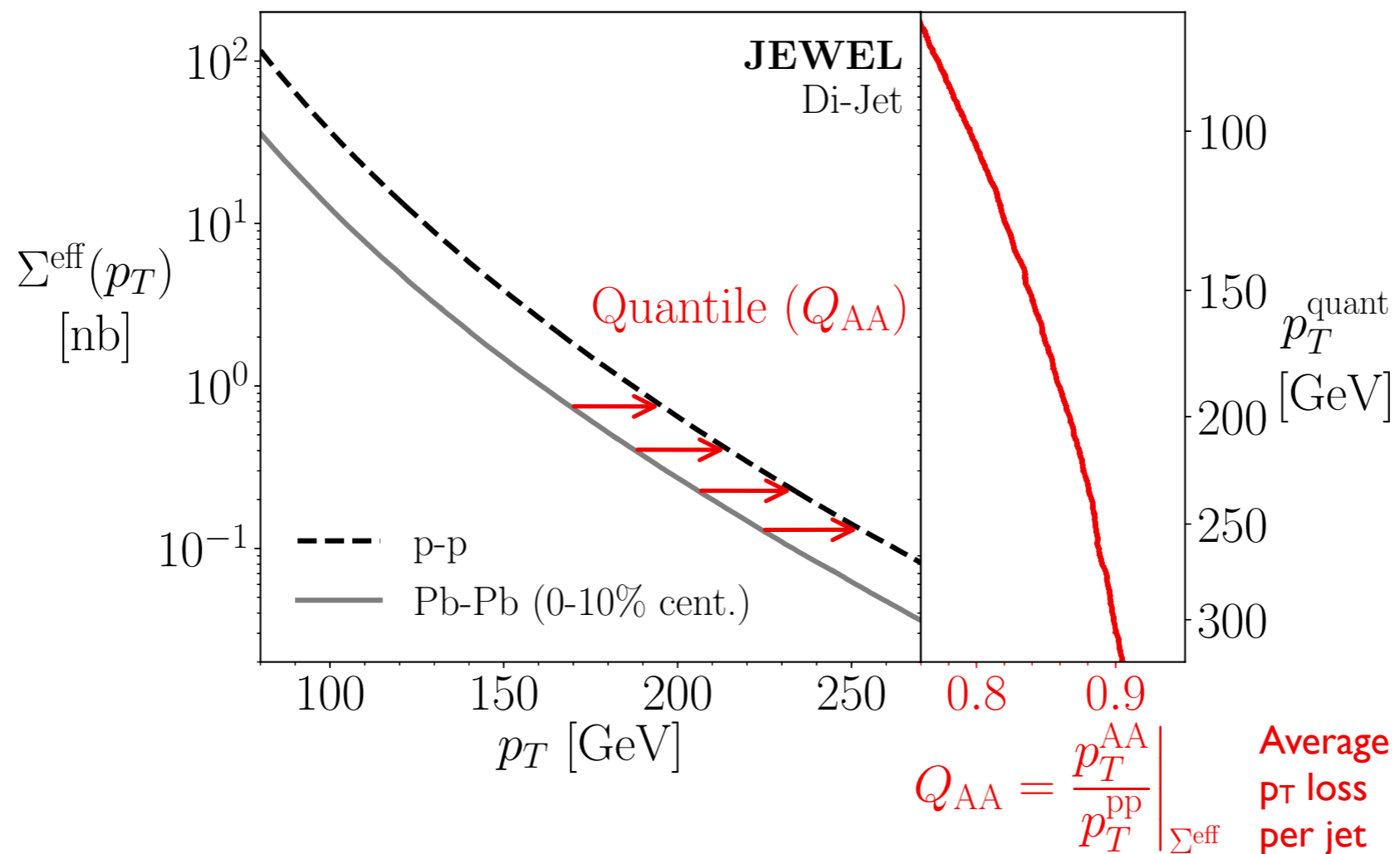


*Optimal transportation plan defines mapping
between in-medium jets and vacuum jets!*



Jet Quenching via Quantile Matching

Equivalent to following a geodesic in theory space (!)



[Brewer, Milhano, JDT, PRL 2019]



Backup Slides

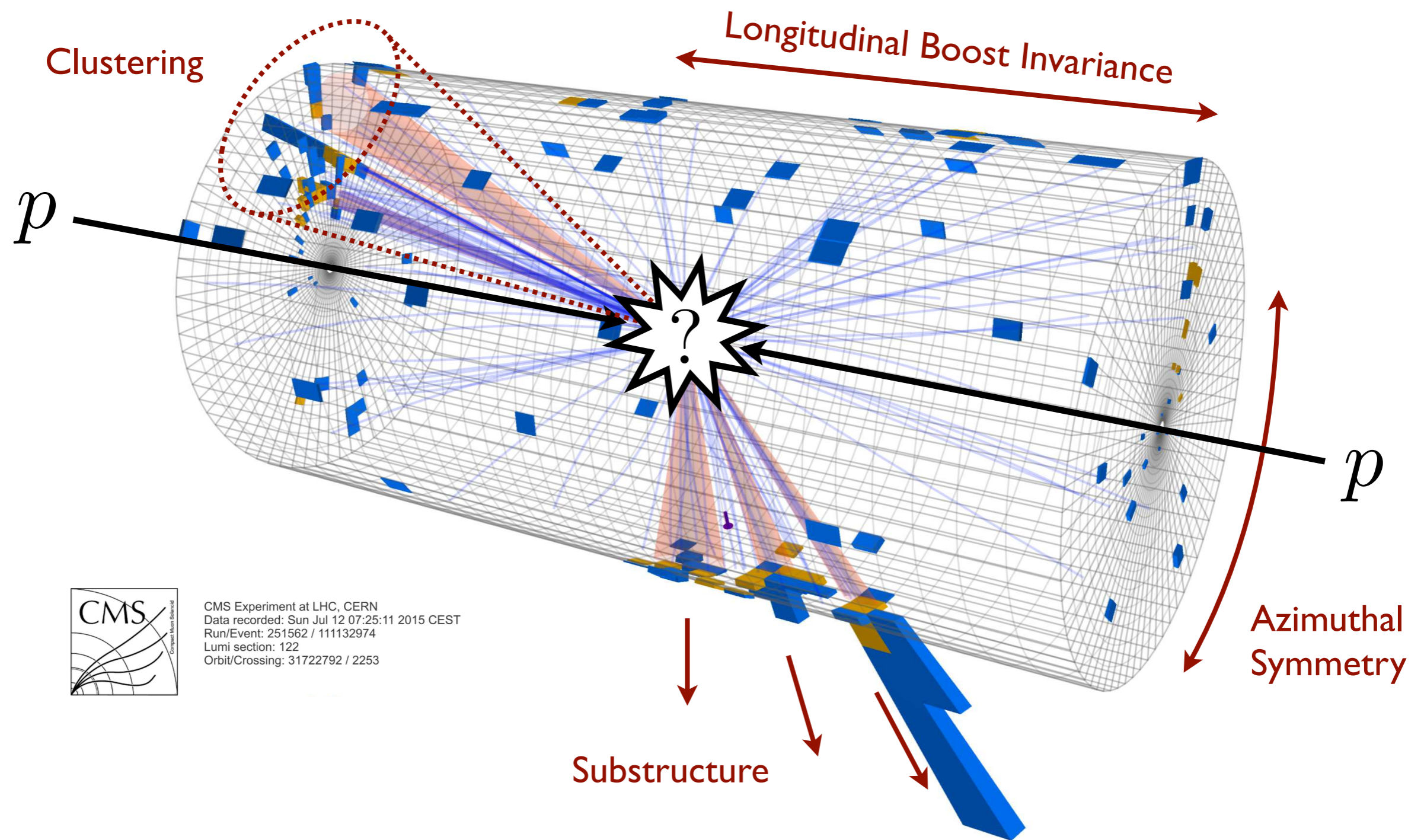
AI²: Ab Initio Artificial Intelligence



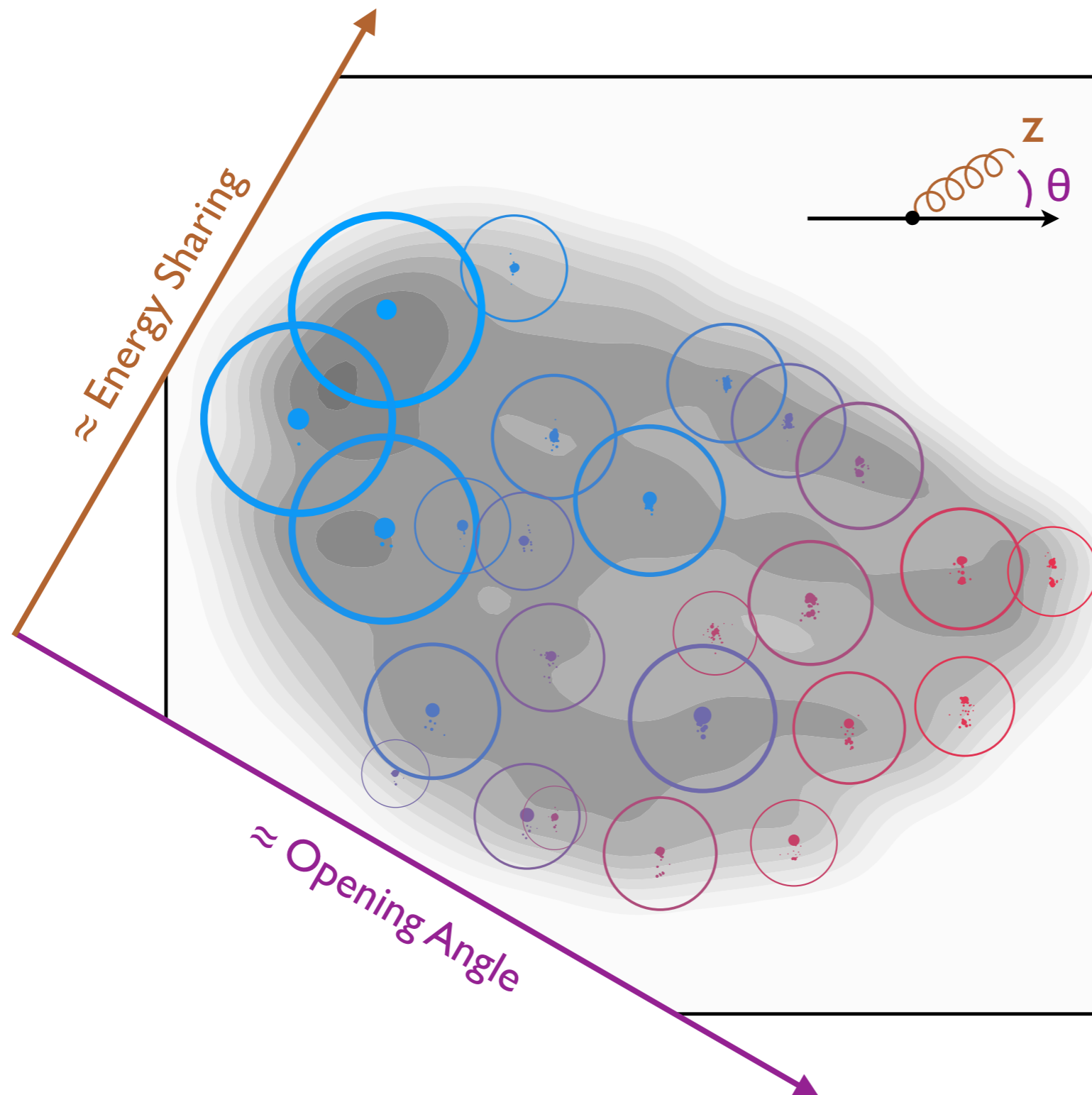
Machine learning that incorporates first principles, best practices, and domain knowledge from fundamental physics

Symmetries, conservation laws, scaling relations, limiting behaviors, locality, causality, unitarity, gauge invariance, entropy, least action, factorization, unit tests, exactness, systematic uncertainties, reproducibility, verifiability, ...

The Manifest Geometry of One Collision



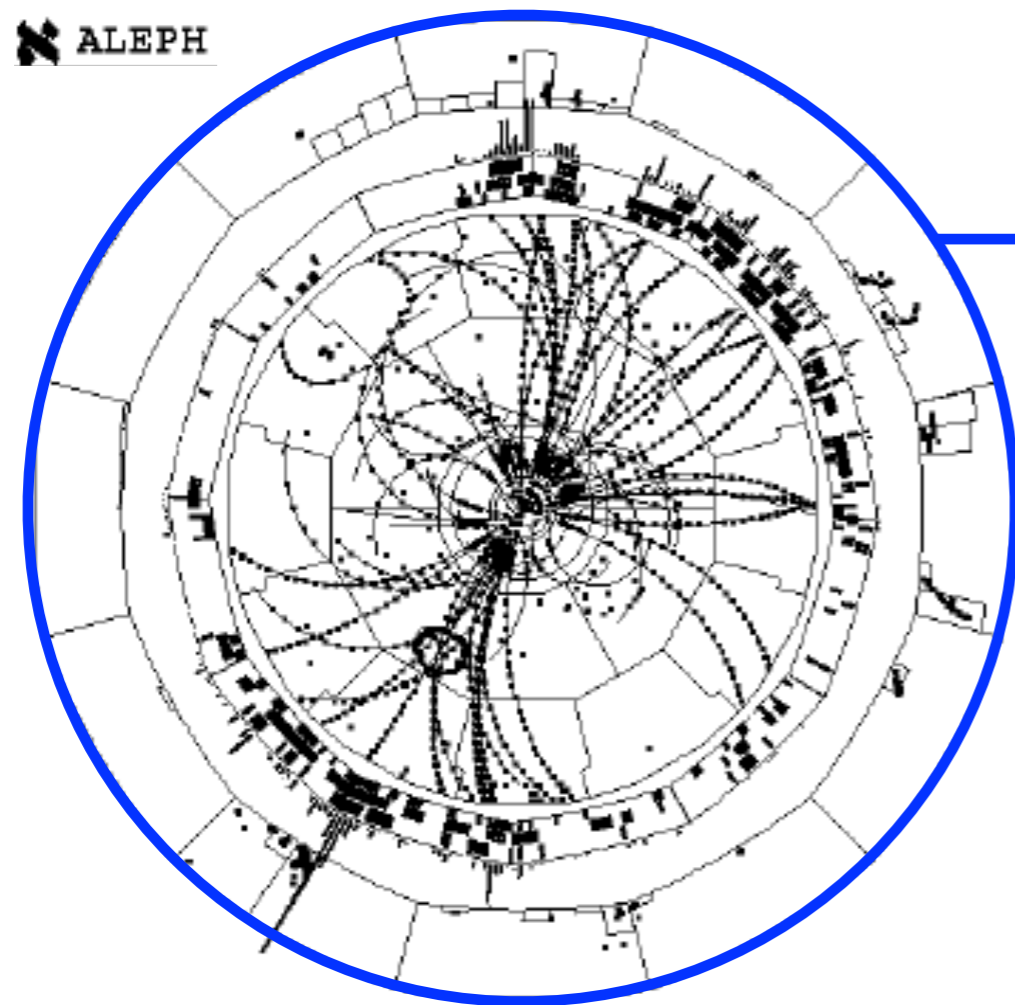
The Emergent Geometry of Many Collisions



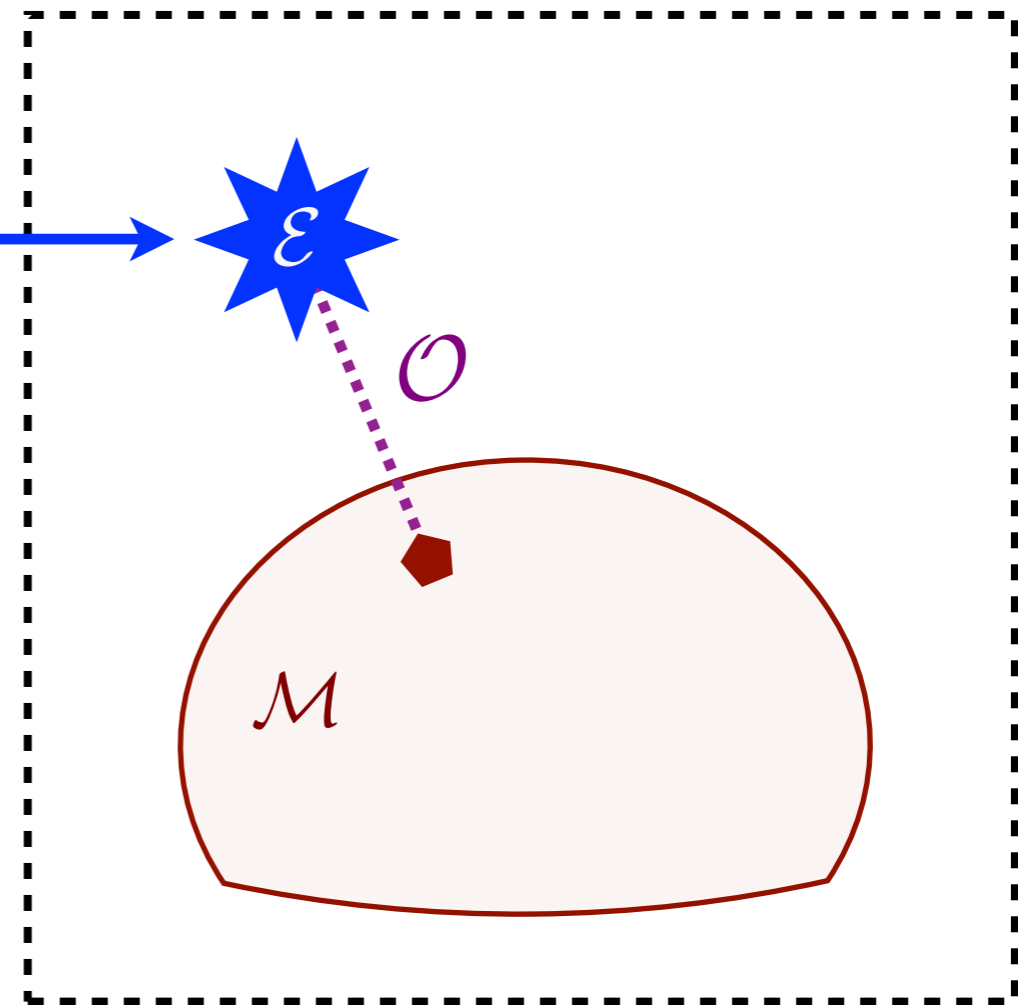
[Komiske, Mastandrea, Metodiev, Naik, JDT, PRD 2020;
based on Komiske, Metodiev, JDT, PRL 2019; using EnergyFlow and CMS Open Data]

The Hidden Geometry of Particle Collisions

E.g. Classic QCD Event Shapes



One Electron-Positron Event



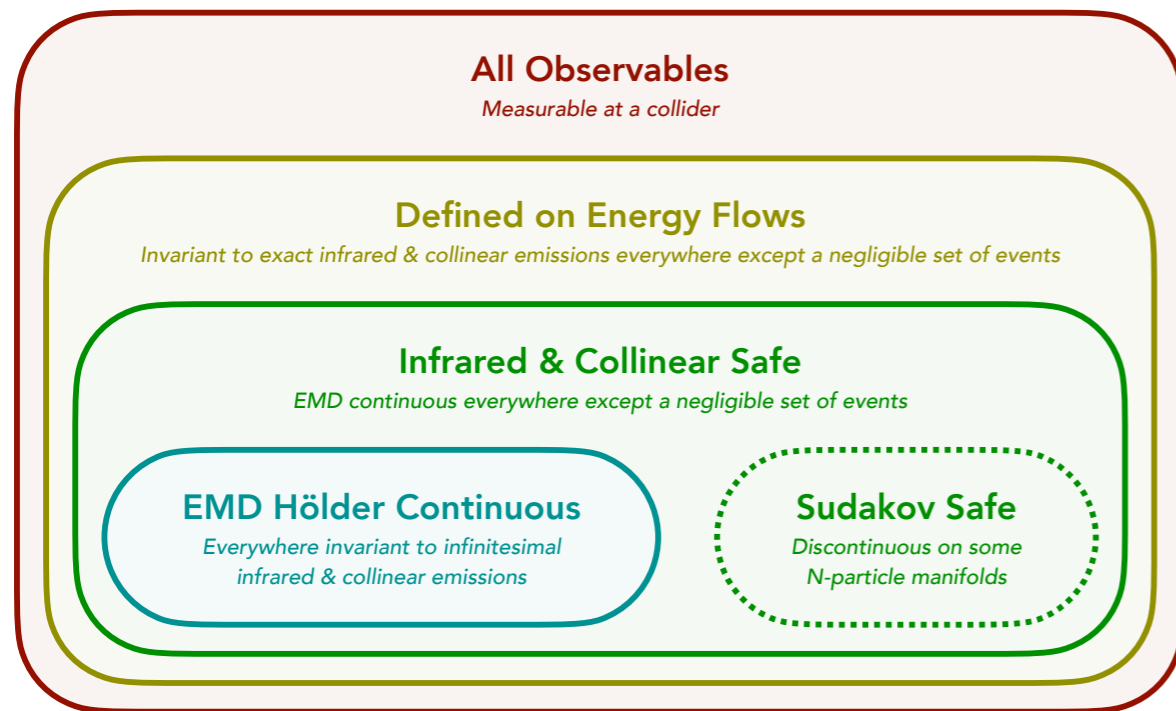
Distance to a Manifold in Event Space

[Komiske, Metodiev, JDT, JHEP 2020]

[Brandt, Peyrou, Sosnowski, Wroblewski, PL 1964; Farhi, PRL 1977]



Observable Taxonomy



All Observables	Comments
Multiplicity ($\sum_i 1$)	IR unsafe and C unsafe
Momentum Dispersion [65] ($\sum_i E_i^2$)	IR safe but C unsafe
Sphericity Tensor [66] ($\sum_i p_i^\mu p_i^\nu$)	IR safe but C unsafe
Number of Non-Zero Calorimeter Deposits	C safe but IR unsafe

Defined on Energy Flows	
Pseudo-Multiplicity ($\min\{N \mid \mathcal{T}_N = 0\}$)	Robust to exact IR or C emissions

Infrared & Collinear Safe	
Jet Energy ($\sum_i E_i$)	Disc. at jet boundary
Heavy Jet Mass [67]	Disc. at hemisphere boundary
Soft-Dropped Jet Mass [38, 68]	Disc. at grooming threshold
Calorimeter Activity [69] (N_{95})	Disc. at cell boundary

Sudakov Safe	
Groomed Momentum Fraction [39] (z_g)	Disc. on 1-particle manifold
Jet Angularity Ratios [37]	Disc. on 1-particle manifold
N -subjettiness Ratios [47, 48] (τ_{N+1}/τ_N)	Disc. on N -particle manifold
V parameter [36] (Eq. (2.11))	Hölder disc. on 3-particle manifold

EMD Hölder Continuous Everywhere	
Thrust [40, 41]	
Sphericity [42]	
Angularities [70]	
N -jettiness [44] (\mathcal{T}_N)	
C parameter [71–74]	Resummation beneficial at $C = \frac{3}{4}$
Linear Sphericity [72] ($\sum_i E_i n_i^\mu n_i^\nu$)	
Energy Correlators [36, 75–77]	
Energy Flow Polynomials [15, 17]	

[Komiske, Metodiev, JDT, JHEP 2020; cf. Sterman, PRD 1979; Banfi, Salam, Zanderighi, JHEP 2005; Larkoski, Marzani, JDT, PRD 2015]