EW corrections at very high energies

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Based on
1601.07190, 1703.08562
as well as 1712.xxxx

Comparison to LHC data requires precise theoretical calculations

- To match the precision of experimental uncertainties NLO_{QCD} calculations are required for most observables, and NNLO calculations are becoming more and more important
- In some cases even N^3LO calculations are available
- While EW corrections are typically much smaller, NLO_{EW} are of same order as $NNLO_{QCD}$ effects
- EW corrections become more and more important as energy of collision increases.

Size of EW corrections grow due to EW Sudakov logarithms and can become dominant effect

For every power of α_{EW} there are two powers of

$$\log(s / m_W^2)$$

This means that the EW expansion is not in α_{EW} but in

$$\alpha_{EW} \log(s / m_W^2)$$

This means that for $s \gg m_W$ EW corrections can become more important than QCD corrections

- Quick overview of EW Sudakov logarithms

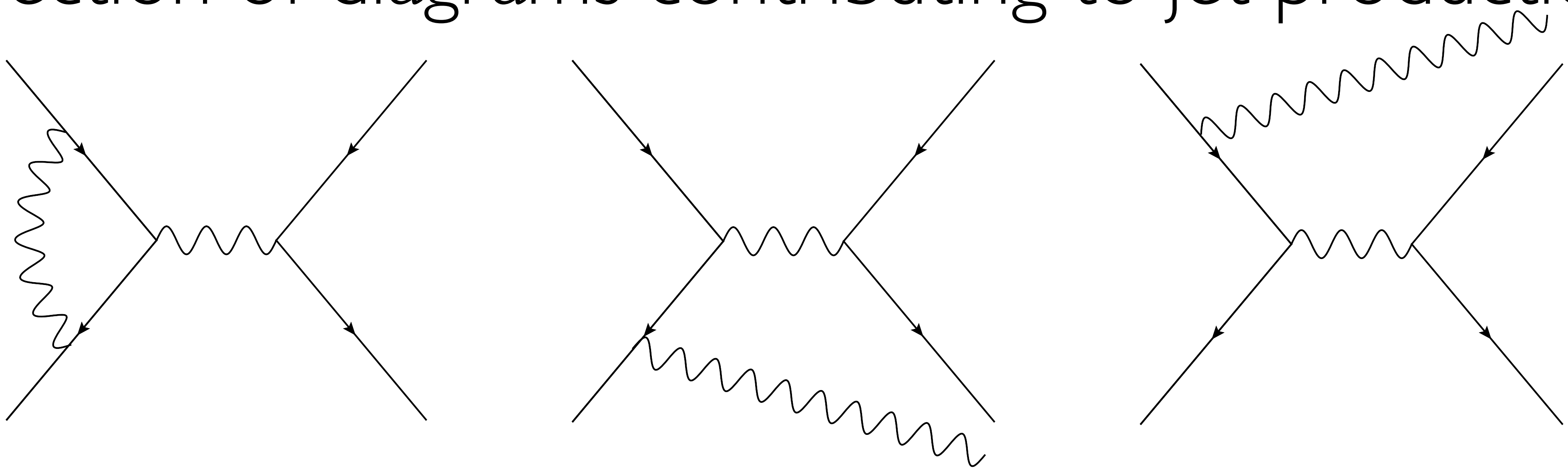
- Large logs in virtual and real contributions

- DGLAP evolution in the full SM

- Combining NLO and LL electroweak calculations

Higher order QCD calculations involve IR divergent contributions that cancel when calculating observables

Selection of diagrams contributing to jet production



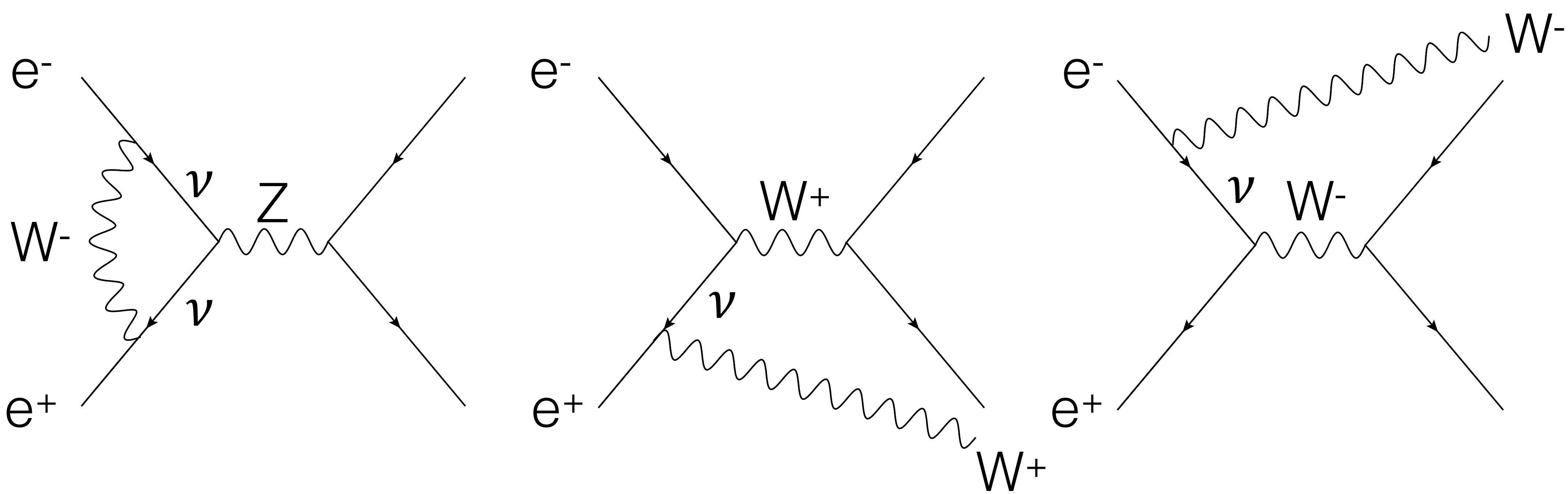
Any observable gets contributions from virtual and real corrections

Both virtual and real are separately IR divergent

All divergences cancel when virtual and real are properly combined (KLN theorem)

Electroweak Sudakov logarithms arise from exchanges of electroweak gauge bosons

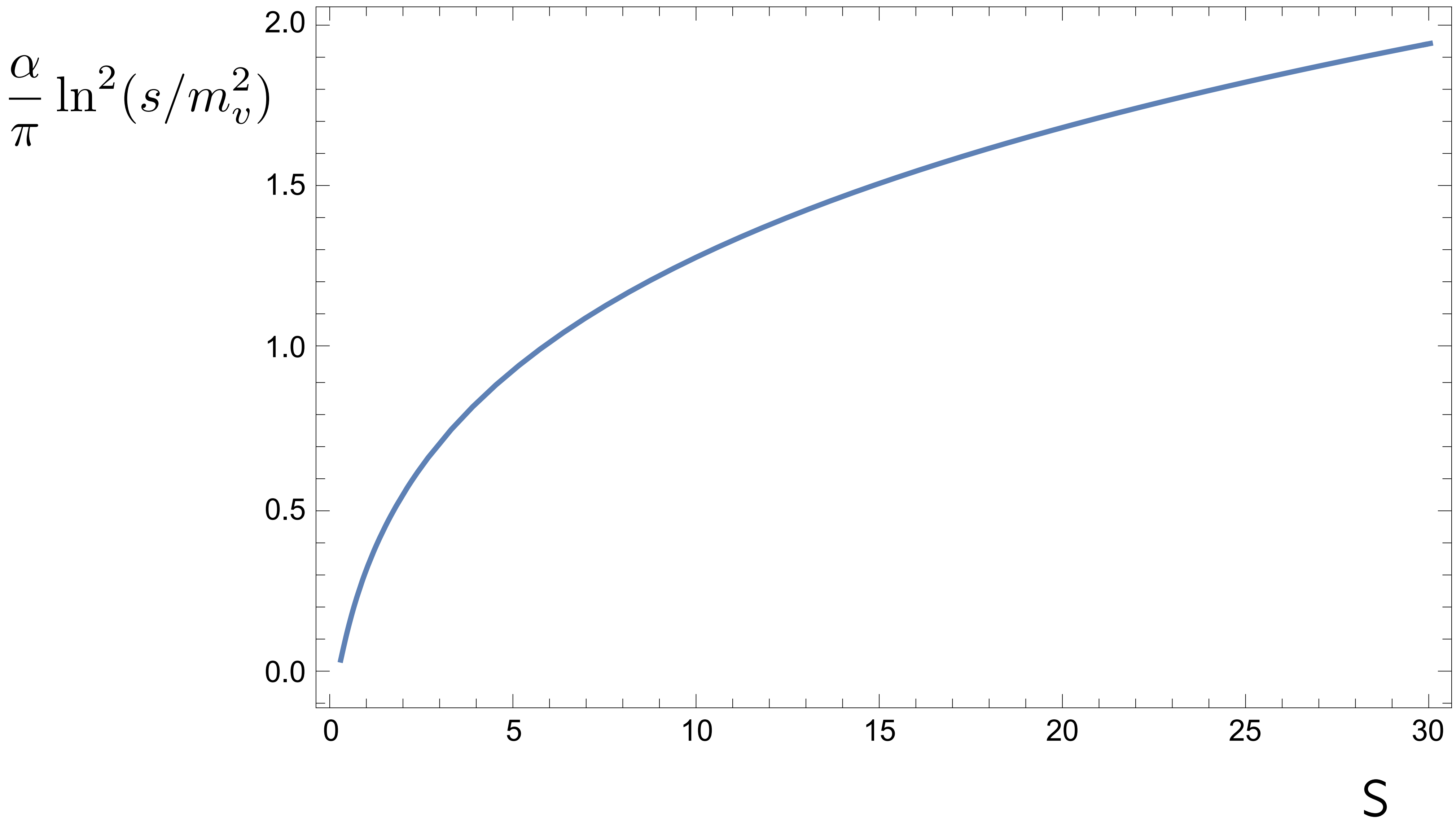
Similar set of diagrams for EW corrections, but with W/Z instead of gluons



For massive W, IR divergences turn into $\log(m_W^2/s)$, and generally have two powers per power of alpha

Both virtual and real sensitive to $\log(m_W^2/s)$

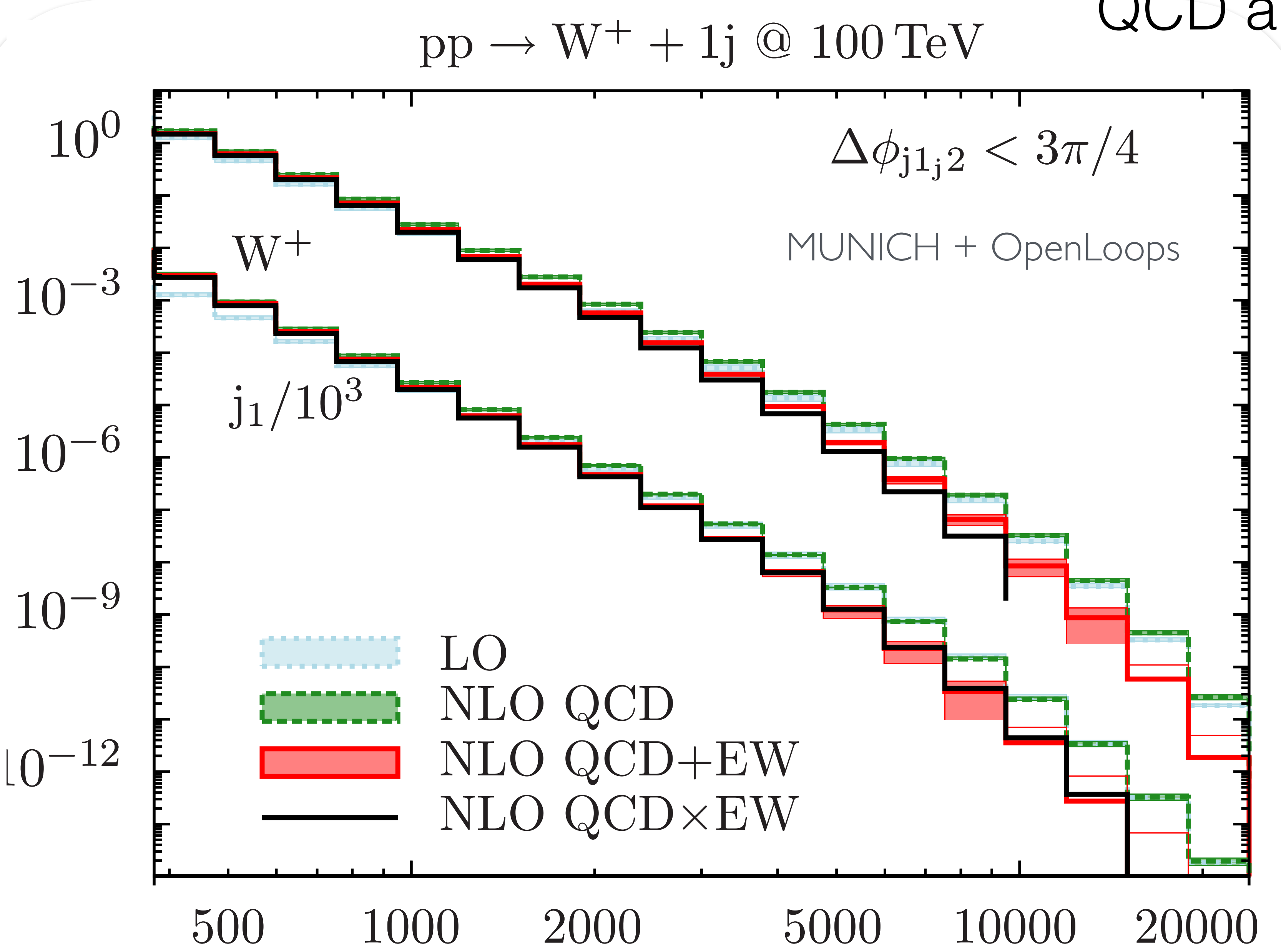
The numerical effect of EW Sudakov logarithms becomes large at high energies



No sense in which electroweak corrections are small

Fixed order results at a future 100 TeV machine show that EW corrections are much larger than QcD corrections

Lindert,
QCD and EW at 100 TeV colliders



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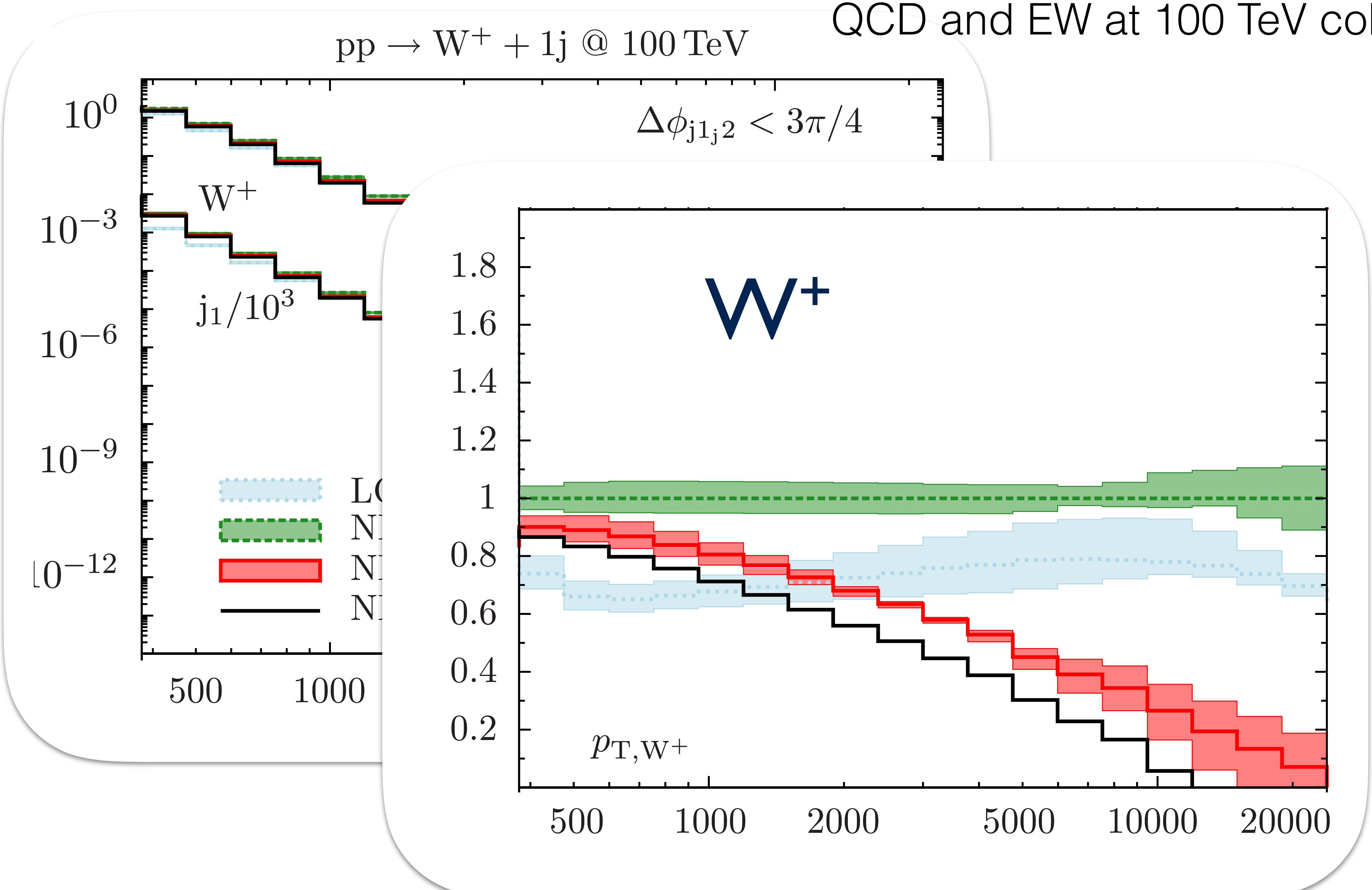
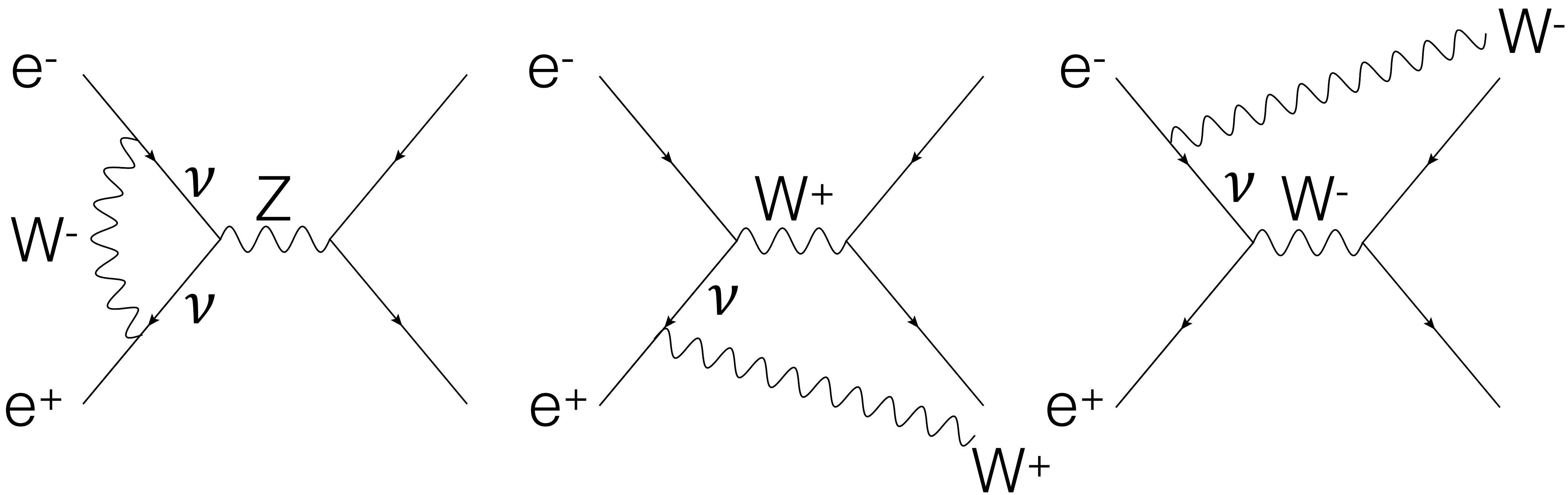


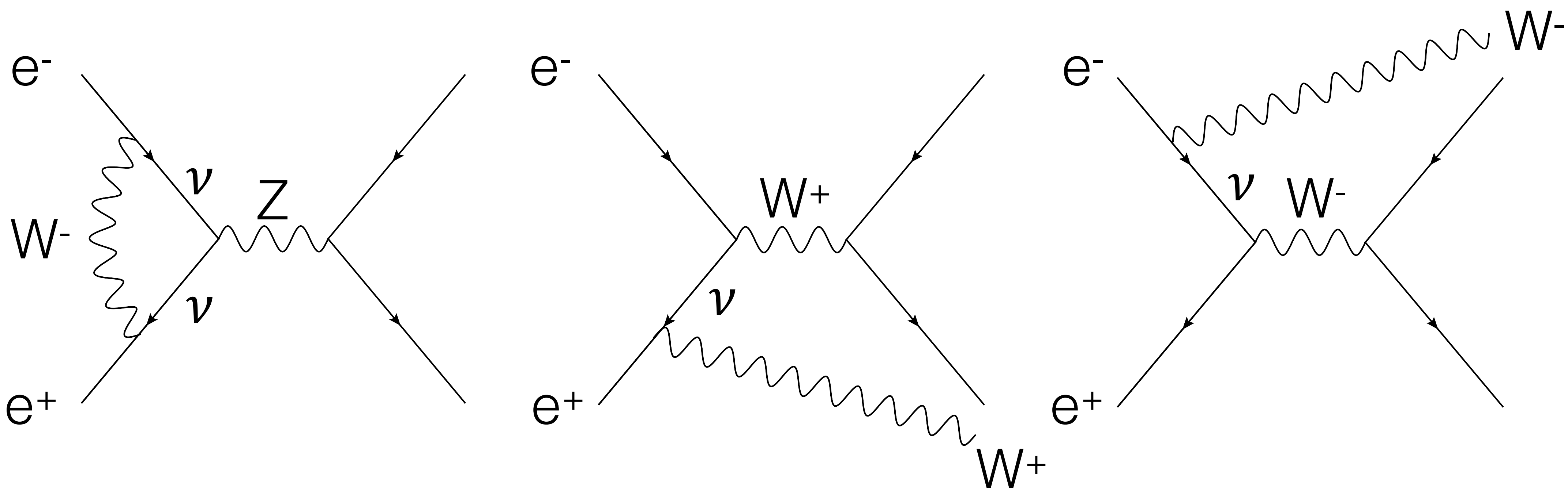
Since no existing experiment collides SU(2) singlets, cancellation between virtual and real logs incomplete



Incomplete cancellation since the collider only collides electrons, not neutrinos.

For proton colliders, SU(2) breaking since $f_{u/p}(x,q) \neq f_{d/p}(x,q)$

Lesson 1



Electroweak correction give rise to logarithmic terms in any observable

Logarithmic effects in virtual corrections have been resummed in SCET quite a while ago

Chiu, Golf, Kelley, Manohar, ('08)

Full theory

$\mu = Q$ ———————— $C(Q,\mu)$

$\text{SCET}_{W,Z,\gamma}$ (M=0)

↓ γ_{SCET}

$\text{SCET}_{W,Z,\gamma}$ (M≠0)

$\mu = m_V$ ———————— $D(m_V,\mu)$

SCET_{γ}

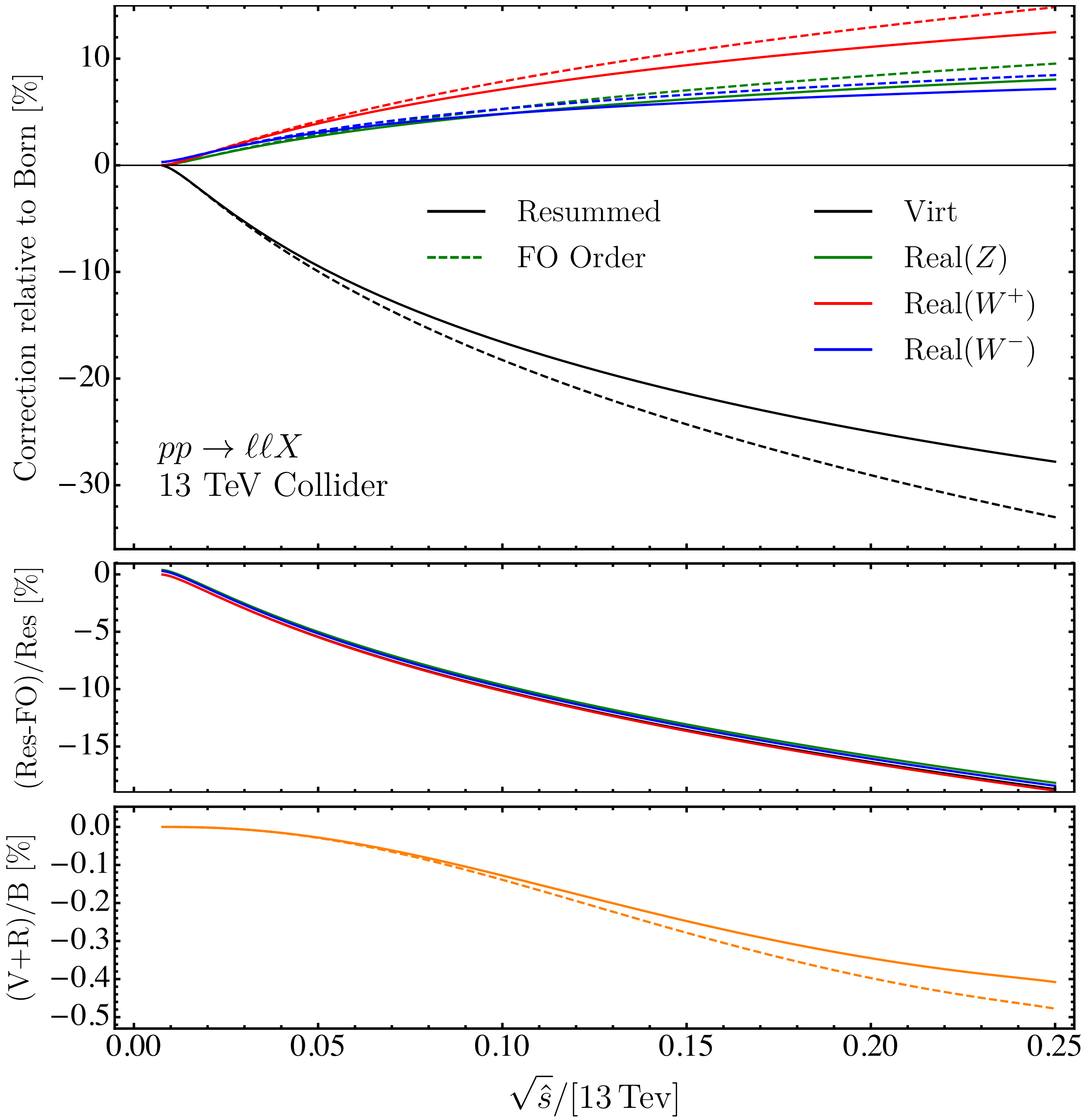
Problem is completely solved at NLL'
for any process

Resummation of LL logarithms in real radiation can be obtained using analogy with parton shower

CWB, Ferland ('16)

- Parton shower properly resums LL
- Analytically compute first emission of parton shower
- From virtual results know Sudakov factors
- Combining with splitting function get resummed emission probability
- Integrating that result gives the 1-emission cross section integrated over phase space

This gives appreciable effects at 13 TeV LHC and future 100 TeV collider



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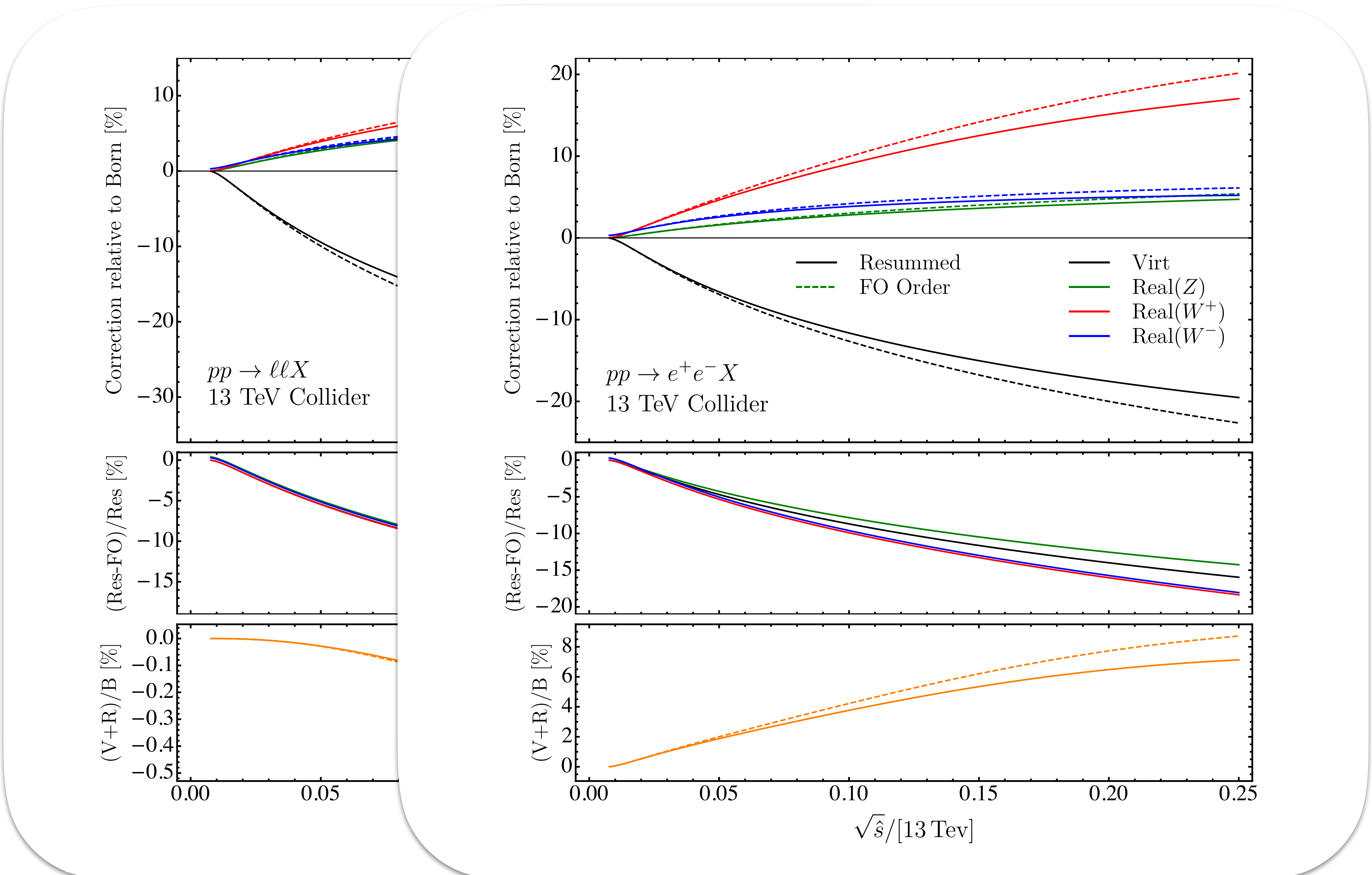

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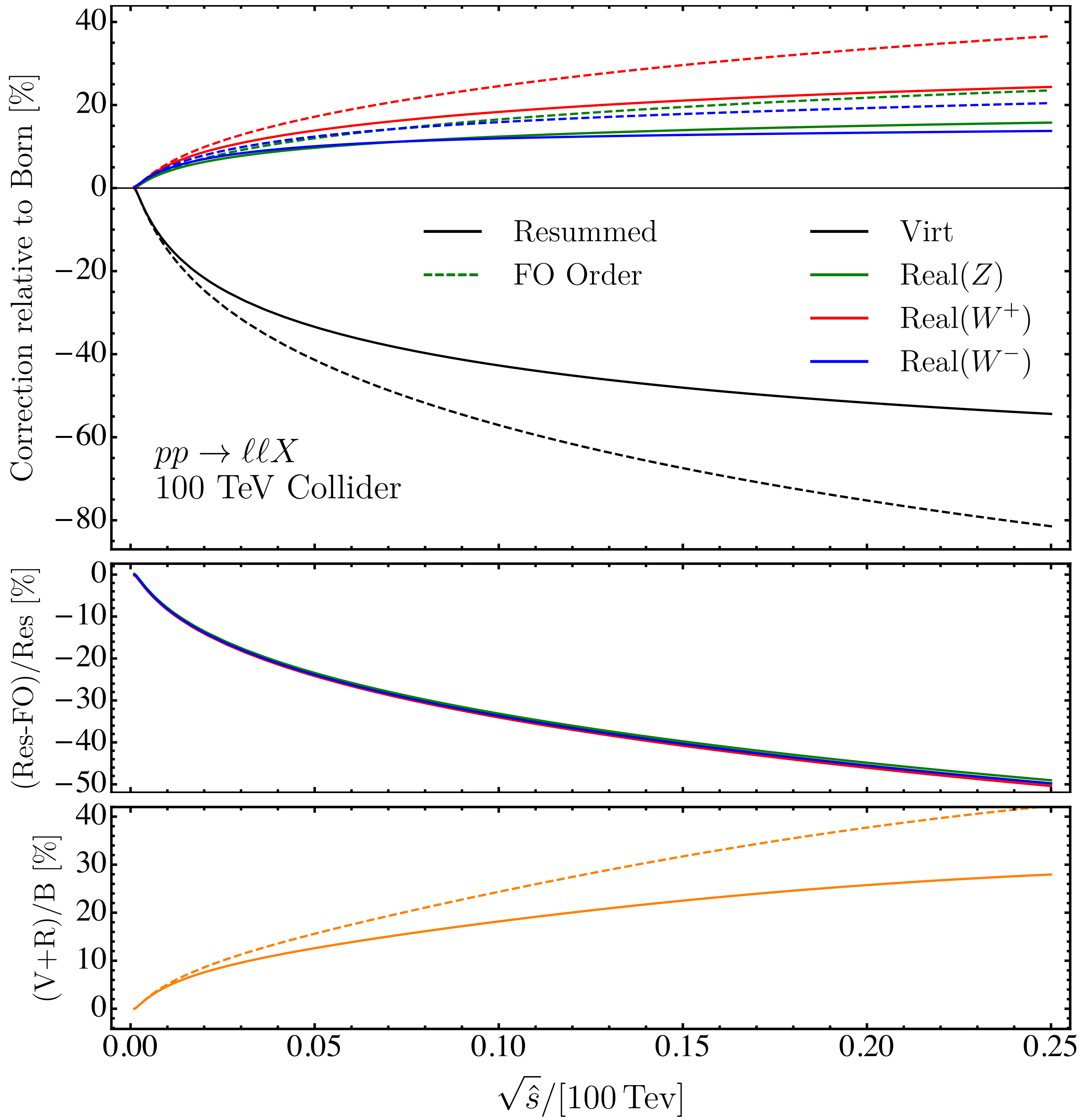

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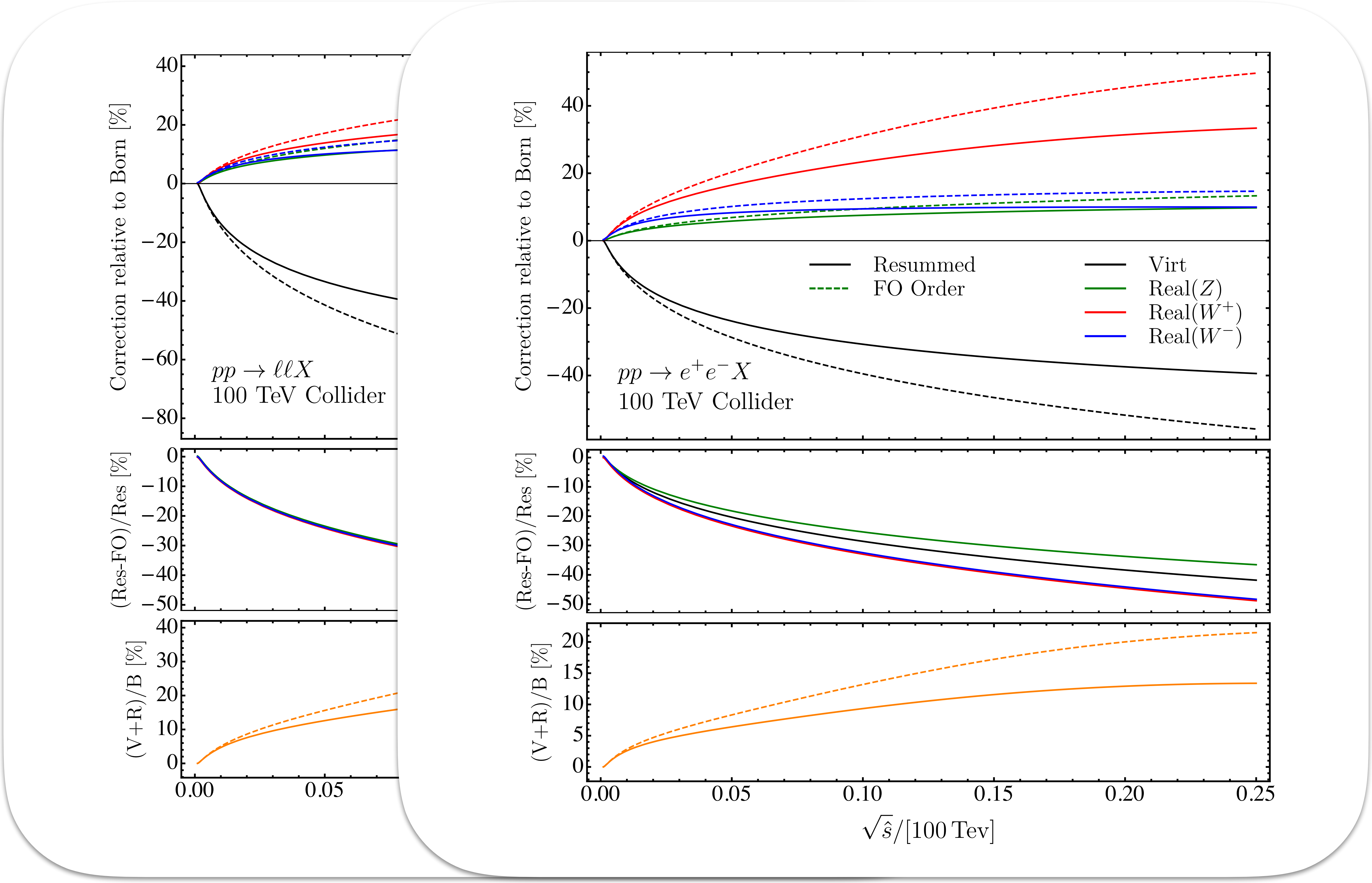
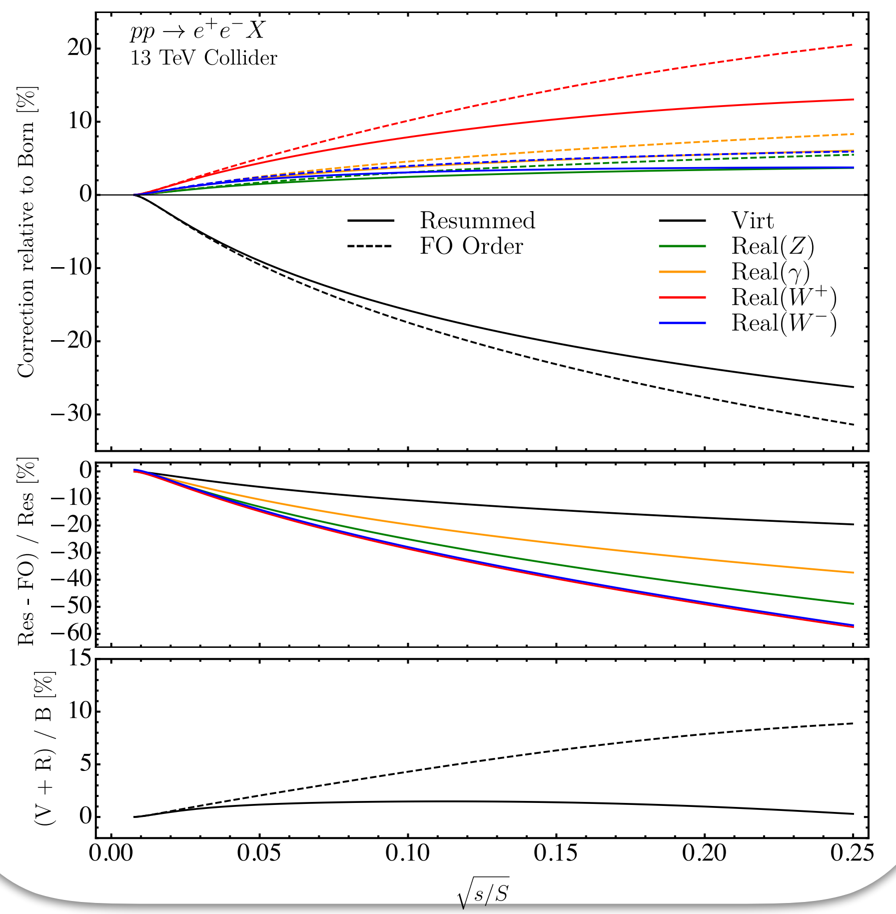
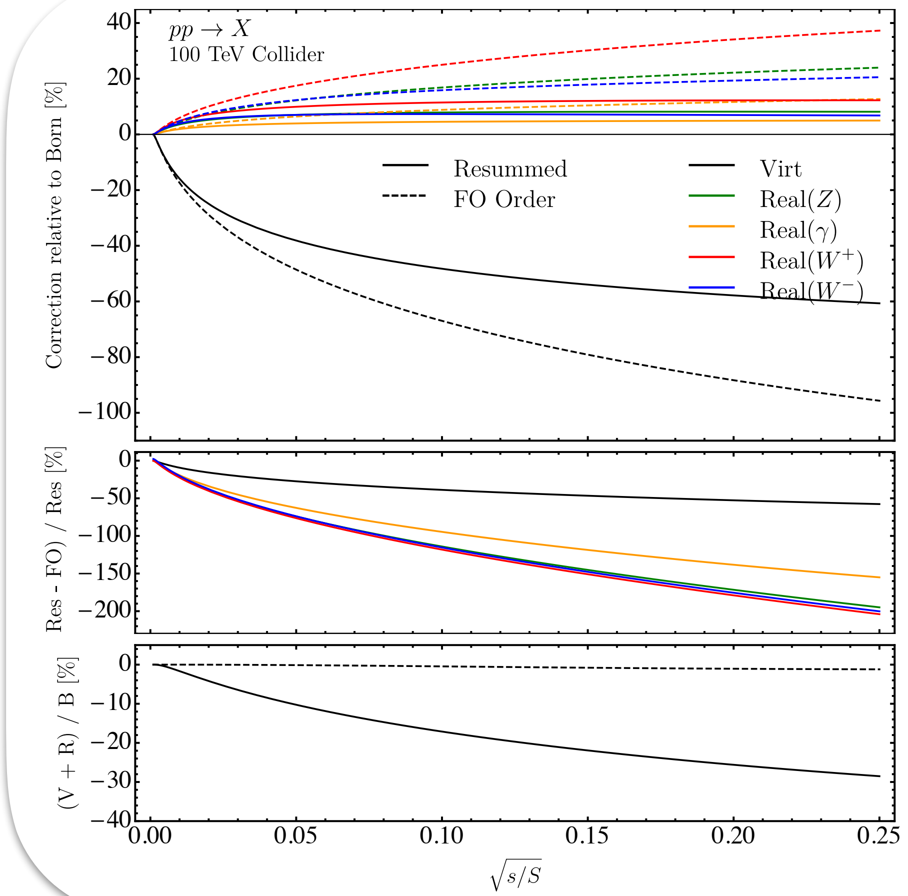


Lesson 2





Resummation of electroweak corrections can lead to large effects, especially at 100 TeV

To understand the irreducible EW logarithms one needs to understand PDF evoltuion

- As already discussed, irreducible logarithms arise from the fact that initial states are not SU(2) singlets
- For pp colliders, this manifests itself in the fact that $f_u \neq f_d$ etc
- However, logarithms from initial state radiation can be resummed using DGLAP evolution

Parton distribution functions are matrix elements of collinear bi-local operators

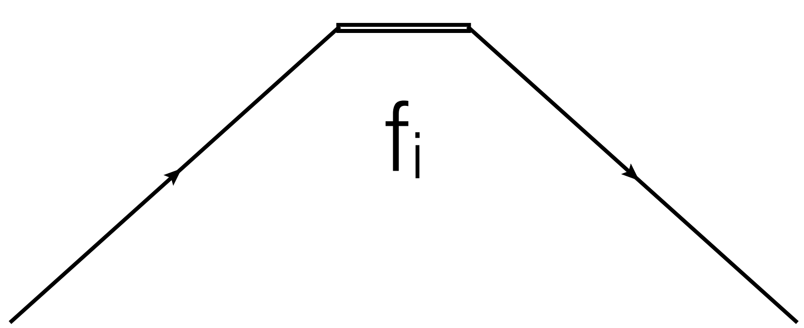
CWB, Ferland, Webber ('17)
see also Ciafaloni, Comelli ('00-'05)

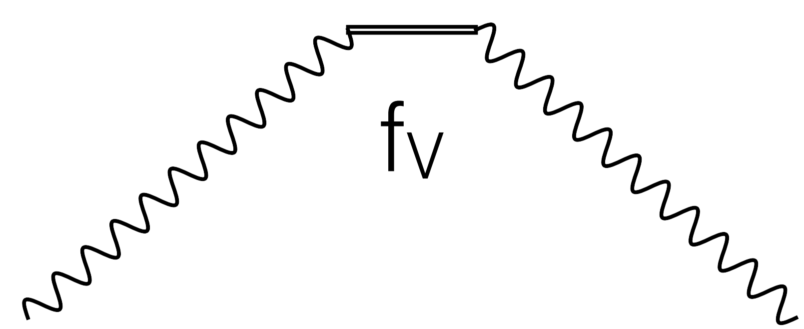
Parton distribution functions are matrix elements of collinear operators of field separated along the light-cone

$$f_i(x) = x \int \frac{dy}{2\pi}\, e^{-i\,2x\bar{n}\cdot p\, y} \langle p \big| \, \bar{\psi}^{(i)}(y)\, \not{\bar{n}}\, \psi^{(i)}(-y) \big| p \rangle$$

$$f_V(x) = \frac{2}{\bar{n}\cdot p} \int \frac{dy}{2\pi}\, e^{-i\,2x\bar{n}\cdot p\, y}\, \bar{n}_\mu \bar{n}^\nu \langle p \big| \, V^{\mu\lambda}(y) V_{\lambda\nu}(-y) \big| p \rangle \Big|_{\text{spin avg.}}$$

Diagramatically, can think of them as



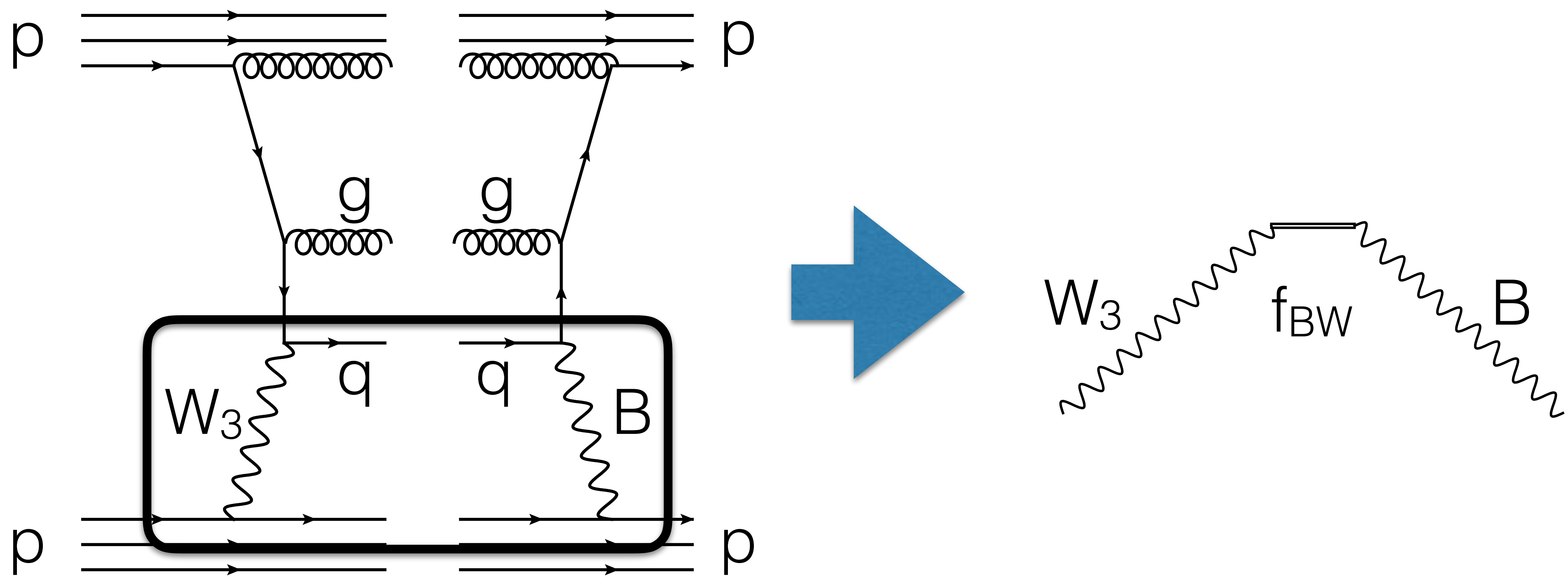


Once full SM evolution is considered, need pdf for every particle (including Higgs)

Besides these “standard” forward pdf’s, one also needs to consider non-forward, mixed pdf’s

$$f_{BW}(x) = \frac{1}{2}\left(\frac{2}{\bar{n}\cdot p}\int\frac{dy}{2\pi}\, e^{-i\,2x\bar{n}\cdot p\, y}\, \bar{n}_\mu \bar{n}^\nu \langle p|\, B^{\mu\lambda}(y) W^3_{\lambda\nu}(-y)|p\rangle\Big|_{\text{spin avg.}} + \text{h.c.}\right)$$

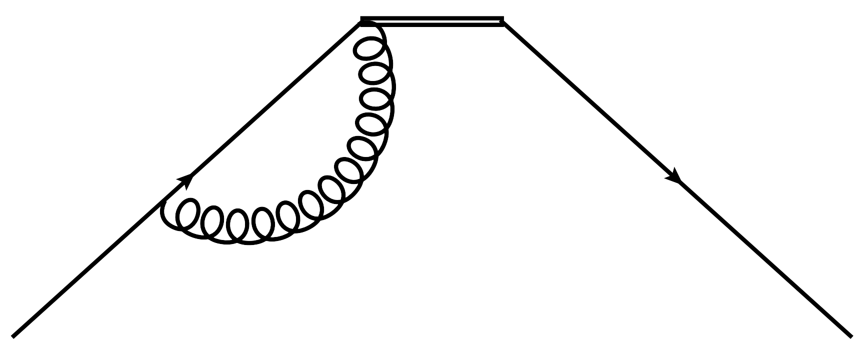
This pdf is required to describe mixed processes with Z or gamma in initial state

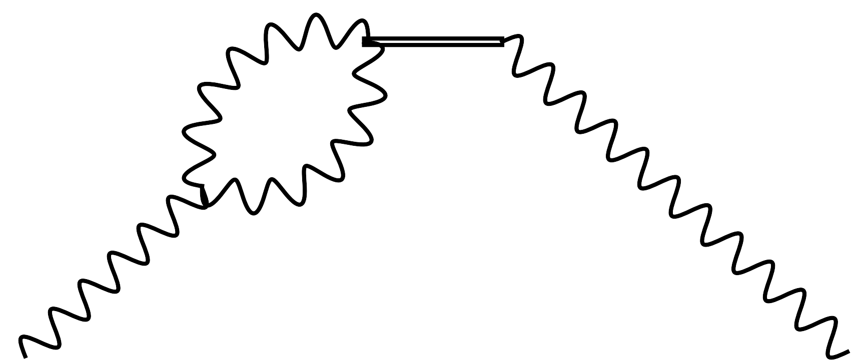


DGLAP equations are simply renormalization group equations of these operators

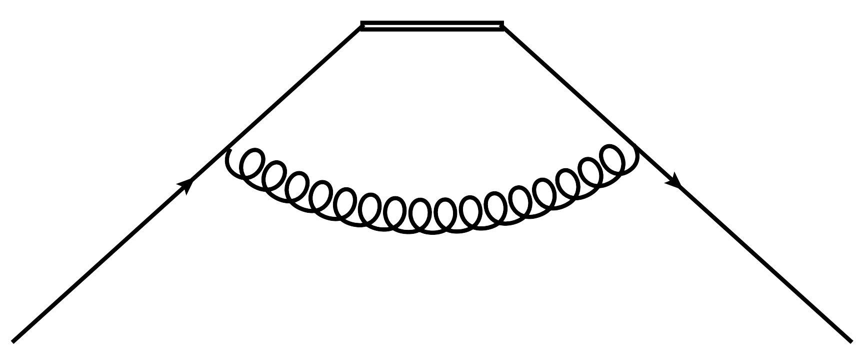
As for any operator in field theory depend on renormalization scale, and RGE is derived from divergent structure of loops

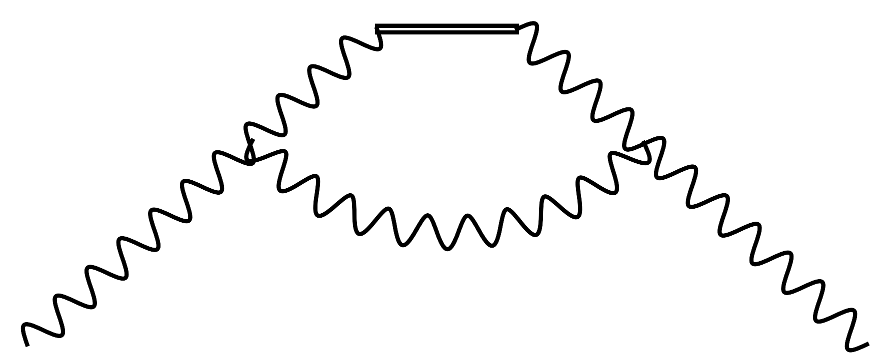
Virtual contributions have loop stay on same side of operator





Real contributions have loop go from one side to other





There most general form of the DGLAP equation has a very simple form

$$q\frac{\partial}{\partial q} f_i(x,q) = \sum_I \frac{\alpha_I(q)}{\pi}\left[P^V_{i,I}(q)\, f_i(x,q) + \sum_j C_{ij,I}\int_x^{z^{ij,I}_{\max}(q)} \mathrm{d}z\, P^R_{ij,I}(z) f_j(x/z,q)\right]$$

Can define a Sudakov factor by exponentiating virtual piece

$$\Delta_{i,I}(q) = \exp\left[\int_{q_0}^{q} \frac{\mathrm{d}q'}{q'}\frac{\alpha_I(q')}{\pi} P^V_{i,I}(q')\right]$$

Allows to write a slightly simpler form of the DGLAP equation

$$\Delta_i(q)\, q\frac{\partial}{\partial q}\left[\frac{f_i(x,q)}{\Delta_i(q)}\right] = \sum_I \frac{\alpha_I(q)}{\pi}\sum_j C_{ij,I} P^R_{ij,I} \otimes f_j$$

DGLAP equations are very similar to form used in QCD, just with different coefficients (and some new splitting functions)

$$\Delta_i(q)\, q \frac{\partial}{\partial q}\left[\frac{f_i(x,q)}{\Delta_i(q)}\right] = \sum_I \frac{\alpha_I(q)}{\pi} \sum_j C_{ij,I} P^R_{ij,I} \otimes f_j$$

In QCD reduces to the well known results

$$\left[\Delta_{q,3}\, q \frac{\partial}{\partial q} \frac{f_q}{\Delta_{q,3}}\right]_3 = \frac{\alpha_3}{\pi}\left[C_F P^R_{ff,G} \otimes f_q + T_R P^R_{fV,G} \otimes f_g\right],$$

$$\left[\Delta_{g,3}\, q \frac{\partial}{\partial q} \frac{f_g}{\Delta_{g,3}}\right]_3 = \frac{\alpha_3}{\pi}\left[C_A P^R_{VV,G} \otimes f_g + \sum_f C_F P^R_{Vf,G} \otimes f_q\right]$$

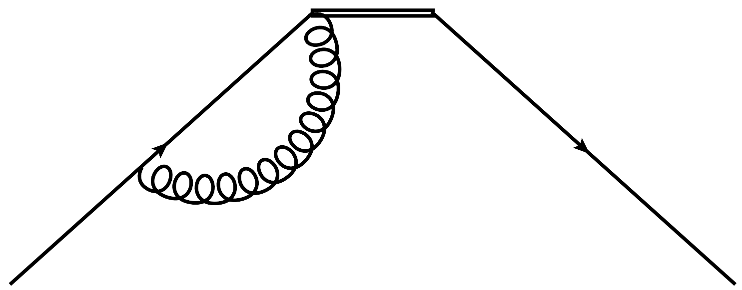
Lesson 3

$$\Delta_i(q)\, q \frac{\partial}{\partial q}\left[\frac{f_i(x,q)}{\Delta_i(q)}\right] = \sum_I \frac{\alpha_I(q)}{\pi} \sum_j C_{ij,I} P^R_{ij,I} \otimes f_j$$

DGLAP equation same as QCD, just need more splitting functions and new coefficients

For usual QCD evolution of PDF's solution to DGLAP is only single logarithmic

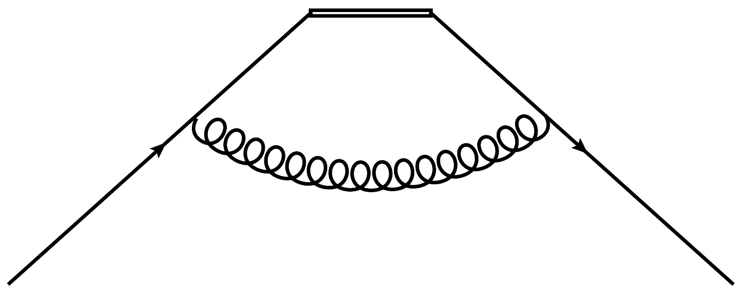
Consider evolution of quark pdf:



Virtual

$$t\frac{\mathrm{d}}{\mathrm{d}t}f_u(x,t) = \frac{\alpha\, C_F}{\pi}P_q^V(t) f_u(x,t)$$

$$P_q^V(t) = -\int_0^{z_{\max}(t)} \mathrm{d}z\, P_{qq}(z)$$



Real

$$t\frac{\mathrm{d}}{\mathrm{d}t}f_q(x,t) = \frac{\alpha\, C_F}{\pi}\int_x^{z_{\max}(t)} \mathrm{d}z\, P_{qq}(z) f_q(x/z,t)$$

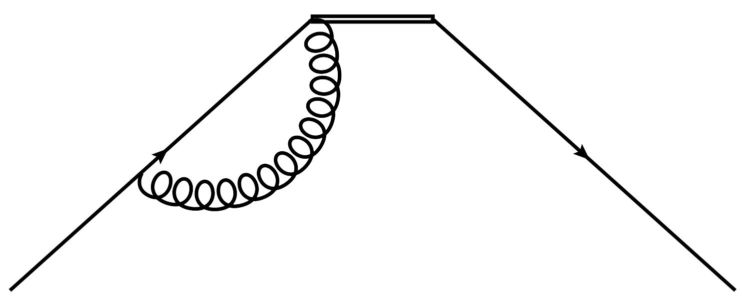
Combination

$$t\frac{\mathrm{d}}{\mathrm{d}t}f_q(x,t) = \frac{\alpha\, C_F}{\pi}\int_0^{z_{\max}(t)} \mathrm{d}z\, P_{qq}(z)\left[f_q(x/z,t) - f_q(x,t)\right] + \ldots$$

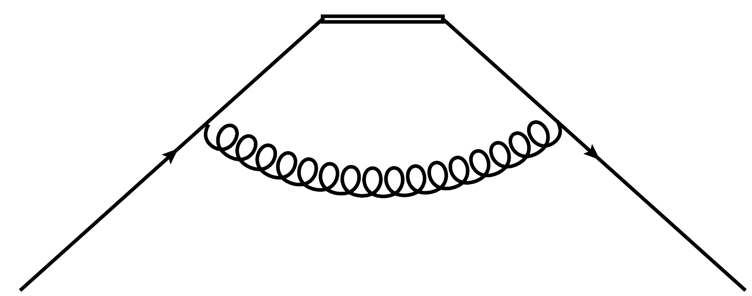
Logarithmic singularity as z→1 vanishes

Since charged W bosons can change the flavor of the fermions, cancellation between virtual and real broken

Consider evolution of an up-type pdf:



Virtual



Real

$$t\frac{\mathrm{d}}{\mathrm{d}t}f_u(x,t) = \frac{\alpha\, C_F}{\pi} P_q^V(t) f_u(x,t)$$

$$t\frac{\mathrm{d}}{\mathrm{d}t}f_u(x,t) = \frac{\alpha\, C_F}{\pi}\int_0^{z_{\max}(t)} \mathrm{d}z\, P_{qq}(z) \times \left[\frac{2}{3}f_d(x/z,t) + \frac{1}{3}f_u(x/z,t)\right]$$

Since $f_u \neq f_d$ (the proton is not SU(2) singlet), real and virtual contributions do not cancel

Double logarithmic terms remain

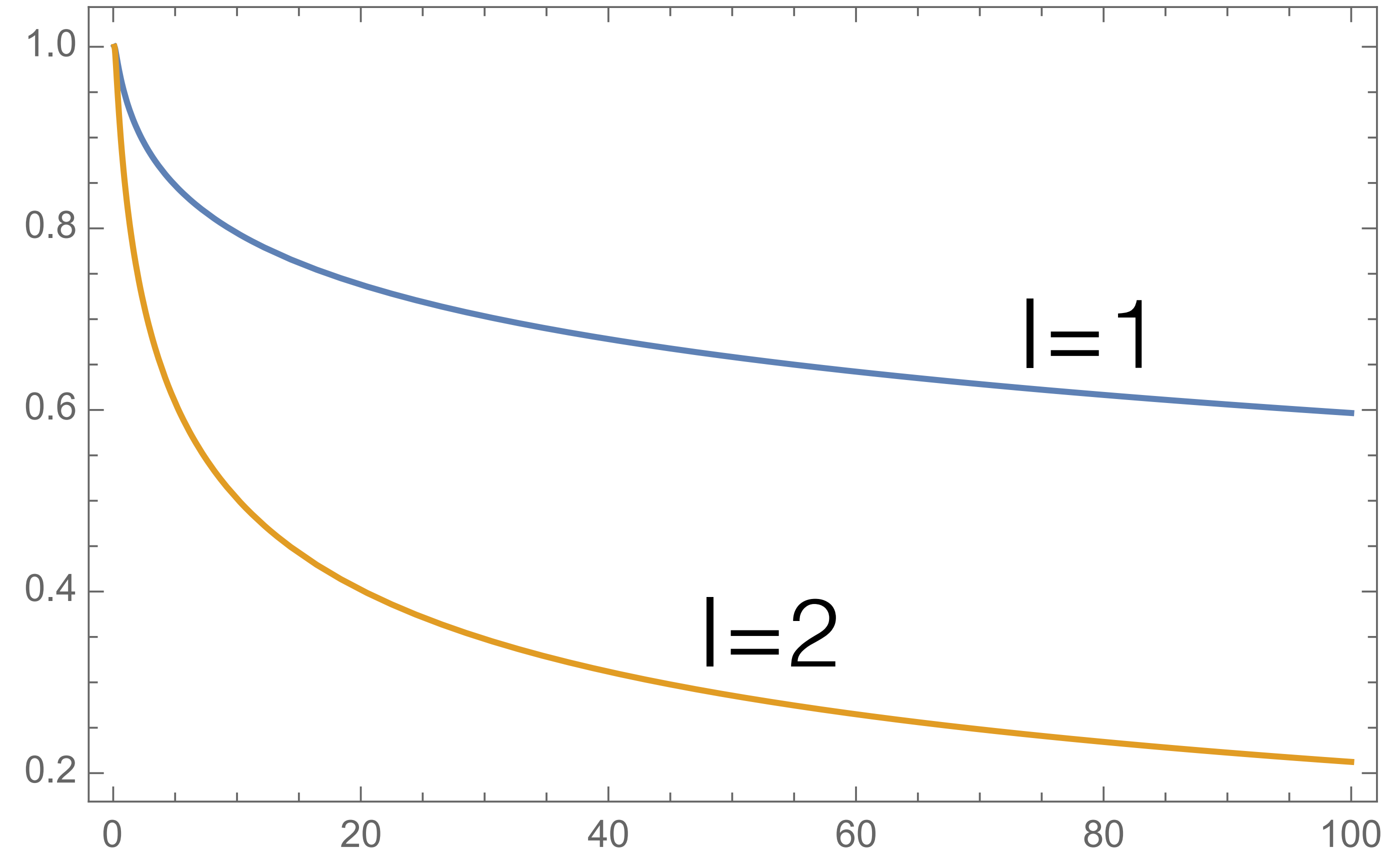
By studying the equations more carefully, one finds that the double logarithms restore electroweak symmetry breaking

By switching from a flavor basis to an isospin basis

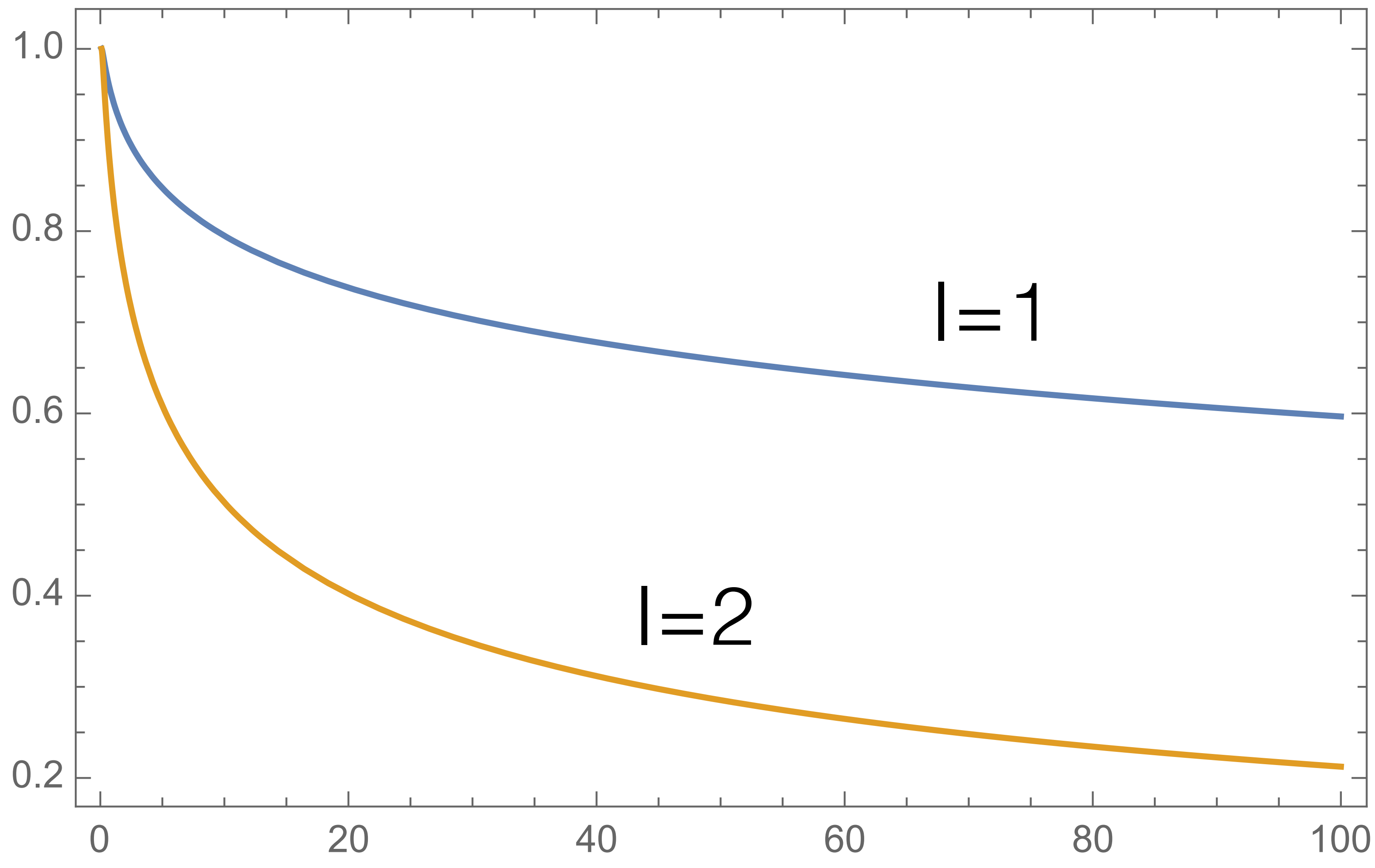
$$f^0(x,t) = \frac{f_u(x,t) + f_d(x,t)}{2} \qquad f^1(x,t) = \frac{f_u(x,t) - f_d(x,t)}{2}$$

States with I ≠ 0 go double logarithmically to zero

$$f^I(x,t) \sim \exp\left[-\frac{I(I+1)}{2}\frac{\alpha_2}{\pi}\ln^2\frac{t}{m_V}\right]$$

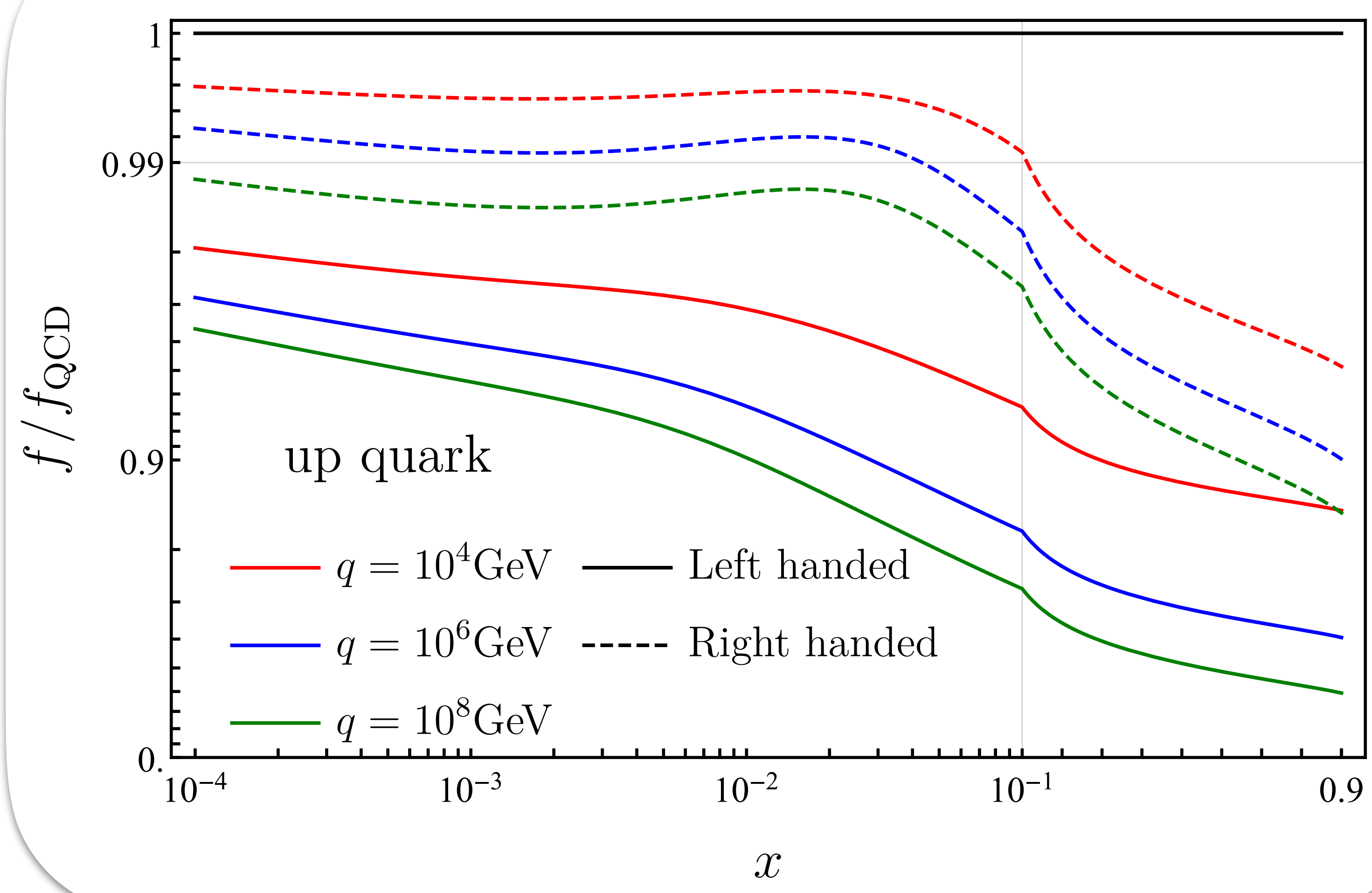


Lesson 4



Contributions violating SU(2) symmetry (l ≠ 0) go to zero as q → 0

Quark pdf's are modified from their value obtained with only QCD evolution included



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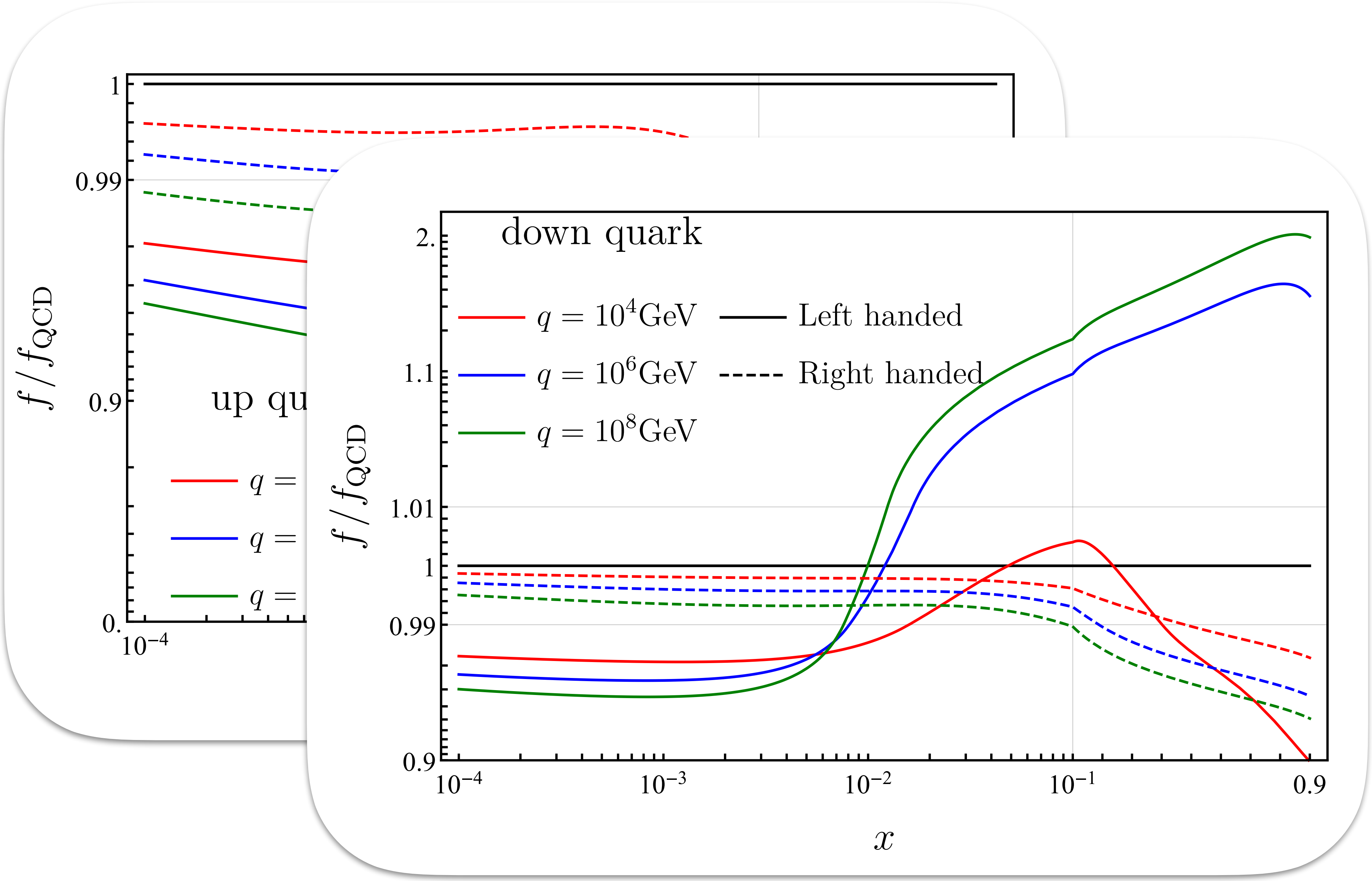

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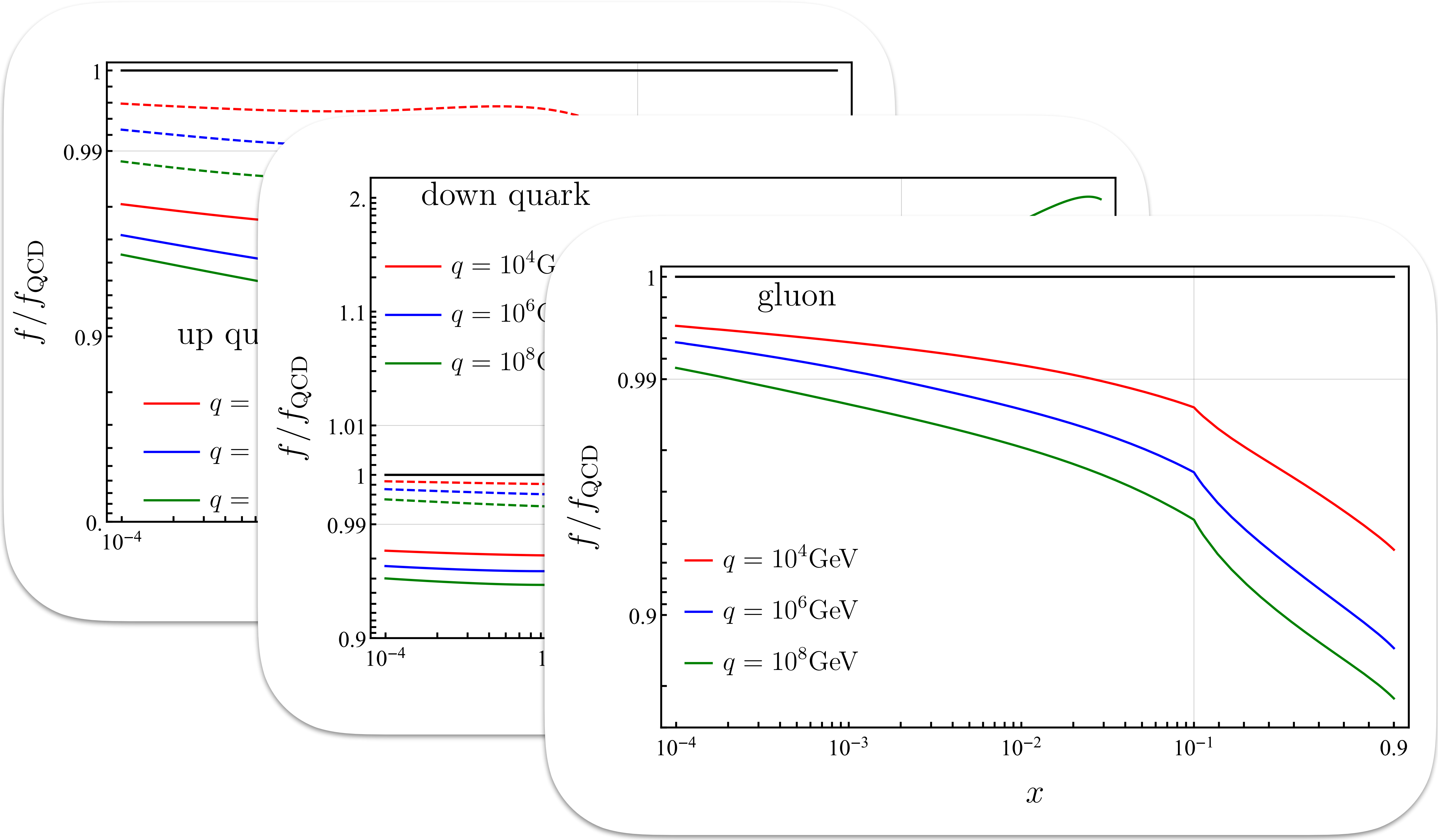
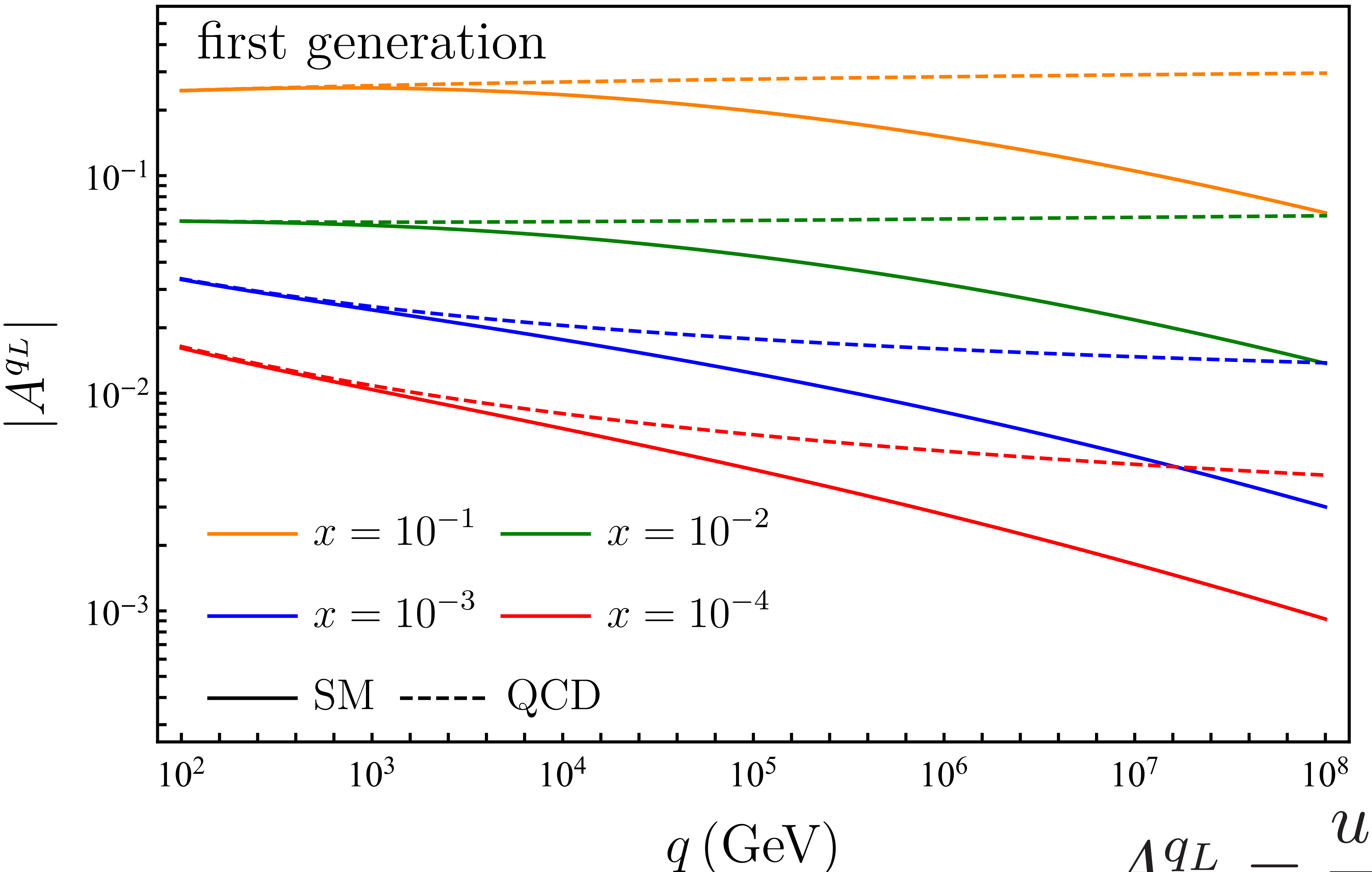
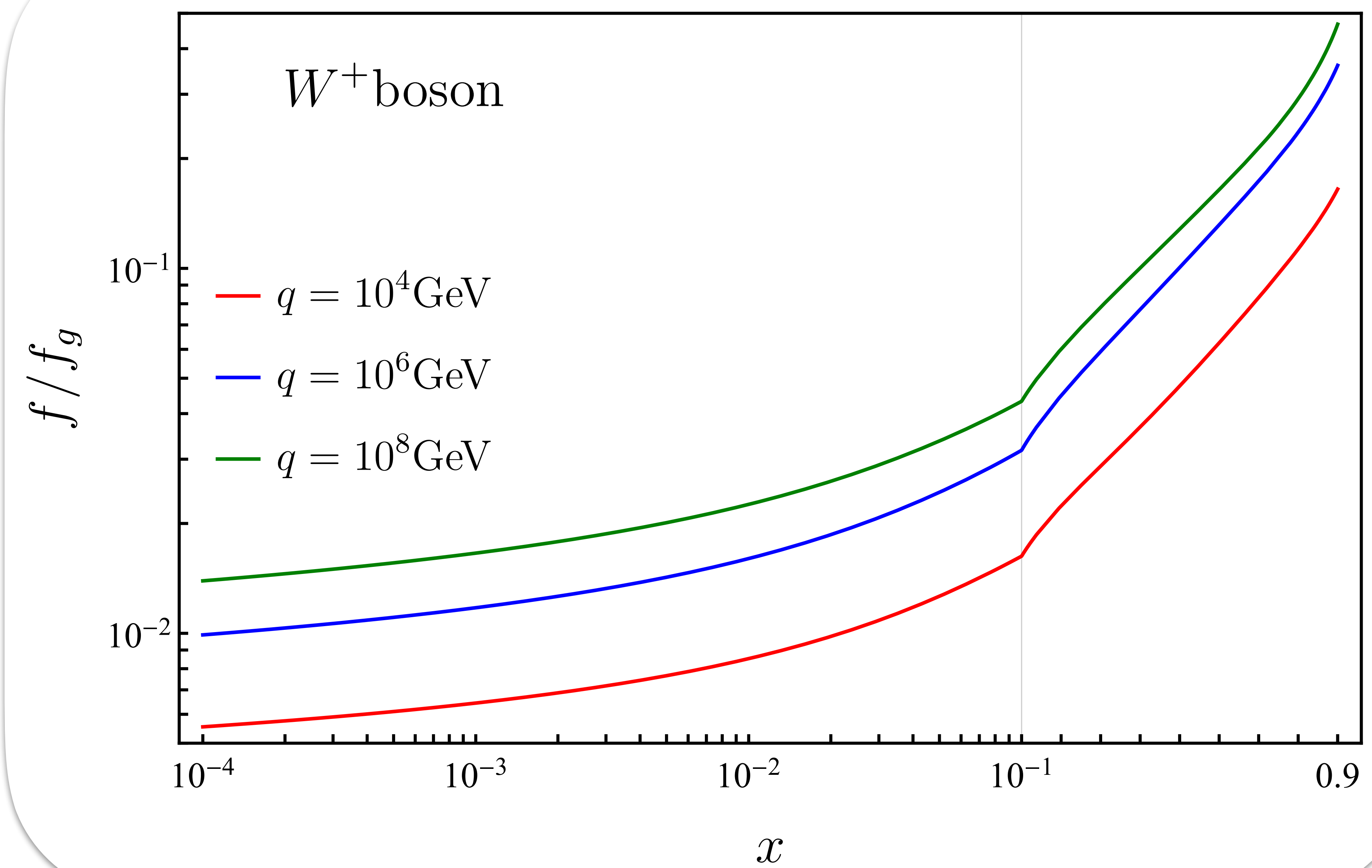


The isospin asymmetry is driven to zero, as predicted earlier



$$A^{q_L} = \frac{u_L - d_L}{u_L + d_L}$$

The probability of finding a vector boson in the proton becomes comparable to that to find a gluon



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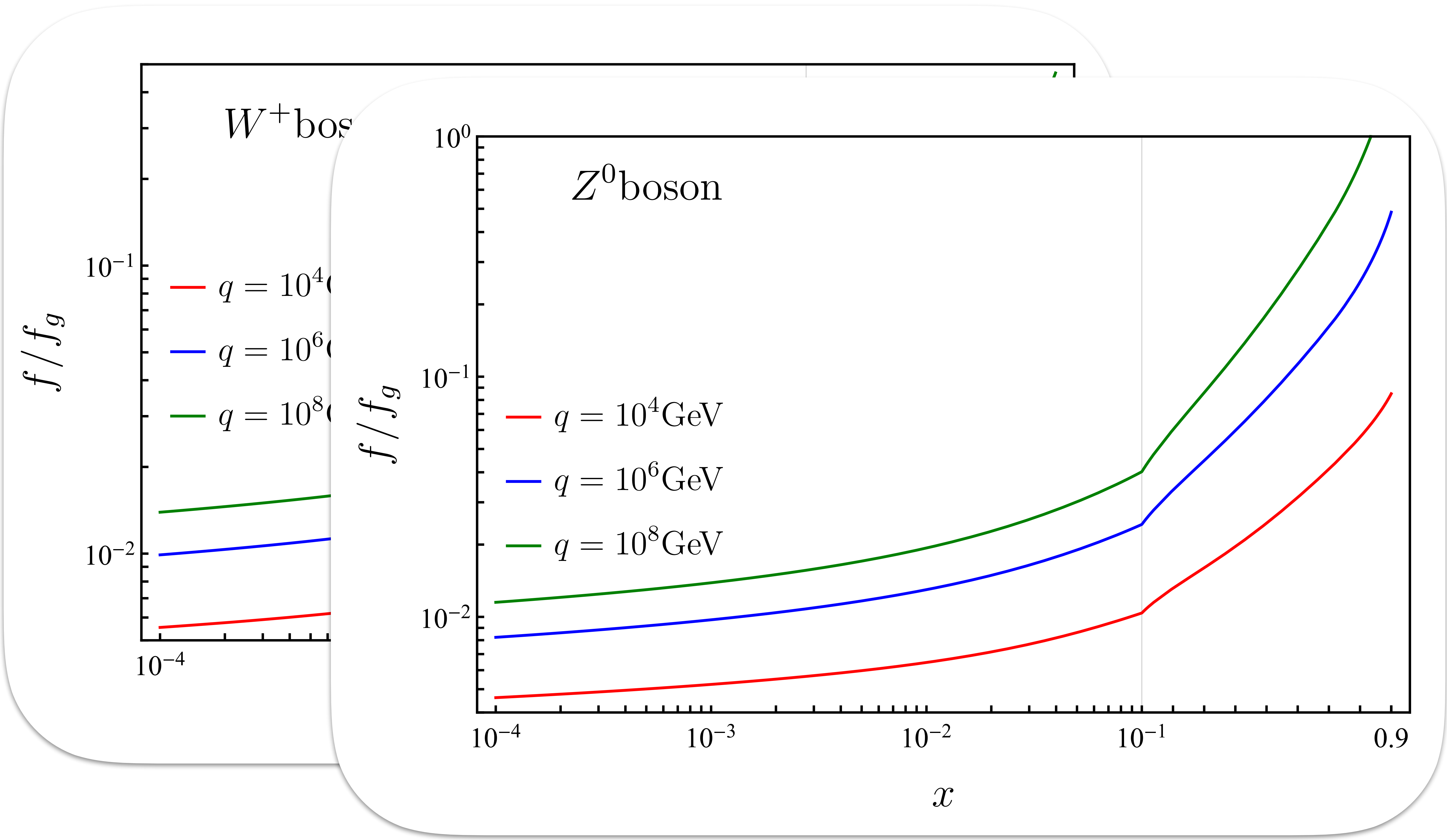
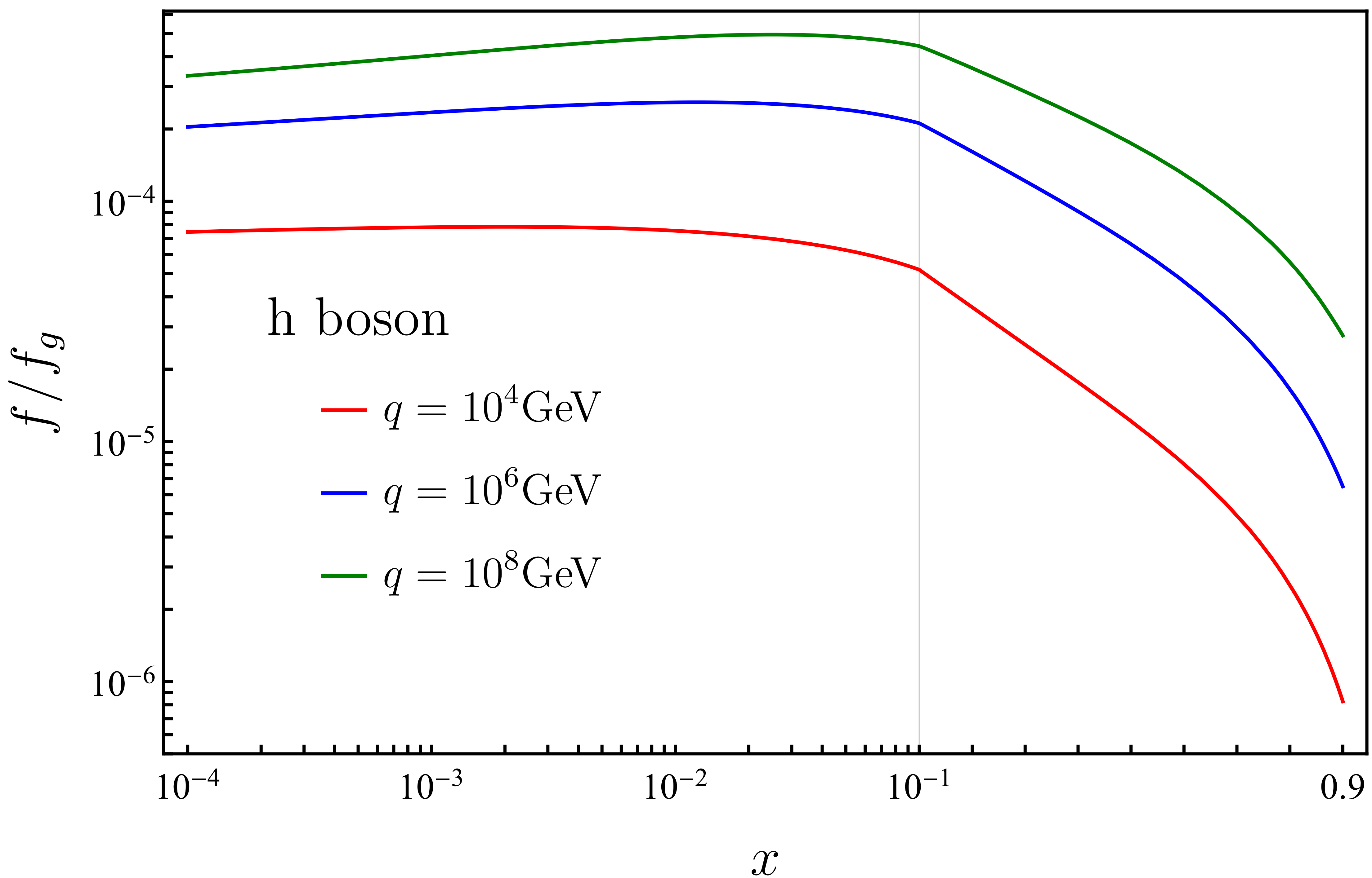
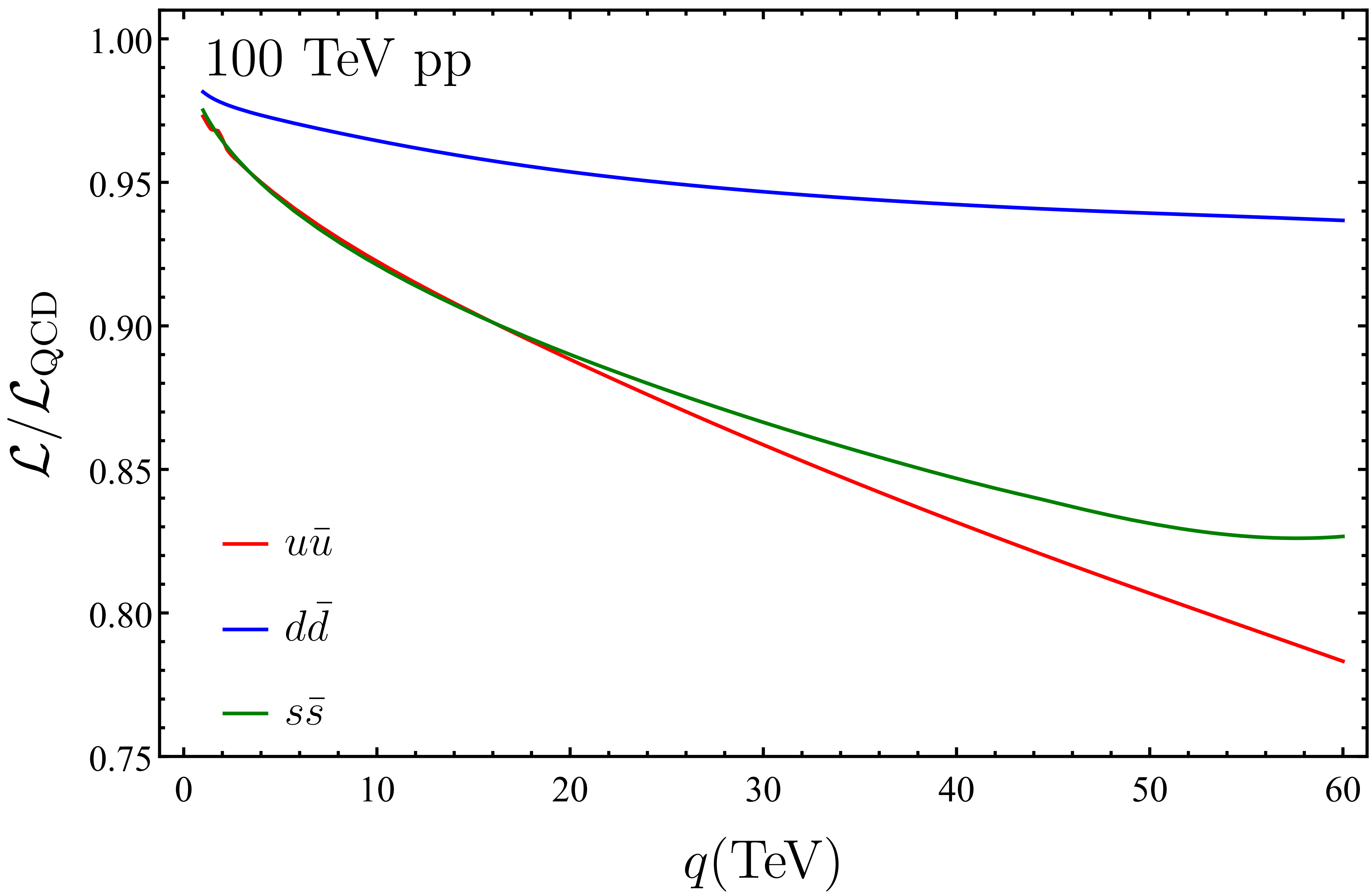


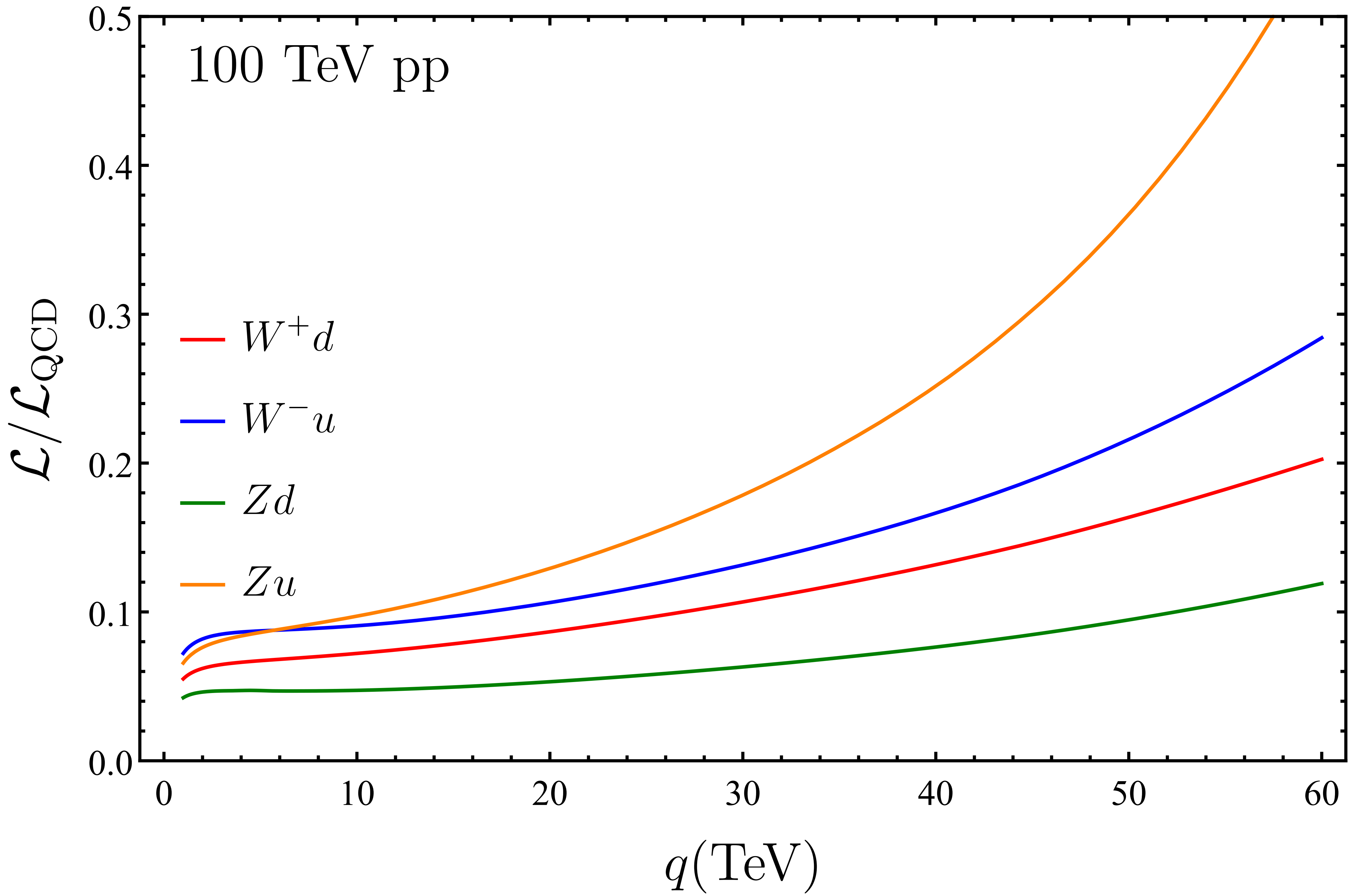
Even have probability of finding a Higgs bosons in proton, but much smaller than gluon



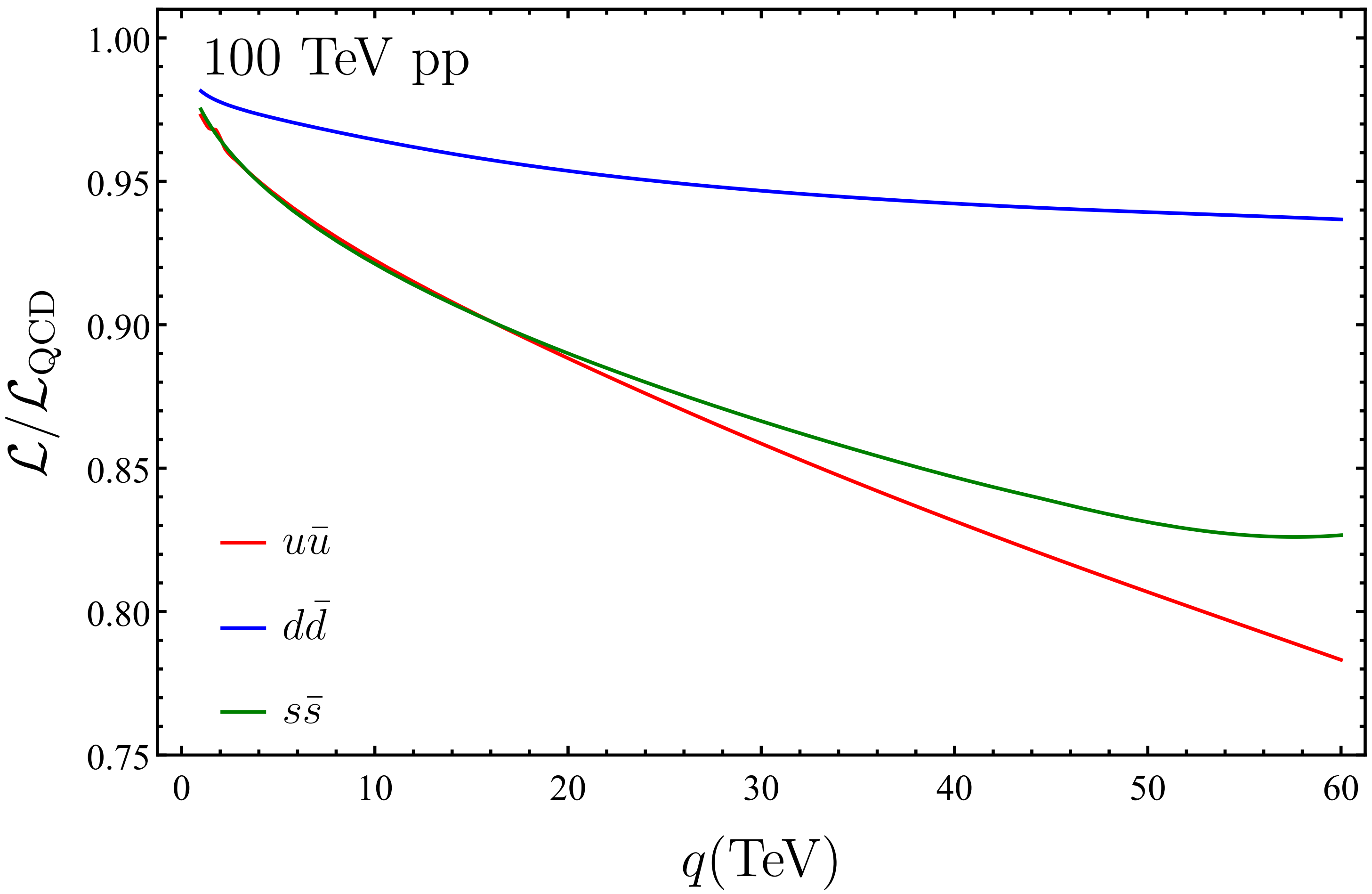
Luminosities at a 100TeV collider are changed noticeably from the values including only QCD running



Luminosities including vector bosons become a significant fraction of more standard luminosities



Lesson 5



As q ≫ m$_V$, get O(1) differences from QCD evolution

Using the PDFs discussed so far, one can resum the LL to any fully inclusive observable

Fully inclusive: final state is completely SU(2) symmetric
(sums complete fermion multiplets
adds extra gauge bosons)

Logarithms arise only from initial state symmetry breaking and are therefore resummed by DGLAP evolution

$$\langle O \rangle_{\mathrm{LL}} = \sum_{AB} \int \mathrm{d}\Phi_n \, O_n(\Phi_n) \, \mathcal{L}_{AB}^{\mathrm{SM}}(x_A, x_B; Q) B_{AB}(\widehat{\Phi}_n)$$

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Observable

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Observable

Partonic cross section

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Observable — Parton Luminosity — Partonic cross section

Can one replace the first order in α with the exact result from fixed order

Since LL resummation does not include subleading logarithms or threshold effects, would improve precision to replace O(α) terms with fixed order expression

Easily accomplished by writing

$$\langle O \rangle_{\mathrm{NLO/LL}} = \langle O \rangle_{\mathrm{NLO}} + \langle O \rangle_{\mathrm{LL}} - [\langle O \rangle_{\mathrm{LL}}]_\alpha$$

Expansion of the LL result defined by

$$[\langle O \rangle_{\mathrm{LL}}]_\alpha = \sum_{AB} \int \mathrm{d}\Phi_n \, O_n(\Phi_n) \, \left[\mathcal{L}^{\mathrm{SM}}_{AB}(x_A, x_B; Q)\right]_\alpha B_{AB}(\widehat{\Phi}_n)$$

Only need expansion of parton luminosity

The a DGLAP equation for the expanded PDFs can be easily derived from the previous results

To perform the expansion, we write

$$f_i^{\mathrm{SM,Exp}}(x,q) = f_i^{\mathrm{noEW}}(x,q) + g_i(x,q)$$

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$$f_i^{\mathrm{SM,Exp}}(x,q) = f_i^{\mathrm{noEW}}(x,q) + \boxed{g_i(x,q)}$$ linear in α

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To perform the expansion, we write

$$f_i^{\rm SM,Exp}(x,q) = f_i^{\rm noEW}(x,q) + \boxed{g_i(x,q)}$$ linear in α

The expanded PDF reproduces all terms up to O(α^2)

$$f_i^{\rm SM}(x,q) = f_i^{\rm SM,Exp}(x,q) + \mathcal{O}(\alpha_I^2)$$

For precise definition of $f^{\rm SM,Exp}$, need to define $f^{\rm noEW}$

Most natural choice:

$$f_i^{\rm noEW}(x,q) = \begin{cases} \text{QCED evolution} & q < q_V \\ \text{QCD evolution} & q > q_V \end{cases}$$

The a DGLAP equation for the expanded PDFs can be easily derived from the previous results

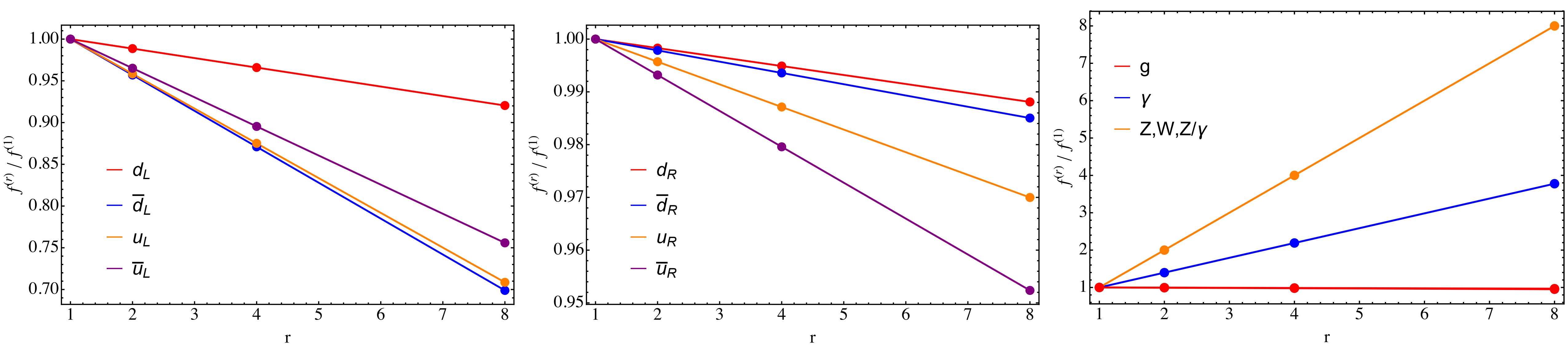
By performing simple expansion of previous DGLAP equation to linear order, one finds

$$q\frac{\partial}{\partial q} g_i(x,q)$$

$$= \frac{\alpha_3(q)}{\pi}\left[P^V_{i,3}(q)\, g_i(x,q) + \sum_j C_{ij,I}\int_x^1 \mathrm{d}z\, P^R_{ij,3}(z) g_j(x/z,q)\right]$$

$$+ \sum_{I\in 1,2,M}\frac{\alpha_I(q)}{\pi}\left[P^V_{i,I}(q)\, f_i^{\mathrm{noEW}}(x,q) + \sum_j C_{ij,I}\int_x^{z_{\max}^{ij,I}(q)} \mathrm{d}z\, P^R_{ij,I}(z) f_j^{\mathrm{noEW}}(x/z,q)\right]$$

Linearity with EW coupling can easily be verified



Perturbative expansion verifies the breakdown of perturbation theory at large Q

Perturbative expansion best studied by defining

$$f_i^{\rm SM}(x,q) = f_i^{\rm noEW}(x,q) + g_i(x,q) + h_i(x,q)$$

If $f^{\rm noEW} \neq 0$, have the perturbative scaling

$$r_f^{\rm noEW}(x,q) = 1 - \frac{g_i(x,q) + h_i(x,q)}{f_i^{\rm SM}(x,q)} \sim 1 + \mathcal{O}(\alpha_I)$$

$$r_f^{\rm SM,Exp}(x,q) = 1 - \frac{h_i(x,q)}{f_i^{\rm SM}(x,q)} \sim 1 + \mathcal{O}(\alpha_I^2)$$

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PDFs with $f^{\mathrm{noEW}} \neq 0$

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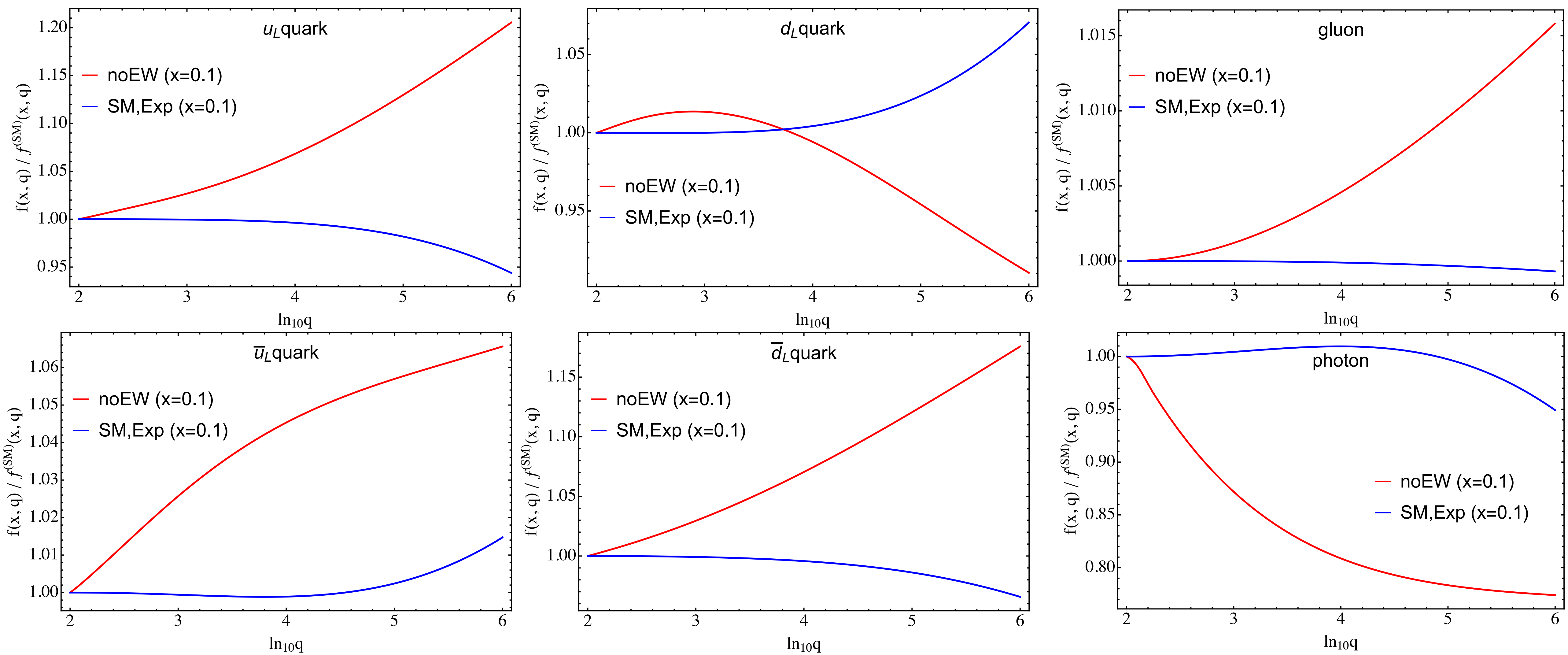

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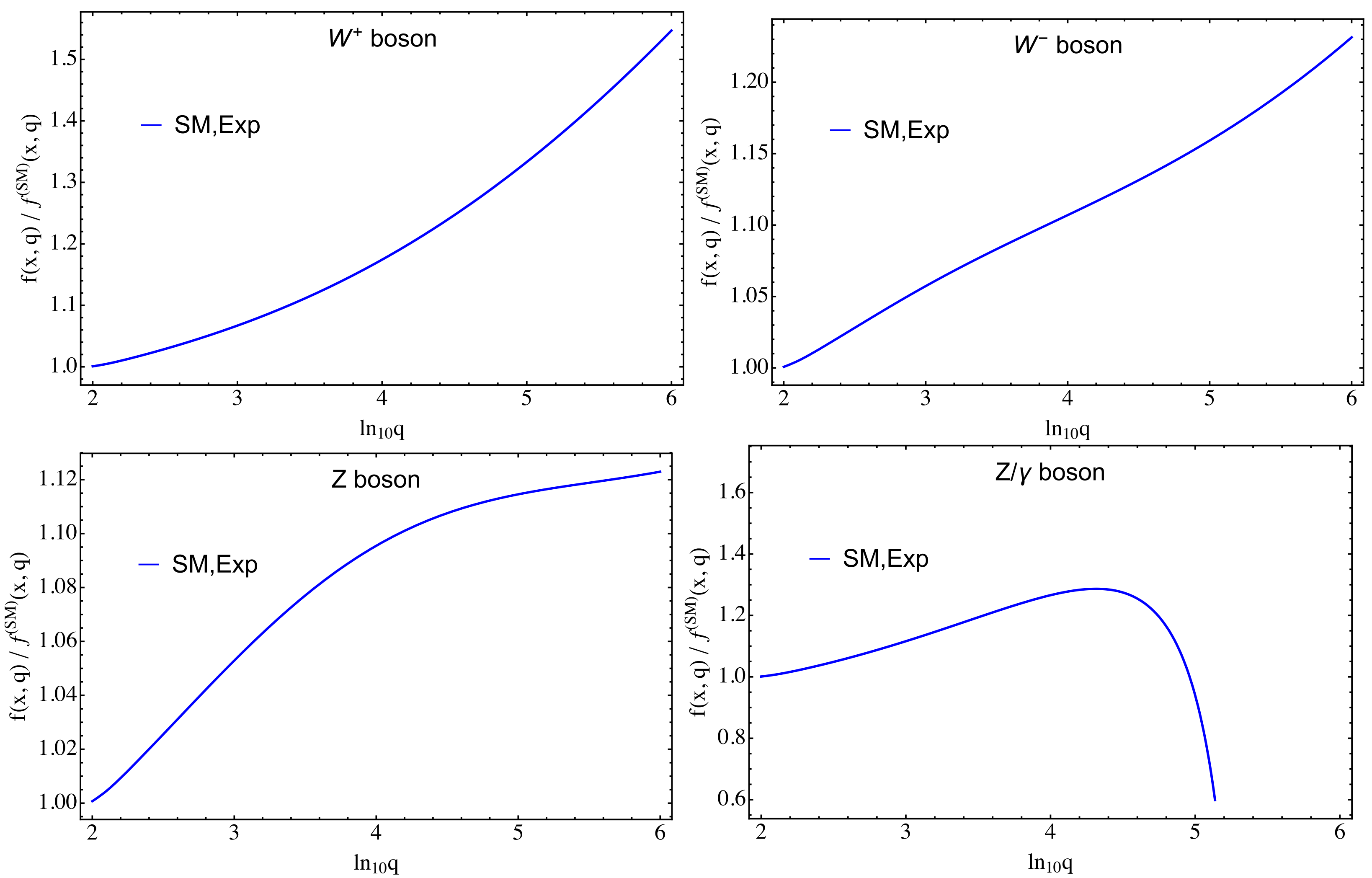

This leads to very interesting effects at for fully inclusive di-lepton production at a future 100 TeV collider

Inclusive di-lepton production defined by summing over following final states [to be SU(2) symmetric]

$$\ell^+\ell^-(+V),\ \ell^+\nu_\ell(+V),\ \bar{\nu}_\ell\ell^-(+V),\ \bar{\nu}_\ell\nu_\ell(+V)$$

Which initial states do we need?

PDF	leading α power	log scaling
$q,\ g$	0	$\alpha^n \mathrm{Ln}_Q^{2n}$
γ	1	$\alpha^n \mathrm{Ln}_Q^{2n-1}$
V_T	1	$\alpha^n \mathrm{Ln}_Q^{2n-1}$
V_L	2	$\alpha^n \mathrm{Ln}_Q^{2n-2}$
ℓ	2	$\alpha^n \mathrm{Ln}_Q^{2n-2}$
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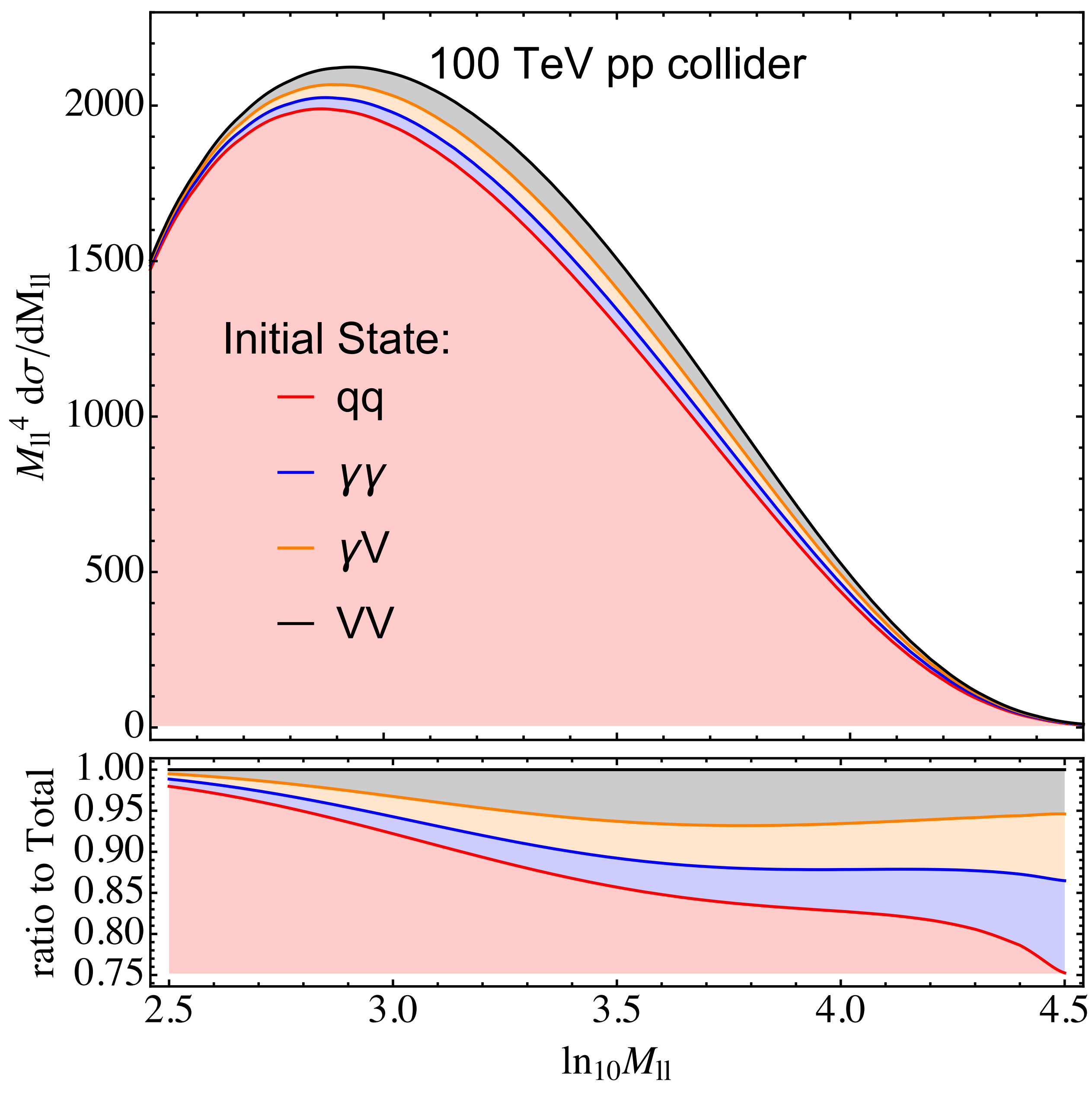
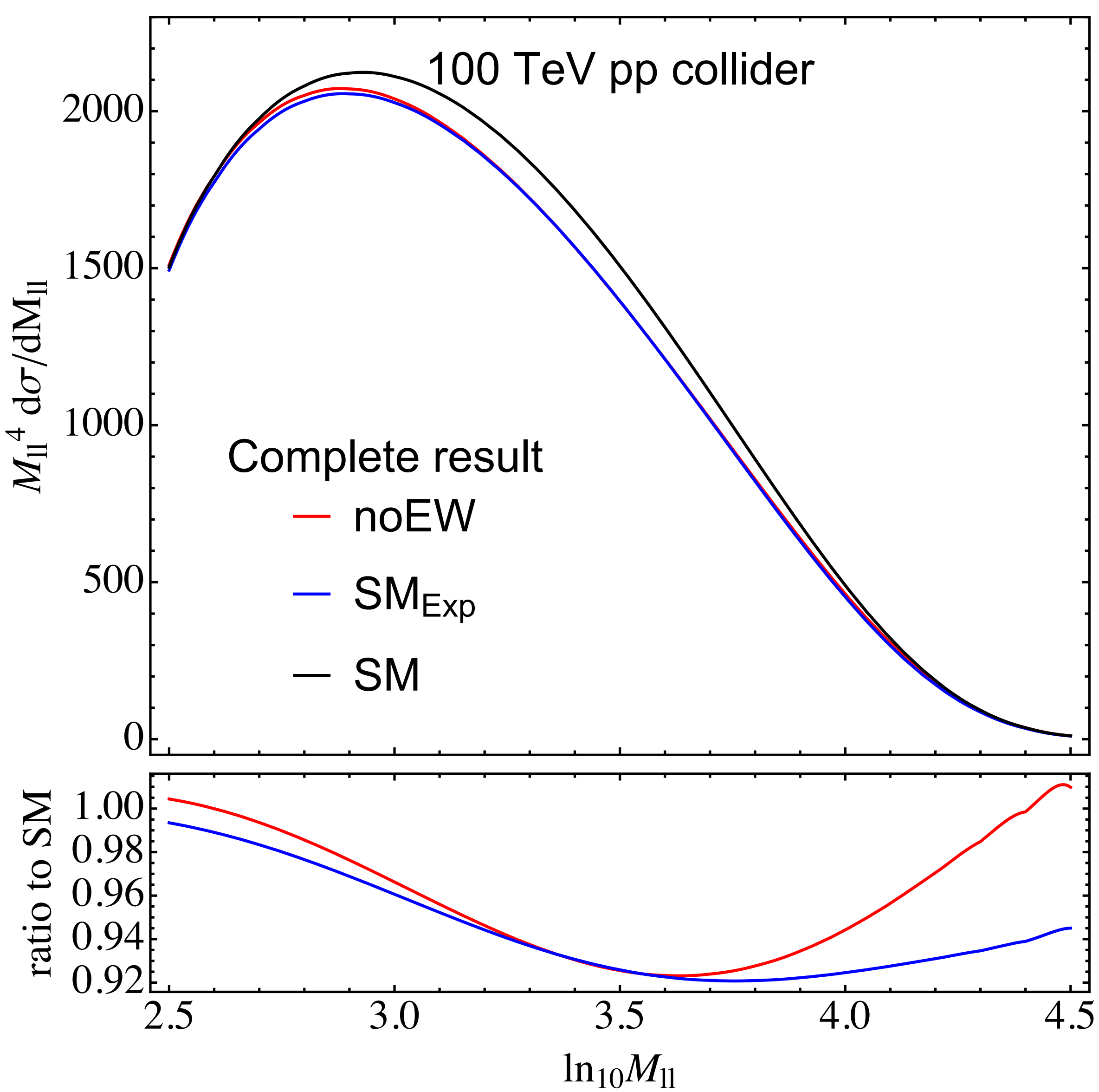
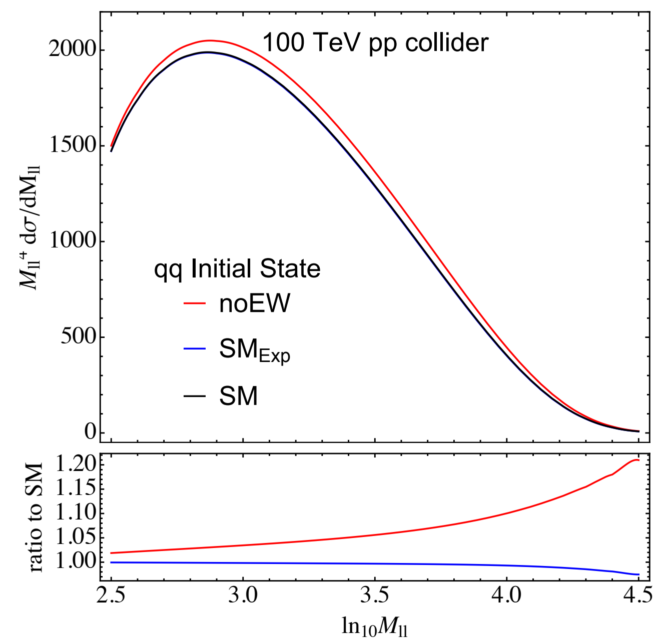
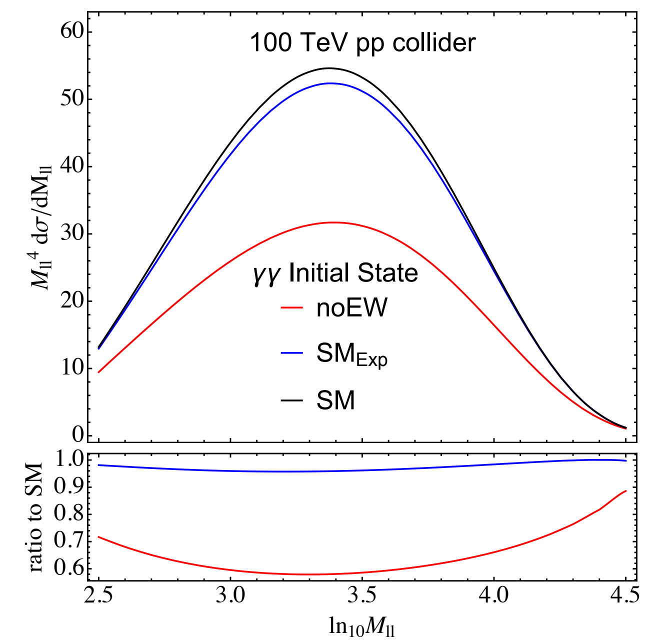


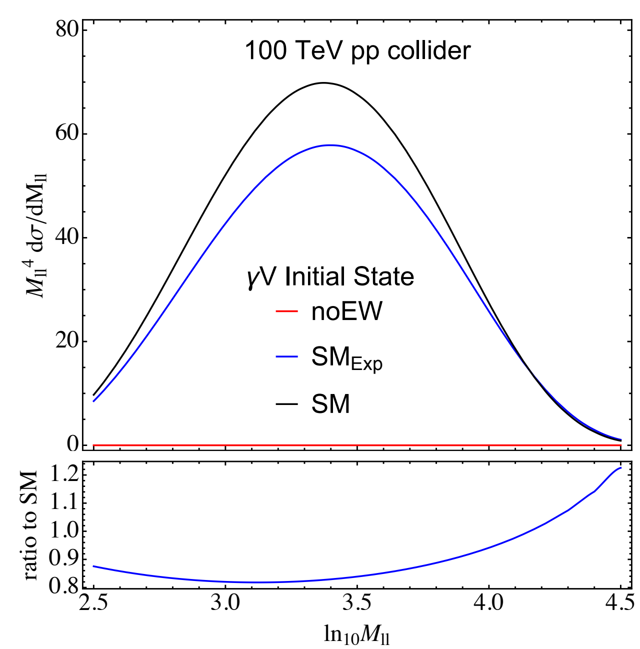
This leads to very interesting effects at for fully inclusive di-lepton production at a future 100 TeV collider

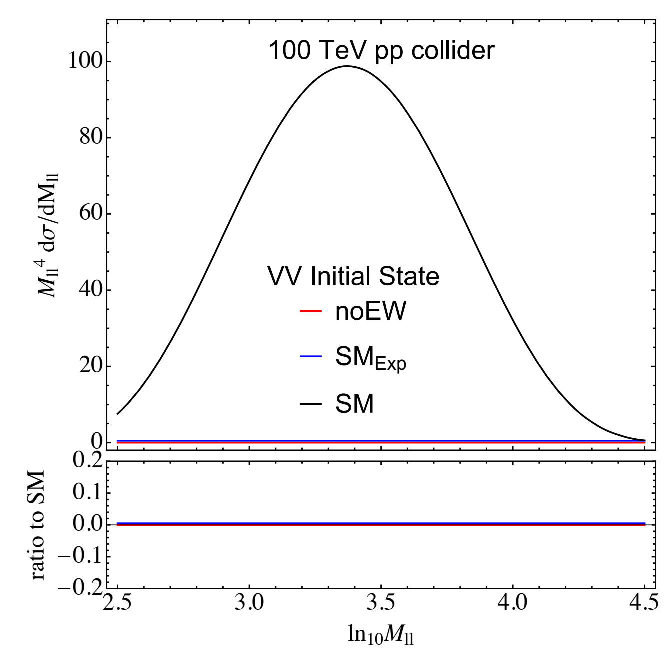
The perturbative expansion reveals that there are large corrections at O(α^2)



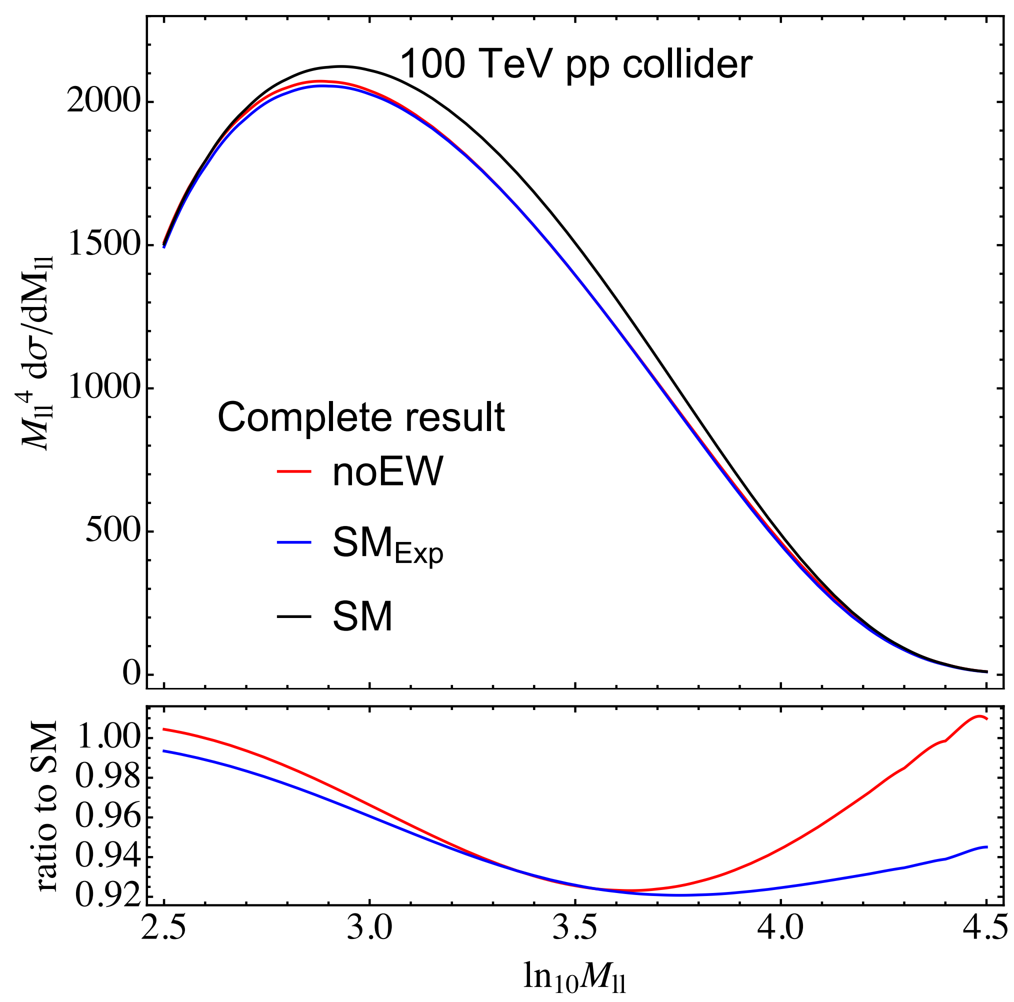








Lesson 6



At a 100 TeV collider, corrections beyond NLO are important, but can be understood using resummation

Lesson 1

Electroweak correction give rise to logarithmic terms in any observable

Lesson 2

Resummation of electroweak corrections can lead to large effects

Lesson 3

DGLAP equations same as QCD, just need more splitting functions and coeffs

Questions?

Lesson 4

Contribution violating SU(2) symmetry go to zero as q gets large

Lesson 5

As $q \gg m_V$, get O(1) differences from QCD evolutions

Lesson 6

Corrections beyond NLO important, but can be understood using resummation