The package `TopoID` and its applications to Higgs physics

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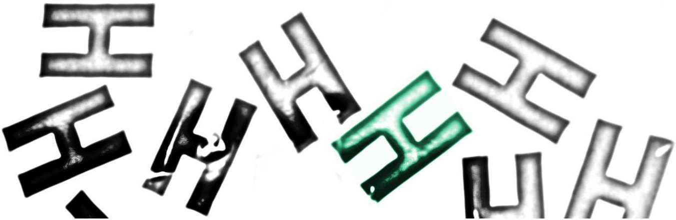

1. `TopoID`
   - Polynomial ordering
   - Topology classification
   - Relations among Feynman integrals
   - Topology "merging"
   - Partial fractioning

2. **N^3LO Higgs production: qq'-channel**
   - Optical theorem and Cutkosky's rules
   - Calculation
   - Results

3. **NLO and NNLO Higgs pair production**
   - Differential factorization
   - Soft-virtual approximation
   - Asymptotic expansion
   - Calculation
   - Results

TopoID

Topology IDentification

```
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```

Idea: Generic, process independent `Mathematica` package

| Feynman diagrams $\longrightarrow$ reduced result | (Laporta not included)

- Topology construction
  (identification, minimal sets; partial fractioning; factorization, ...)
- Handle properties
  (completeness, linear dependence; subtopologies, scalelessness, symmetries; graphs, unitarity cuts, ...)
- `FORM` code generator
  (diagram mapping, topology processing, integral reduction, ...)
- Master integral identification
  (base changes, non-trivial relations, ...)

Bring the polynomial P with m terms into unique form $\hat{P}$ by renaming the n variables $\{x_j\}$: [Pak; '12]

1. Convert P into $m \times (n+1)$ matrix $M^{(0)}$
   (row: term, 1st column: coefficient, remaining columns: powers of $\{x_j\}$)
2. Start with considering the above $M^{(0)}$ and the 2nd column ($k = 1$)
3. Compute for all considered matrices $M^{(k),\sigma}$ all transpositions of columns k and $k+1, \ldots$ (and collect perputations σ)
4. Sort rows in each matrix lexicographically by the first k columns
5. Extract in columns k the lexicographically largest vector
6. Keep only matrices with this maximal vector;
   If $k < n-1$: $k \to k+1$ and goto Step 3
7. Each remaining matrix encodes the same unique $\hat{P}_\sigma$ and a permutation of variables σ

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(row: term, 1st column: coefficient, remaining columns: powers of $\{x_j\}$)
2. Start with considering the above $M^{(0)}$ and the 2nd column ($k = 1$)

$$P = x_1^2 + 2x_1x_2 + x_2^2 + x_3^2 \quad \rightarrow \quad M^{(0)} = \begin{pmatrix} 1 & 2 & 0 & 0 \\ 2 & 1 & 1 & 0 \\ 1 & 0 & 2 & 0 \\ 1 & 0 & 0 & 2 \end{pmatrix} \qquad \text{(Step 1)}$$

$$S^{(1)} = \left\{ M^{(0)(123)} = M^{(0)} \right\}, \quad k = 1 \qquad \text{(Step 2)}$$

3 Compute for all considered matrices $M^{(k),\sigma}$ all transpositions of columns k and $k+1,\ldots$ (and collect perputations σ)

$$S'^{(1)}: \quad M'^{(1)(123)} = \left(\begin{array}{cc|cc} 1 & 2 & 0 & 0 \\ 2 & 1 & 1 & 0 \\ 1 & 0 & 2 & 0 \\ 1 & 0 & 0 & 2 \end{array}\right), \quad M'^{(1)(213)} = \left(\begin{array}{cc|cc} 1 & 0 & 2 & 0 \\ 2 & 1 & 1 & 0 \\ 1 & 2 & 0 & 0 \\ 1 & 0 & 0 & 2 \end{array}\right),$$

$$M'^{(1)(321)} = \left(\begin{array}{cc|cc} 1 & 0 & 0 & 2 \\ 2 & 0 & 1 & 1 \\ 1 & 0 & 2 & 0 \\ 1 & 2 & 0 & 0 \end{array}\right)$$

(Step 3-1)

4 Sort rows in each matrix lexicographically by the first k columns

$$S''^{(1)}: \quad M''^{(1)(123)} = \begin{pmatrix} 1 & \mathbf{0} & 0 & 2 \\ 1 & \mathbf{0} & 2 & 0 \\ 1 & \mathbf{2} & 0 & 0 \\ 2 & \mathbf{1} & 1 & 0 \end{pmatrix}, \quad M''^{(1)(213)} = \begin{pmatrix} 1 & \mathbf{0} & 2 & 0 \\ 1 & \mathbf{0} & 0 & 2 \\ 1 & \mathbf{2} & 0 & 0 \\ 2 & \mathbf{1} & 1 & 0 \end{pmatrix},$$

$$M''^{(1)(321)} = \begin{pmatrix} 1 & 0 & 0 & 2 \\ 1 & 0 & 2 & 0 \\ 1 & 2 & 0 & 0 \\ 2 & 0 & 1 & 1 \end{pmatrix}$$

(Step 4-1)

5. Extract in columns k the lexicographically largest vector
6. Keep only matrices with this maximal vector;
   If $k < n-1$: $k \to k+1$ and goto Step 3

$$\hat{M}''^{(1)} = \begin{pmatrix} 0 \\ 0 \\ 2 \\ 1 \end{pmatrix} \qquad \text{(Step 5-1)}$$

$$S^{(2)} = \left\{ M''^{(1)(123)}, M''^{(1)(213)} \right\}, \quad k = 2 \qquad \text{(Step 6-1)}$$

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$$M'^{(2)(213)} = \left(\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 1 & 0 & 0 & 2 \\ 1 & 2 & 0 & 0 \\ 2 & 1 & 1 & 0 \end{array}\right), \quad M'^{(2)(231)} = \left(\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 1 & 0 & 2 & 0 \\ 1 & 2 & 0 & 0 \\ 2 & 1 & 0 & 1 \end{array}\right)$$

(Step 3-2)

4 Sort rows in each matrix lexicographically by the first k columns

$$S''^{(2)}: \quad M''^{(2)(123)} = \begin{pmatrix} 1 & 0 & \mathbf{0} & 2 \\ 1 & 0 & \mathbf{2} & 0 \\ 1 & 2 & \mathbf{0} & 0 \\ 2 & 1 & \mathbf{1} & 0 \end{pmatrix}, \quad M''^{(2)(132)} = \begin{pmatrix} 1 & 0 & 0 & 2 \\ 1 & 0 & 2 & 0 \\ 1 & 2 & 0 & 0 \\ 2 & 1 & 0 & 1 \end{pmatrix},$$

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7. Each remaining matrix encodes the same unique $\hat{P}_\sigma$ and a permutation of variables σ

$$\hat{P} = P = x_3^2 + x_2^2 + x_1^2 + 2x_1x_2\,, \quad \hat{\sigma} = \{(123), (213)\} \qquad \text{(Step 7)}$$

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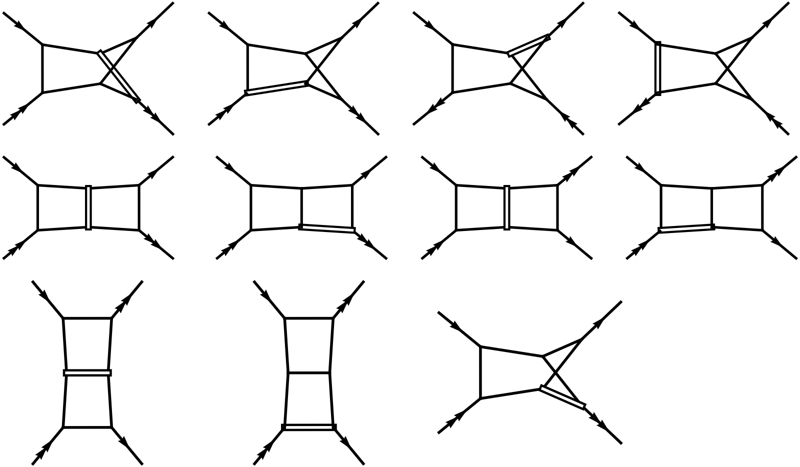
- P already in canonical form; two permutations $\{(123), (213)\}$
- (213) denotes $(x_1, x_2, x_3) \to (x_2, x_1, x_3)$; symmetry of P under $x_1 \leftrightarrow x_2$

Application to Feynman integrals

- Use $\mathcal{U} + \mathcal{F}$ from the Feynman representation
- Unique identifier $\hat{\mathcal{U}} + \hat{\mathcal{F}}$; independent of momentum space representation
- Returned permutations: symmetries of Feynman integrals

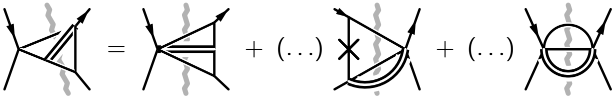
$\Rightarrow$ Many useful applications

Minimal set for NNLO Higgs production:



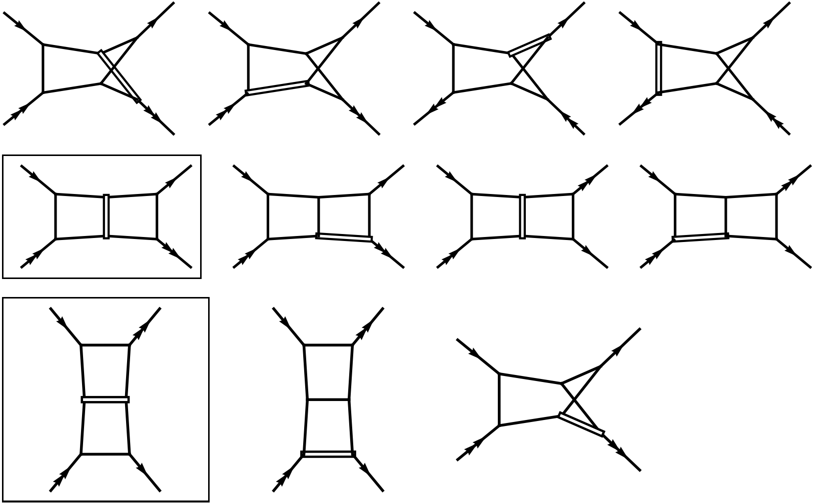
Note: Sufficient for all 2 946 diagrams

Non-trivial relation for NNLO Higgs production:



- Cross-topology relations; not from Laporta reduction
- Simplify calculation
- Usefull cross-checks

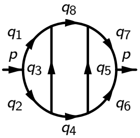
Minimal set for NNLO Higgs production:



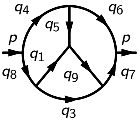
3-loop massless propagators:

$$
\begin{aligned}
q_1 &= k_1, & q_4 &= p - k_1 - k_2, & q_7 &= k_1 + k_2 + k_3,\\
q_2 &= p - k_1, & q_5 &= k_3, & q_8 &= k_1 + k_2,\\
q_3 &= k_2, & q_6 &= p - k_1 - k_2 - k_3, & q_9 &= k_1 + k_3.
\end{aligned}
$$

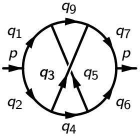
$P_3(a_1, \ldots, a_8, 0) =$



$P_3(a_1, 0, a_3, \ldots, a_9) =$



$P_3(a_1, \ldots, a_7, 0, a_9) =$

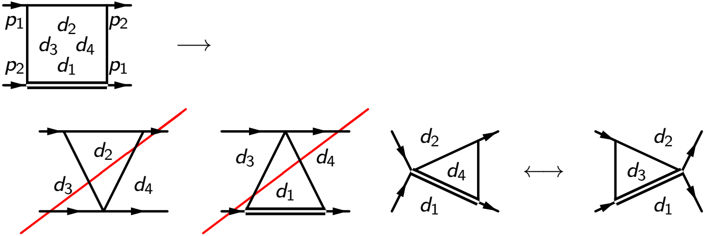


- 1 external, 3 internal momenta

⇒ 9 scalar products

- 3 incomplete topologies with 8 lines
- Identify "greatest common subtopology"
- Find "supertopology" with these 3 different graphs as subtopologies

Partial fractioning for NLO Higgs production:



Via Gröbner basis:

$$d_4 \to -m_H^2 + s + d_1 + d_2 - d_3$$

$$\frac{d_3}{d_4} \to \frac{1}{d_4}\left(-m_H^2 + s + d_1 + d_2 - d_4\right)$$

$$\frac{d_2}{d_3\,d_4} \to \frac{1}{d_3\,d_4}\left(m_H^2 - s - d_1 + d_3 + d_4\right)$$

$$\frac{d_1}{d_2\,d_3\,d_4} \to \frac{1}{d_2\,d_3\,d_4}\left(m_H^2 - s - d_2 + d_3 + d_4\right)$$

$$\frac{1}{d_1\,d_2\,d_3\,d_4} \to \frac{1}{\left(m_H^2 - s\right)\,d_1\,d_2\,d_3\,d_4}\left(d_1 + d_2 - d_3 - d_4\right)$$

N^3LO Higgs production: qq'-channel

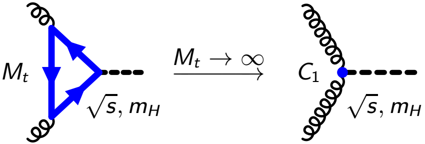
Motivation and introduction

- Higgs production at the LHC dominated by gluon fusion
- After [Anastasiou, Duhr, Dulat, Furlan, Gehrmann, Herzog, Mistlberger; '14, '15]:
  2.2% corrections, 3% uncertainty at N^3LO

$\Rightarrow$ Cross-check

- Loop-induced process, dominated by top quark mass
- Effective field theory with top quark integrated out:

$$\mathcal{L}_{Y,\text{eff}} = -\frac{H}{v} C_1 \mathcal{O}_1 \quad \text{and} \quad \mathcal{O}_1 = \frac{1}{4} G^a_{\mu\nu} G^{a,\mu\nu}$$



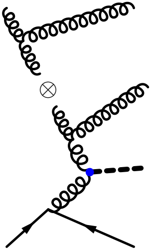
- Reduced numbers of scales and loops:
  single dimensionless variable $x = m_H^2/s$ (soft: $x \to 1$)
- Finite matching coefficient C_1 needed to 4-loop

[Chetyrkin, Kniehl, Steinhauser; '98] [Schröder, Steinhauser; '06] [Chetyrkin, Kühn, Sturm; '06]

N^3LO Higgs production: qq'-channel

Status

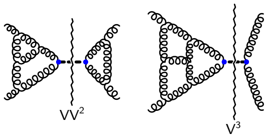
- LO calculation (exact)
  [Ellis et al.; '76] [Wilczek et at.; '77] [Georgi et al.; '78] [Rizzo; '80]
- NLO (exact)
  [Dawson; '91] [Djouadi, Spira, Zerwas; '91]
- NNLO (EFT) $\Rightarrow$ soft expansion to 3rd order valid to $\mathcal{O}(1\%)$
  [Harlander, Kilgore; '02] [Anastasiou, Melnikov; '02] [Ravindran, Smith, van Neerven; '03]
- NNLO $\mathcal{O}\left(1/M_t^2\right)$ corrections $\approx$ NNLO $+1\%$
  [Pak, Rogal, Steinhauser; '09-'11] [Harlander, Mantler, Marzani, Ozeren; '09-'10]
- N^3LO IR counterterms
  - 3-loop splitting functions
    [Moch, Vermaseren, Vogt; '02]
  - NNLO master integrals to higher orders in ϵ
    [Pak, Rogal, Steinhauser; '11] [Anastasiou, Bühler, Duhr, Herzog; '12]
  - Cross sections and convolution integrals
    [Höschele, JH, Pak, Steinhauser, Ueda; '12, '13] [Bühler, Lazopoulos; '13]
- N^3LO scale variation $\Rightarrow$ $\mathcal{O}(2\% - 8\%)$
  [Bühler, Lazopoulos; '13]



N^3LO Higgs production: qq'-channel

Status

- N^3LO corrections
  - VV^2 and V^3 – 3-loop gluon form factor
    [Baikov, Chetyrkin, Smirnov^2, Steinhauser; '09] [Gehrmann, Glover, Huber, Ikizlerli, Studerus; '09]
  - VRV exact in x
    [Anastasiou, Duhr, Dulat, Herzog, Mistlberger; '13] [Kilgore; '14]
  - V^2R exact in x
    [Dulat, Mistlberger; '14] [Duhr, Gehrmann, Jaquier; '14]
  - VR^2 expansion in $x \to 1$
    [Anastasiou, Duhr, Dulat, Furlan, Gehrmann; '14] [Li, von Manteuffel, Schabinger, Zhu; '14], [Anastasiou, Duhr, Dulat, Furlan, Herzog, Mistlberger; '15]
  - R^3 expansion in $x \to 1$
    [Anastasiou, Duhr, Dulat, Mistlberger; '13]
  - 37 terms in the $x \to 1$ expansion
    [Anastasiou, Duhr, Dulat, Furlan, Gehrmann, Herzog, Mistlberger; '14, '15]



$\Rightarrow$ Sufficient for phenomenology

  - qq'-channel exact in x (VR^2, R^3)
    [Höschele, JH, Ueda; '14] [Anzai, Hasselhuhn, Höschele, JH, Kilgore, Steinhauser, Ueda; '15]

$\Rightarrow$ Independent cross-check

- Many different resummations
  [Catani et al.] [Moch et al.] [Ahrens et al.] [de Florian et al.] [Ball et al.] [. . .]

N^3LO Higgs production: qq'-channel

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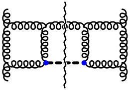
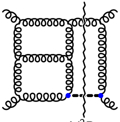

VRV


V^2R

$\Rightarrow$ Sufficient for phenomenology

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N^3LO Higgs production: qq'-channel

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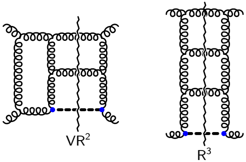

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N^3LO Higgs production: qq'-channel

Generalities

1. Reduction of integrals with full x-dependence
   $\longrightarrow$ Only contributing cuts
2. Construct differential equations for master integrals
   $\longrightarrow$ Canonical basis (in general: coupled system)
3. Soft limit $x \to 1$ as boundary condition
   $\longrightarrow$ Leading term using Mellin-Barnes representation

Canonical differential equations:

$$\frac{\mathrm{d}}{\mathrm{d}x} m_i(x,\epsilon) = \epsilon\, A_{ij}(x)\; m_j(x,\epsilon) \quad \text{with} \quad d = 4 - 2\,\epsilon$$

[Henn et al.; '13-...]

- ϵ- and x-dependence factorize
- Solve order-by-order in ϵ
- A_{ij}: alphabet of appearing functions

E.g. Harmonic Polylogarithms (HPLs):

$$\mathrm{H}_{\vec{w}}(x) = \int_0^x \mathrm{d}x'\, f_{w_1}\left(x'\right) \mathrm{H}_{\vec{w}_{n-1}}\left(x'\right) \quad \text{and} \quad f_0(x) = \frac{1}{x},\; f_{\pm 1}(x) = \frac{1}{1 \mp x}$$

Optical theorem and Cutkosky's rules

Higher-order corrections:

Virtual More loop integrations

Real Additional final state particles (different phase spaces)

Optical theorem:

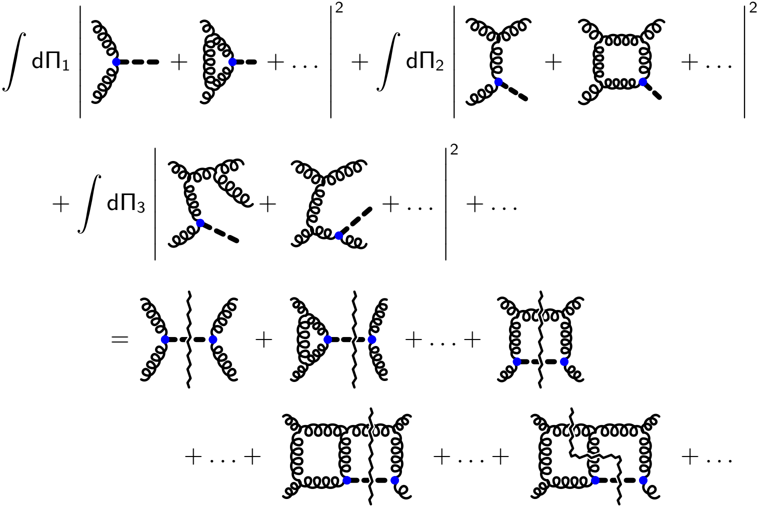
$$\sigma(i \to f) \quad \sim \quad \sum_f \int \mathrm{d}\Pi_f \; |\mathcal{M}(i \to f)|^2 \quad \sim \quad \mathrm{Disc}\,\mathcal{M}(i \to i)$$

Cutkosky's rules: Consider only valid diagrammatic cuts for Disc

- Two connectivity components
- Separate in- and outgoing momenta (s-channel)
- Contribute to the process (1 or 2 Higgs, 0 to 3 parton lines)

Optical theorem and Cutkosky's rules

E.g. to NNLO:



Optical theorem and Cutkosky's rules

Optical theorem:

$$\sigma(i \to f) \quad \sim \quad \sum_f \int \mathrm{d}\Pi_f \; |\mathcal{M}(i \to f)|^2 \quad \sim \quad \mathrm{Disc}\, \mathcal{M}(i \to i)$$

Pros
- Forward scattering $\Rightarrow$ simplified kinematics
- Common treatment of loop and phase space integrals
- Calculation of Disc only for master integrals

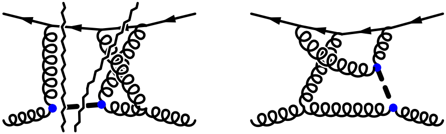
Cons
- More loops and diagrams
- Only total cross section (naively)

Approach first used in [Anastasiou, Melnikov; '02]

Optical theorem and Cutkosky's rules

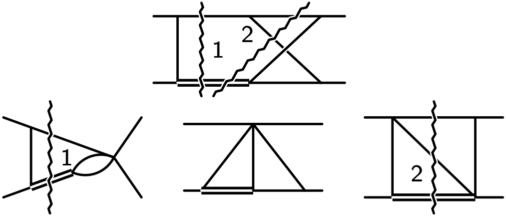
Handling cut-diagrams:

Filter diagrams



- Fast graph-based algorithm build into `Perl` script to process `QGRAF` output
- N^3LO Higgs: 860 118 $\to$ 174 938
- NNLO Higgs pair (SV): 17 667 600 $\to$ 42 252

Assist reduction



- Build also into `TopoID` $\Rightarrow$ pass to Laporta reduction
- Typically: only $\mathcal{O}(10\%)$ of subtopologies (or relations)

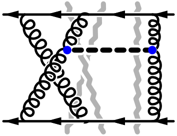
N^3LO Higgs production: qq'-channel

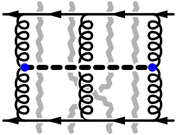
Calculation: toolchain

Reduction

1. Generate Feynman diagrams
   `QGRAF` [Nogueira; '93]
2. Select diagrams with specific cuts
   filter [JH, Pak; (unpublished)]
3. Map diagrams to topologies (← graph information)
   `exp` [Harlander, Seidensticker, Steinhauser; '98]
   `reg` [Pak; (unpublished)]
4. Reduction to scalar integrals (← generic topologies)
   `FORM` [Kuipers, Ueda, Vermaseren, Vollinga; '13]
5. Reduction to master integrals (← basic topologies)
   `rows` [JH, Pak; (unpublished)]
   `FIRE` [Smirnov]
6. Minimal basis of master integrals
   `TopoID` [JH, Pak; (unpublished)]

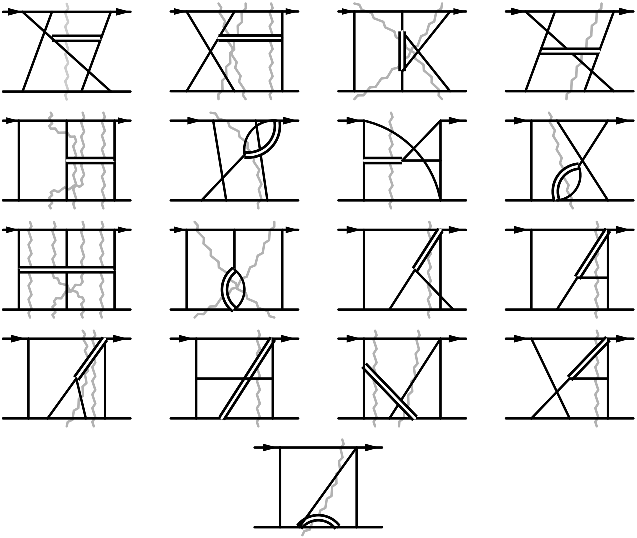
220 diagrams, e.g.





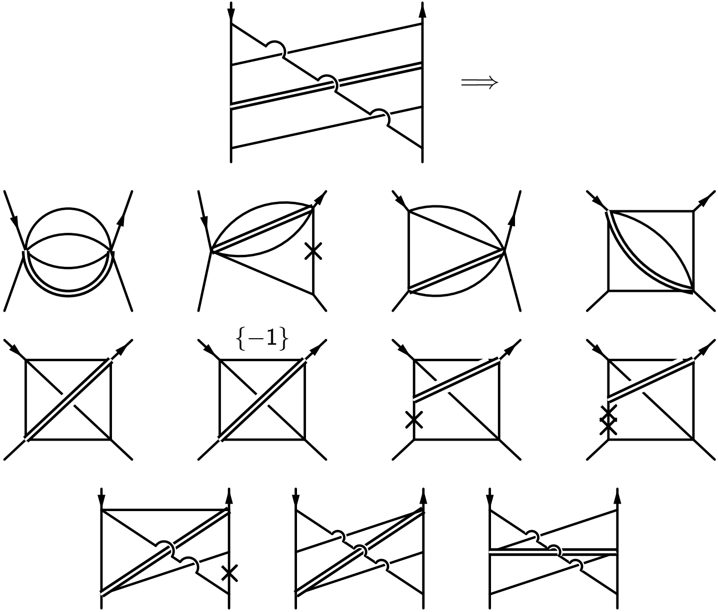
N^{3}LO Higgs production: qq'-channel

Calculation: 17 **topologies with** 3- **and** 4-**particle cuts**



N^{3}LO Higgs production: qq'-channel

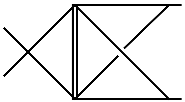
Calculation: e.g. "sea snake" topology



N^3LO Higgs production: qq'-channel

Results: functions beyond HPLs

- Laporta: 332 master integrals; `TopoID`: 108 and cancellation of ξ
- Some Feynman integrals generate functions with alphabet beyond HPLs: $\left\{\frac{1}{x}, \frac{1}{1-x}, \frac{1}{1+x}, \frac{1}{1+4\,x}, \frac{1}{x\sqrt{1+4\,x}}\right\}$
- Traced back to common subtopology:



- Numerical implementation in `Mathematica`:
  - Change alphabet to $\left\{\frac{1}{x}, \frac{1}{1-x}, \frac{1}{1+x}, \frac{1}{1+4\,x}, \frac{1}{x}\left(\frac{1}{\sqrt{1+4\,x}} - 1\right)\right\}$
  - Use series expansions $x \to 0$ and $x \to 1$ $\Rightarrow$ 10 digits in 1 second
- Note: $x \to (1-x)/x^2$ $\Rightarrow$ "Cyclotomic Polylogarithms"
  - Representation as "Goncharov Polylogarithms" (linear denominators)
  - Letters of 6th roots of unity; here only: $\left\{\frac{1}{x}, \frac{1}{1-x}, \frac{1}{(-1)^{\frac{1}{3}} - x}\right\}$

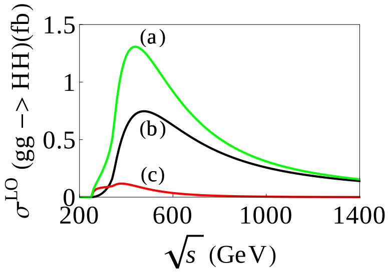
NLO and NNLO Higgs pair production

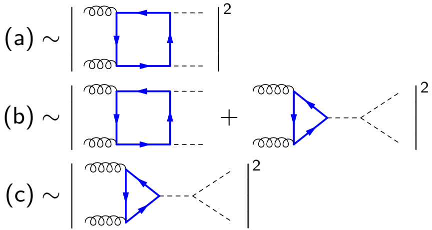
Motivation

Higgs Potential in the Standard Model:

$$V(H) = \frac{1}{2} m_H^2 H^2 + \lambda v H^3 + \frac{1}{4} \lambda H^4, \quad \lambda^{\text{SM}} = \frac{m_H^2}{2v^2} \approx 0.13, \quad v\text{: Higgs vev.}$$

- Verify mechanism of spontaneous symmetry breaking in the SM
- Measure the Higgs self-coupling $\Rightarrow$ sensitive process





Prospects for the LHC @ 14 TeV:

- $b\bar{b}\gamma\gamma$-channel, 600 fb^{-1}: $\lambda \neq 0$ [Baur, Plehn, Rainwater; '04]
- $b\bar{b}\gamma\gamma$-, $b\bar{b}\tau^+\tau^-$-channels: "promising";
  $b\bar{b}W^+W^-$-channel: "not promising"
  [Baglio, Djouadi, Gröber, Mühlleitner, Quevillon, Spira; '13]
- 600 fb^{-1}: $\lambda > 0$; 3000 fb^{-1}: $\lambda^{+30\%}_{-20\%}$ (ratio with Higgs cross section)
  [Goertz, Papaefstathiou, Yang, Zurita; '13]
- And many others, e.g.:
  [Dolan, Englert, Spannowsky; '12] [Papaefstathiou, Yang, Zurita; '13]
  [Barr, Dolan, Englert, Spannowsky; '13] [Barger, Everett, Jackson, Shaughnessy; '14]
  [Englert, Krauss, Spannowsky, Thompson; '15] [. . .]
- Until now: Higgs pair production not observed in $b\bar{b}b\bar{b}$- and $b\bar{b}\gamma\gamma$-channels (as expected in the SM) [ATLAS; '15] [CMS; '15]

$\Rightarrow$ **Wait for HL-LHC**

NLO and NNLO Higgs pair production

Theory status

Known since long:

- LO result with exact M_t dependence [Glover, van der Bij; '88] [Plehn, Spira, Zerwas; '98]
- NLO result in $M_t \to \infty$ limit [Dawson, Dittmaier, Spira; '98]

$\boxed{\sigma_H \approx 20^{\text{LO}}\,\text{fb} + 20^{\text{NLO},M_t\to\infty}\,\text{fb}}$ for $\sqrt{s_H} = 14\,\text{TeV}$, $\mu = 2m_H$

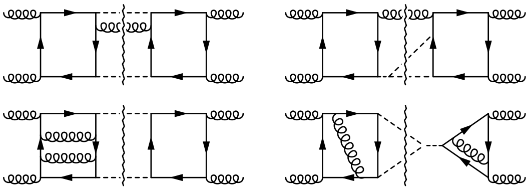
More recently:

- NLO + NNLL $(M_t \to \infty) \approx$ NLO $+20\%$ [Shao, Li, Li, Wang; '13]
- NNLO w/ or w/o soft-virtual approx. $(M_t \to \infty) \approx$ NLO $+20\%$ [de Florian, Mazzitelli; '13]
- $\mathcal{O}\left(1/M_t^8\right)$ corrections at NLO $\approx$ NLO $+10\%$ [Grigo, JH, Melnikov, Steinhauser; '13]
- NLO real exact in M_t, NLO virt. for $M_t \to \infty \approx$ NLO -10% [Maltoni, Vryonidou, Zaro; '14]
- Cross-check of virtual NNLO corrs.; NNLO matching coefficient for $ggHH$-coupling $\approx$ NNLO $+1\%$ [Grigo, Melnikov, Steinhauser; '14]
- Improved $\mathcal{O}\left(1/M_t^{12}\right)$ NLO, $\mathcal{O}\left(1/M_t^4\right)$ NNLO soft-virt. corrections [Grigo, JH, Steinhauser; 15']

NLO and NNLO Higgs pair production

Generalities

- Operate on full-theory diagrams at NLO and NNLO
- Virt. corrs. in two independent calculations:
  - amplitude (differential; 2-/3-loop)
  - forward scattering (total; 4-/5-loop)
- Real corrs.: only via forward scattering at NLO
- Perform expansion for $M_t \to \infty$; improve upon effective theory results for NLO [Dawson, Dittmaier, Spira; '98], NNLO [de Florian, Mazzitelli; '13]
- Laporta reduction to master integrals for the "soft" subdiagrams
- Remaining "hard" massive tadpoles via `MATAD`
- Master integrals as series around $\sqrt{s} = 2m_H$ (not in this talk)



Differential factorization

Factorization of the LO result exact in M_t for:

Total cross section

$$\sigma^{(i)} = \Delta^{(i)} \sigma^{(0)}_{\text{exact}} = \frac{\sigma^{(0)}_{\text{exact}}}{\sigma^{(0)}_{\text{exp}}} \int_{4m_H^2}^{s} \mathrm{d}Q^2 \, \frac{\mathrm{d}\sigma^{(i)}_{\text{exp}}}{\mathrm{d}Q^2}$$

$$\text{with} \quad \Delta^{(i)} = \frac{\sigma^{(i)}_{\text{exp}}}{\sigma^{(0)}_{\text{exp}}}, \quad \sigma^{(i)}_{\text{exp}} = \sum_{n=0}^{N} c_n^{(i)} \rho^n, \quad \rho = \frac{m_H^2}{M_t^2}$$

Differential cross section

$$\sigma^{(i)} = \int_{4m_H^2}^{s} \mathrm{d}Q^2 \, \frac{\left(\frac{\mathrm{d}\sigma^{(0)}_{\text{exact}}}{\mathrm{d}Q^2}\right)}{\left(\frac{\mathrm{d}\sigma^{(0)}_{\text{exp}}}{\mathrm{d}Q^2}\right)} \frac{\mathrm{d}\sigma^{(i)}_{\text{exp}}}{\mathrm{d}Q^2}$$

"Cure" the invalidity of the $M_t \to \infty$ expansion for the large-Q^2 region

- Virt. corrs. via amplitude: access to Q^2-dependence $\sim \delta\left(s - Q^2\right)$
- Real corrs. via optical theorem (naively): only total cross section
  $\Rightarrow$ Use the soft-virtual approximation [de Florian, Mazzitelli; '12]

Soft-virtual approximation

- Split σ into its contributions (works also for $\mathrm{d}\sigma/dQ^2$):

$$\sigma = \sigma^{\text{virt+ren}} + \sigma^{\text{real+split}} = \text{finite}$$
$$= \underbrace{\Sigma_{\text{div}} + \Sigma_{\text{fin}} \quad + \quad \Sigma_{\text{soft}}}_{=\Sigma_{\text{SV}}=\text{finite}} + \underbrace{\Sigma_{\text{hard}}}_{=\Sigma_{\text{H}}=\text{finite}}$$

- Σ_{div} universal for color-less final state [de Florian, Mazzitelli; '12]
- Compute $\sigma^{\text{virt+ren}}$ as ρ-expansion
- Solve $\sigma^{\text{virt+ren}} = \Sigma_{\text{div}} + \Sigma_{\text{fin}}$ for Σ_{fin}
- Σ_{div} and Σ_{soft} (soft coll. counterterms + soft real corrs.) $\sim$ exact σ^{LO} (include M_t effects)

$$Q^2 \frac{\mathrm{d}\sigma}{\mathrm{d}Q^2} = \sigma^{\text{LO}} z G(z) \quad \text{with} \quad z = \frac{Q^2}{s}, \quad G(z) = G_{\text{SV}}(z) + G_{\text{H}}(z)$$

$$\sigma_{(\text{SV})} = \int_{1-\delta}^{1} \mathrm{d}z \sigma^{\text{LO}}(zs)\, G_{(\text{SV})}(z) \quad \text{with} \quad \delta = 1 - \frac{4m_H^2}{s}$$

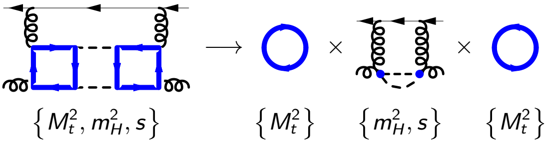
- $G_{\text{SV}}^{(i)}(z)$ constructed from $\sigma_{\text{fin}}^{(i)}$ and σ^{LO} only
  [de Florian, Mazzitelli; '12] [Grigo, JH, Steinhauser; '15]

Asymptotic expansion

- Expand at integrand level for all contributing regions
  $\equiv$ series expansion in analytic result
- Hierarchy: $M_t^2 \gg s, m_H^2 \;\Rightarrow$ series in $\rho = m_H^2/M_t^2$
- Effectively reduce number of loops and scales

- Here: regions correspond to subgraphs (in general more than one)
- Hard mass expansion: subgraphs must contain all heavy lines

Example: NLO real with one region



Asymptotic expansion

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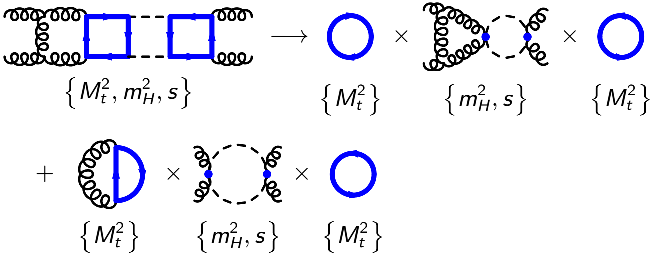
- Effectively reduce number of loops and scales

- Here: regions correspond to subgraphs (in general more than one)
- Hard mass expansion: subgraphs must contain all heavy lines

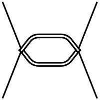
Example: NLO virt. with two regions



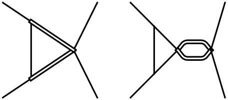
NLO and NNLO Higgs pair production

Calculation (via optical theorem)

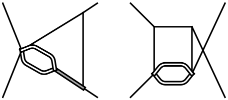
LO topology:



Virt. NLO topologies:



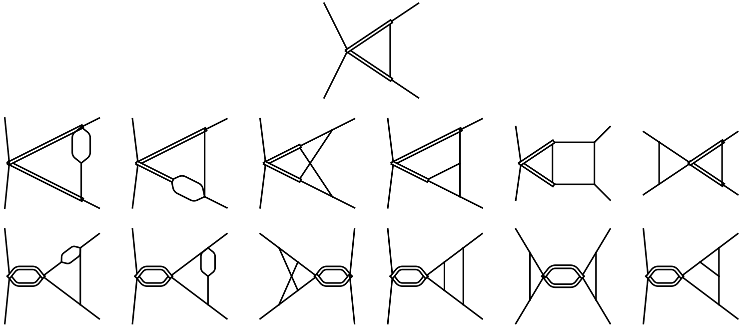
Real NLO topologies:



NLO and NNLO Higgs pair production

Calculation (via optical theorem)

Virt. NNLO topologies:



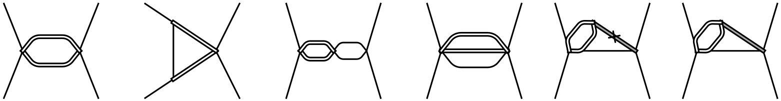
Note:

- Different regions in asymptotic expansion $\Rightarrow$ different loop-orders
- Here: multiplied with 1- to 3-loop massive tadpoles

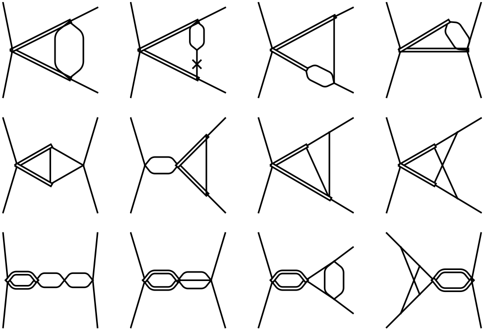
NLO and NNLO Higgs pair production

Calculation (via optical theorem)

Virt. and real LO-NLO master integrals:



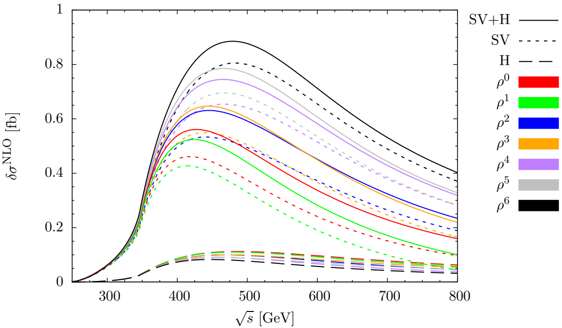
Virt. NNLO master integrals (in addition):



NLO and NNLO Higgs pair production

Splitting in soft-virtual and real contributions at NLO

- Partonic NLO correction; total factorization



Using $\mu = 2m_H$ (also in the following)

- Different behavior for higher orders in ρ expansion:

  SV increasing

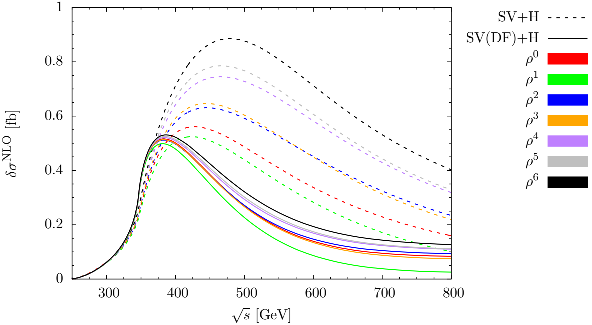
  H descreasing (flat for $\sqrt{s} \gtrsim 400\,\text{GeV}$)

$\Rightarrow$ SV numerically dominant

NLO and NNLO Higgs pair production

Total vs. differential factorization (DF) at NLO

- DF applied only to SV part;
  H treated via total factorization (i.e. identical)



- Maxima of DF curves at lower $\sqrt{s}$; smaller cross sections

$\Rightarrow$ Improvement of convergence:
difference of ρ^0 and corrections to ρ^6 (for $\sqrt{s} = 400\,\mathrm{GeV}$):
0.25 fb vs. 0.05 fb

- (Partonic K-factor: behavior at top quark pair threshold not washed out)

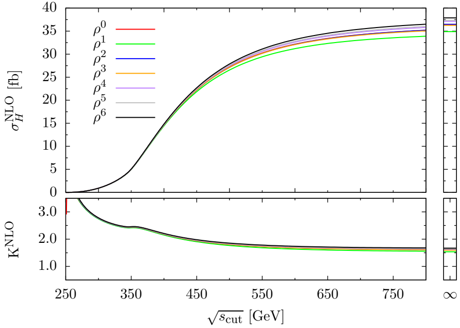
NLO and NNLO Higgs pair production

Hadronic cross section and K-factor

- Technical upper cut on $\sqrt{s}$ (good proxy to Q^2):

$$\sigma_H(s_H, s_{\text{cut}}) = \int_{4m_H^2/s_H}^{1} \mathrm{d}\tau \left(\frac{\mathrm{d}\mathcal{L}_{gg}}{\mathrm{d}\tau}\right)(\tau)\,\sigma(\tau s_H)\,\theta(s_{\text{cut}} - \tau S_H)$$

- $\sqrt{s_{\text{cut}}} \to \infty$: total cross section for 14 TeV



- Spread of ρ-orders $\Rightarrow$ $\pm 10\%$ uncertainty of EFT at NLO due to M_t

Lessons from NLO for NNLO:

- SV approximation contructed for $z \to 1$;
  $G_{SV}(z)$ can be replaced by $f(z)\, G_{SV}(z)$
  Splitting into SV and H not unique
- Tune $f(z)$ at NLO such that $\sigma \approx \Sigma_{SV}$

$\Rightarrow$ $f(z) = z$ accurate to 2%

- Replace RGE logarithms ($\sqrt{s} \approx Q^2$ in the soft limit):

$\Rightarrow$ $\log\left(\mu^2/s\right) \to \log\left(\mu^2/Q^2\right)$

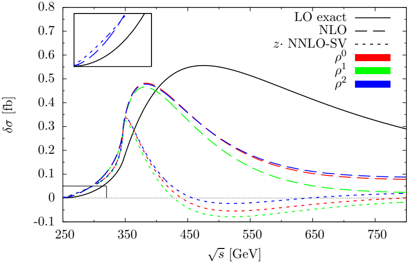
Discrepancy to [Maltoni, Vryonidou, Zaro; '14]:

- Real corrs.: treated exactly; Virt. corrs.: EFT result
- Claim: -10% correction at NLO

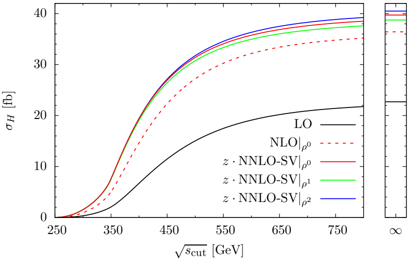
But: Dominant positive shift from virtual $1/M_t$-corrections (c.f. backup slide)

NLO and NNLO Higgs pair production

NNLO SV corrections



- EFT result plus ρ- and ρ^2-terms
- Peaks at smaller $\sqrt{s}$
- Same pattern of ρ-corrections



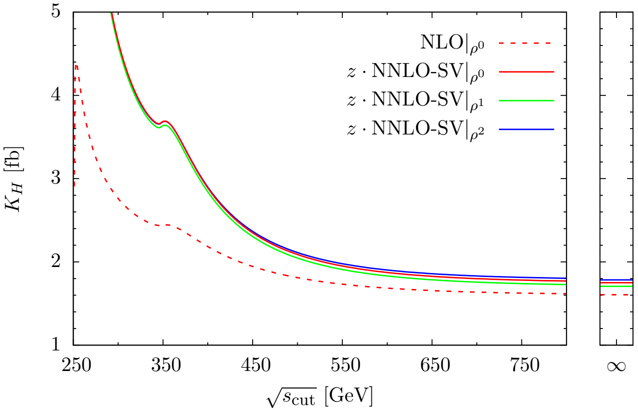
- Conv. up to $\sqrt{s_{\mathrm{cut}}} \approx 400$ GeV
- ρ- and ρ^2-corrections: $\pm 2.5\%$

$\Rightarrow$ **M_t-uncertainty at NNLO: 5%**

(NNLO corrs. $\approx$ 20%)

NLO and NNLO Higgs pair production

NNLO SV K-factor



- Strong raise close to threshold $\Leftarrow$ steeper NNLO correction
- For total cross section: $\boxed{K_H^{\text{NNLO}} \approx 1.7 - 1.8}$

Conclusion

TopoID:

- Generic, process independent Mathematica package for multiloop calculations; especially for many topologies
- Until now: two successful applications
- Works also for 5-loop propagators
- Publication planned for ACAT 2016

qq'-channel in N^{3}LO Higgs production:

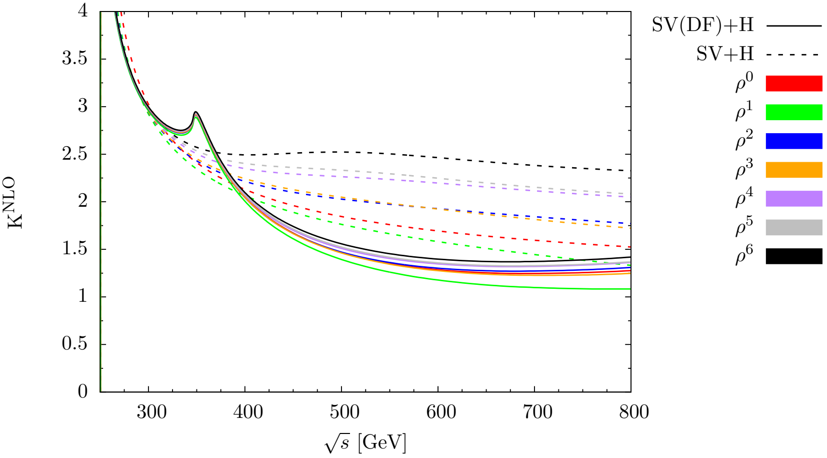
- New iterated integrals beyond HPLs appear
- Agreement of leading logarithms with [Anastasiou, Duhr, Dulat, Furlan, Herzog, Mistlberger; '15]
- Full calculation underway ...

Higgs pair production:

- Real corrections at NLO via opt. theorem; employ SV approx. at NNLO
- Computed expansion to $\mathcal{O}\left(1/M_t^{12}\right)$ at NLO, $\mathcal{O}\left(1/M_t^{4}\right)$ at NNLO
- Uncertainty of EFT results due to M_t: 10% at NLO, 5% at NNLO
- Small-$\sqrt{s}$ behavior as benchmark for exact calculations

Backup: NLO and NNLO Higgs pair production

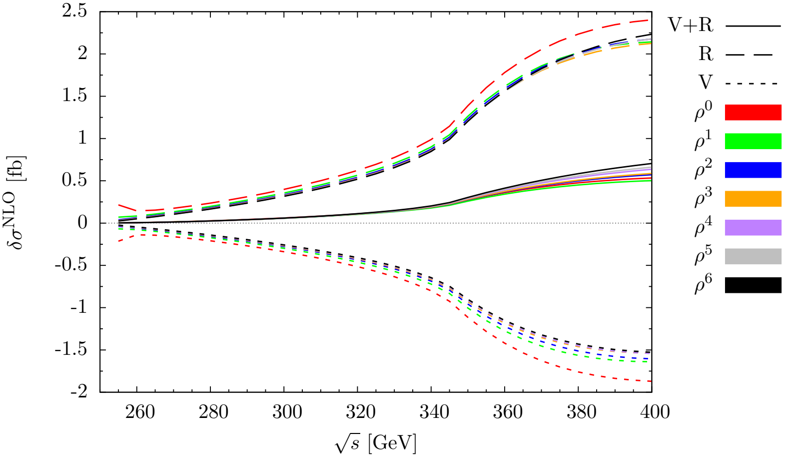
Partonic NLO K-factor



Note: Behavior around top quark pair threshold not washed out

Backup: NLO and NNLO Higgs pair production

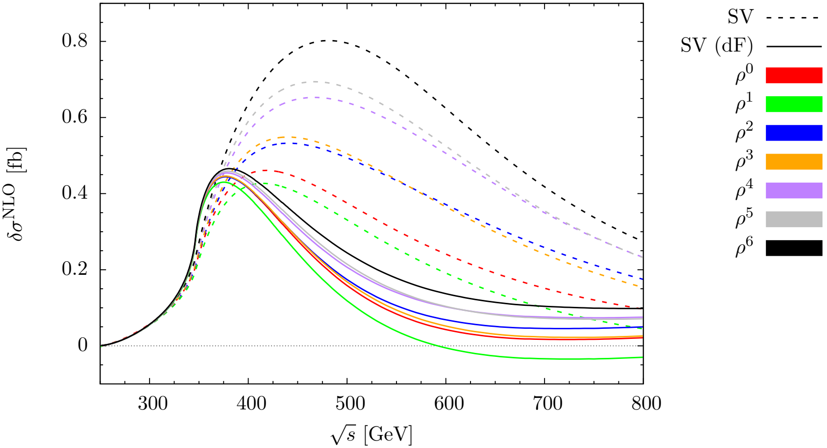
Splitting into real and virtual corrections at NLO



Note: R and V separately divergent; only finite contributions shown

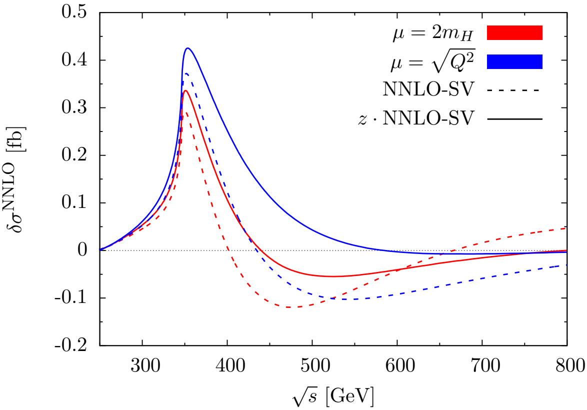
Backup: NLO and NNLO Higgs pair production

Total vs. differential factorization without hard contributions at NLO



Backup: NLO and NNLO Higgs pair production

Partonic NNLO cross section for different scales choices and $f(z)$



Note: $f(z) = z$ (better proxy) leads to higher values