The Non-Perturbative Quantum Vortex in the (2+1)-d $O(2)$ Scalar Field Theory

Joao C. Pinto Barros

High Performance Computational Physics Group, Institute for Theoretical Physics, ETH Zurich

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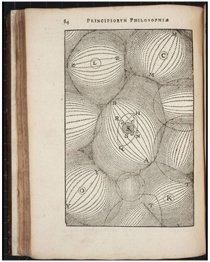
University of Zurich / ETH Zurich

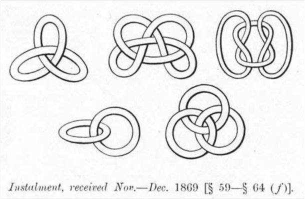
Seminar in Theoretical Particle Physics

Vortices of “ancient” physics

Decartes’ vortex cosmology (1664)



Thomson’s vortex atoms (1867)



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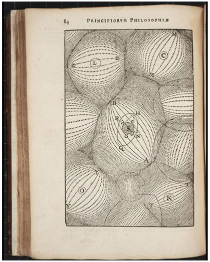

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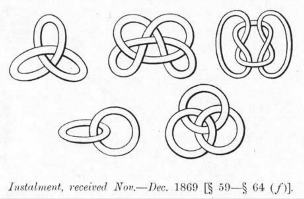

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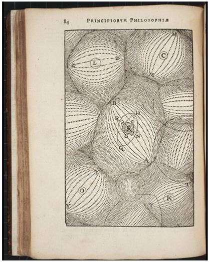

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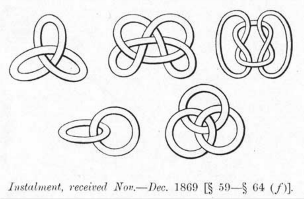

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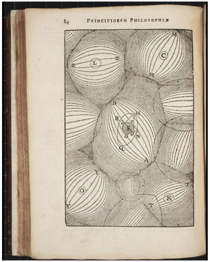

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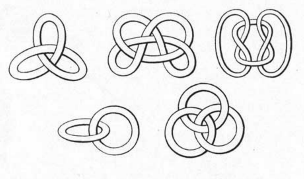

Instalment, received Nov.—Dec. 1869 [§ 59—§ 64 (*f*)].

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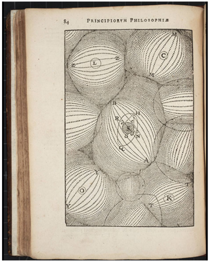

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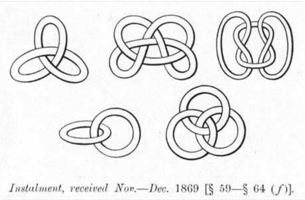

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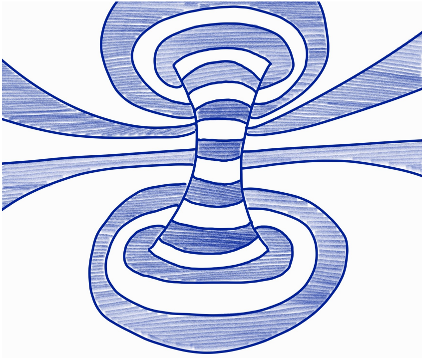
  - Non-perturbative quantization;
- Universal vortex mass and charge of the quantum vortex.

Different dimensionalities:

- point defects in 2d;
- line defects in 3d;



Different physical regimes:

- in classical fluids;
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- (...)

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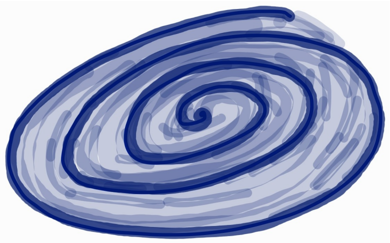

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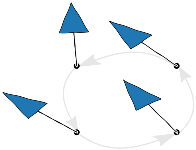
Vorticity in a 2-d vector field



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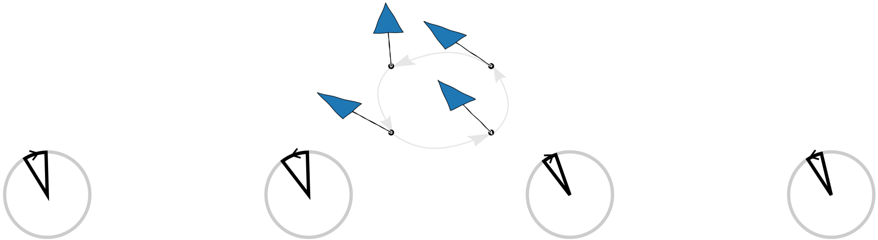

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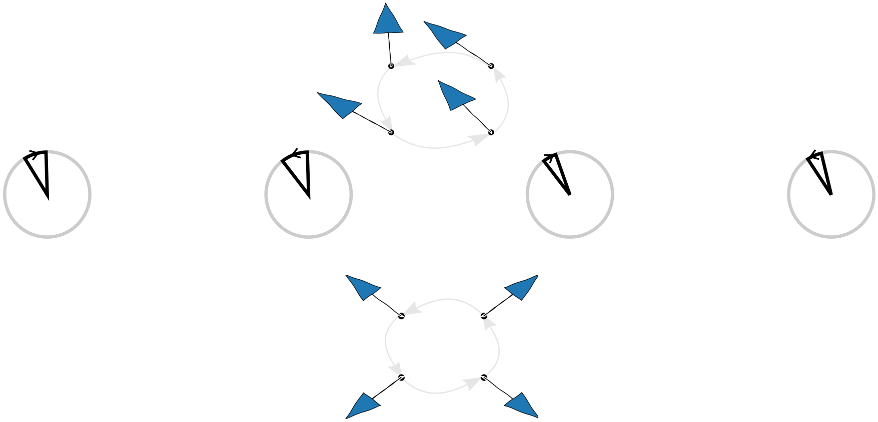

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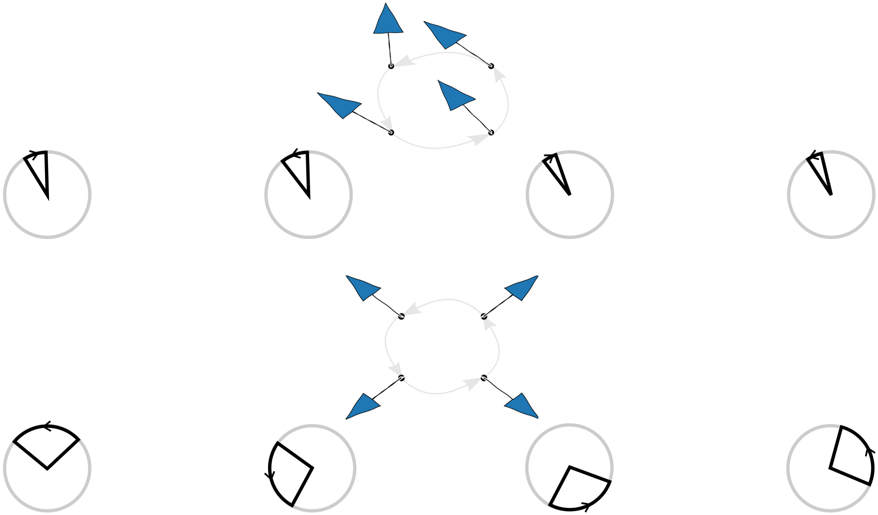
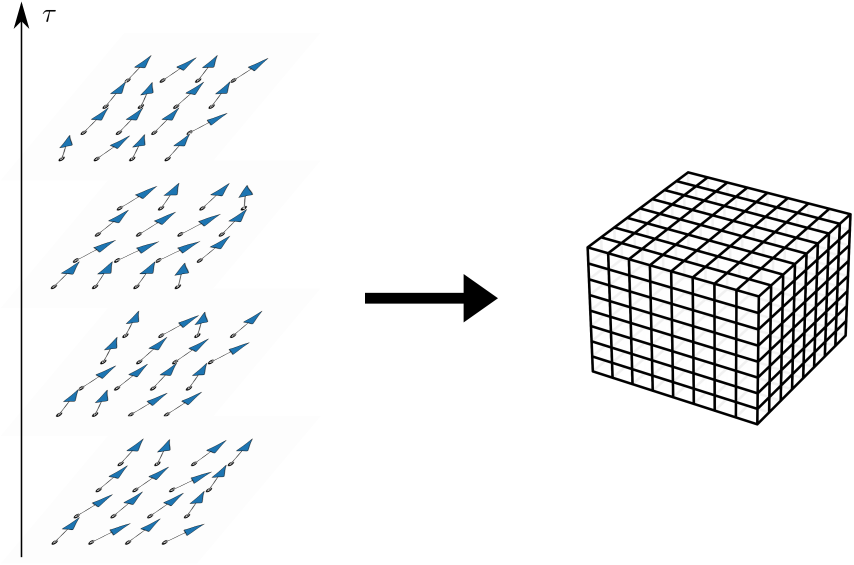
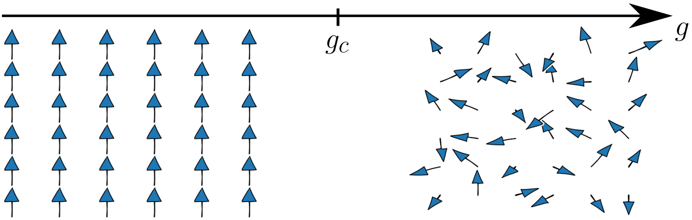


Overview of the (2+1)-d $O(2)$ model



Broken phase

Symmetric Phase



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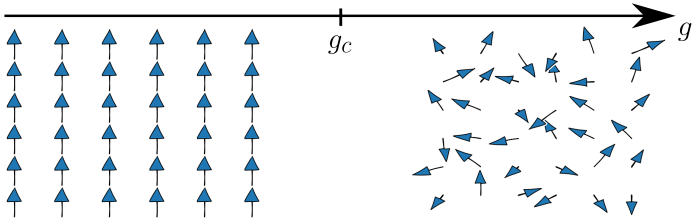
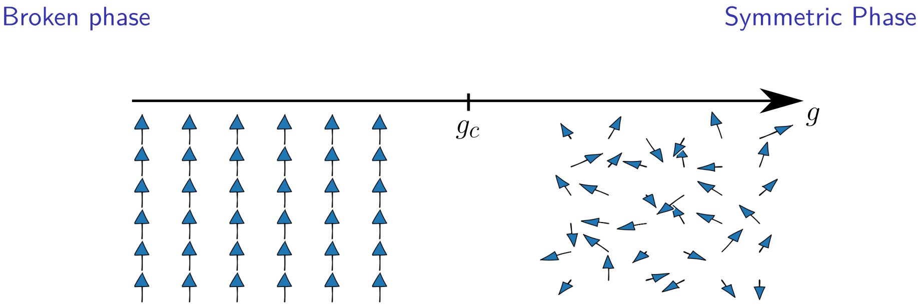
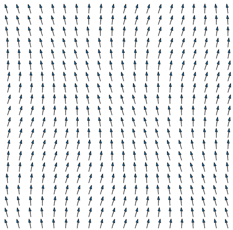


In the broken phase:

- Spontaneous breakdown of the $O(2)$ symmetry;
- Massless modes: Goldstone bosons.





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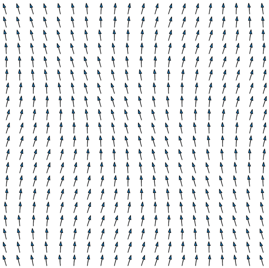
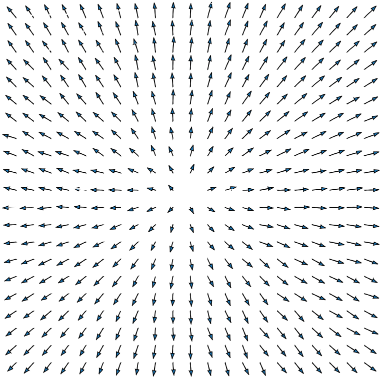
A sea of Goldstone bosons can give rise to a topologically non-trivial excitation





Vortex excitation

The (2+1)-d $O(2)$ scalar field theory:

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Static, rotational invariant solutions:

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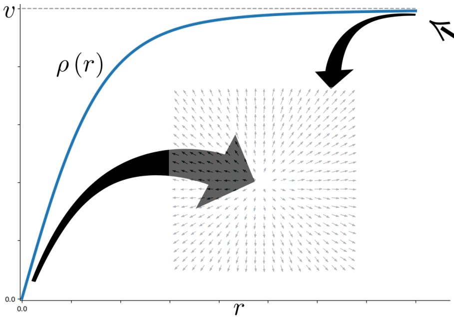
$$\Rightarrow \rho'' + \frac{1}{r}\rho' - \frac{n^2}{r^2}\rho = \frac{\lambda}{6}\rho\left(\rho^2 - v^2\right)$$

The classical vortex: profile and energy

$$\phi(x) = \rho(r)\, e^{i\varphi} \quad \leftrightarrow \quad \rho'' + \frac{1}{r}\rho' - \frac{1}{r^2}\rho = \frac{\lambda}{6}\rho\left(\rho^2 - v^2\right)$$



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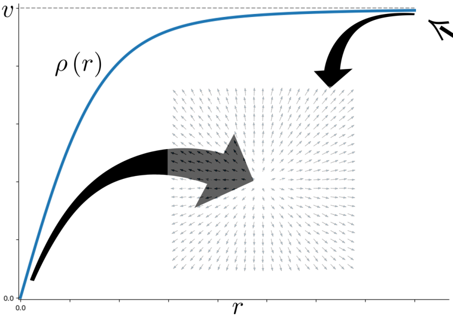

Large distances $\rho\left(r\right)\to v-\frac{1}{6\lambda v r^2}$

$$E\left(R\right)=2\pi\int_0^R dr\ r\mathcal{H}\left(r\right)\ \Rightarrow\ E\left(R\right)\sim\pi v^2\log\frac{R}{R_0}$$

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- Fully non-perturbative approach is non-trivial:
  - vortex correlation not readily amenable to numerical simulations;
  - single vortex never occurs at finite periodic volume.

- Consider the problem in Euclidean time

$$\mathcal{L}_E = \frac{1}{2}\partial_\mu \phi^* \partial^\mu \phi + \frac{\lambda}{4!}\left(|\phi|^2 - v^2\right)^2$$

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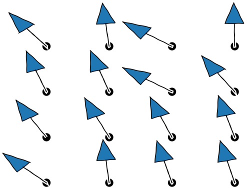

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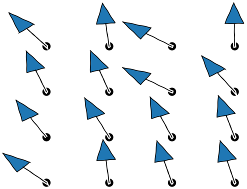

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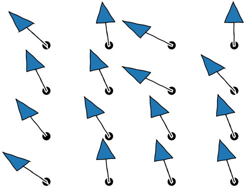

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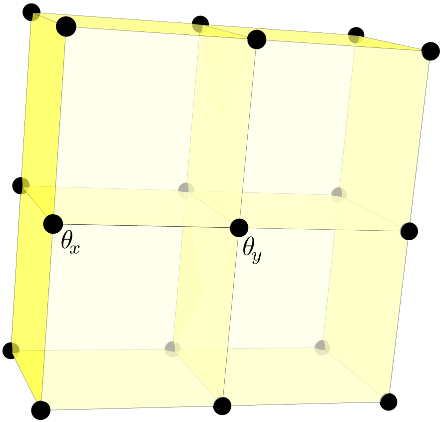
The quantum vortex as a quantum particle



$$s\left(\theta_x - \theta_y\right)$$

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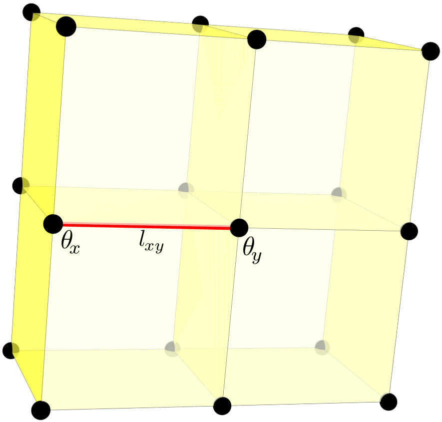
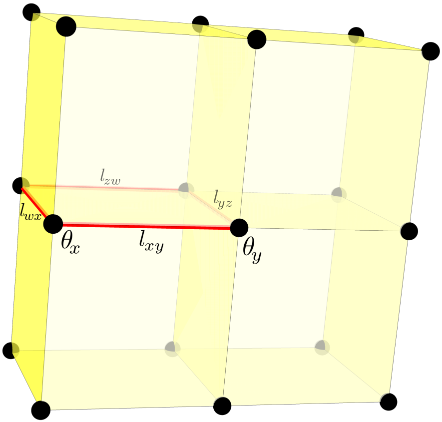
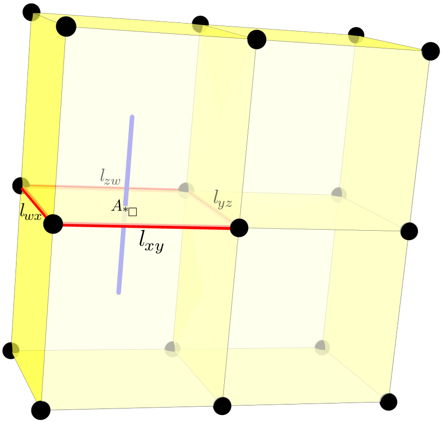


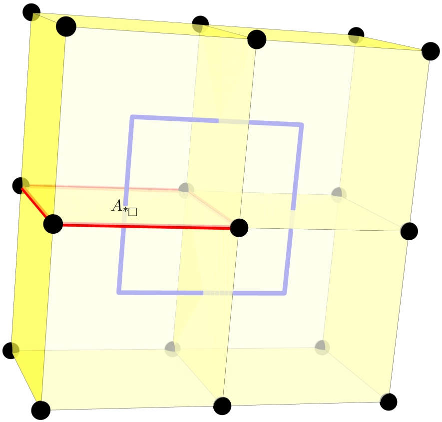
$$s\left(\theta_x - \theta_y\right) \to s\left(l_{xy}\right)$$



$$s\left(l_{xy}\right) \quad \text{with constraints} \quad l_{xy} + l_{yz} + l_{zw} + l_{wx} = 0$$



$$s\left(l_{xy}, A_{*\Box}\right) \quad \text{with} \quad \delta\left(l_{xy} + l_{yz} + l_{zw} + l_{wx}\right) = \sum_{A_{*\Box}\in\mathbb{Z}} e^{iA_{*\Box}(l_{xy}+l_{yz}+l_{zw}+l_{wx})}$$



Integrate out l_{xy} $\Rightarrow$ Gauge Theory : $\tilde{s}\,(A_{*\square})$

The dualized action

- The final gauge theory will depend on the initial action;
  - Villain action

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  - Standard action

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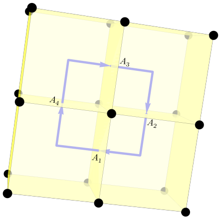
$$F_\square = A_1 + A_2 - A_3 - A_4$$

Gauge Invariance: $A'_{x\mu} = A_{x\mu} + \alpha_{x+\hat{\mu}} - \alpha_x$



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  J. Fröhlich, P. A. Marchetti, Euro. Phys. Lett. 2 (1986) 933

$$\text{Scalar QED}: \bar{A}_{x\mu} \in \mathbb{R}$$

$$S_{\mathrm{QED}}\left(\{\bar{A}\},\{\chi\}\right) = \sum_{\Box} s\left(\bar{F}_{\Box}\right) - \frac{\kappa}{2}\sum_{x,\mu}\left(\chi^*_{x+\hat{\mu}} e^{i\bar{A}_{x\mu}}\chi_x + \chi^*_x e^{-i\bar{A}_{x\mu}}\chi_{x+\hat{\mu}}\right)$$

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Unitary gauge: $-\frac{\kappa}{2} \sum_{x,\mu} \left(\chi^*_{x+\hat{\mu}} e^{i\bar{A}_{x\mu}} \chi_x + \chi^*_x e^{-i\bar{A}_{x\mu}} \chi_{x+\hat{\mu}}\right) \to -\kappa \sum_{x,\mu} \cos \bar{A}_{x\mu}$

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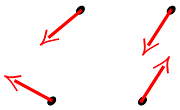
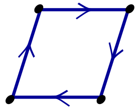
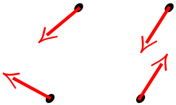
Limit $\kappa \to +\infty$: $-\kappa\sum_{x,\mu}\cos\bar{A}_{x\mu} \to \bar{A}_{x\mu} \in 2\pi\mathbb{Z}$

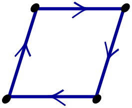




Continuous spin model	DUAL	Scalar QED
$O(2)$ global symmetry	$\rightleftarrows$	$\mathbb{R}$ local symmetry

The dualization picture





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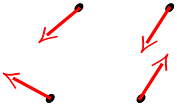

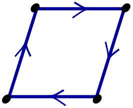

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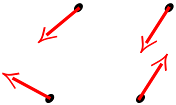

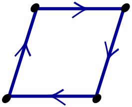

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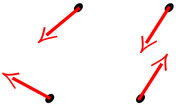

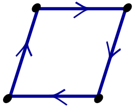

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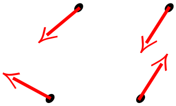

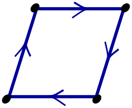

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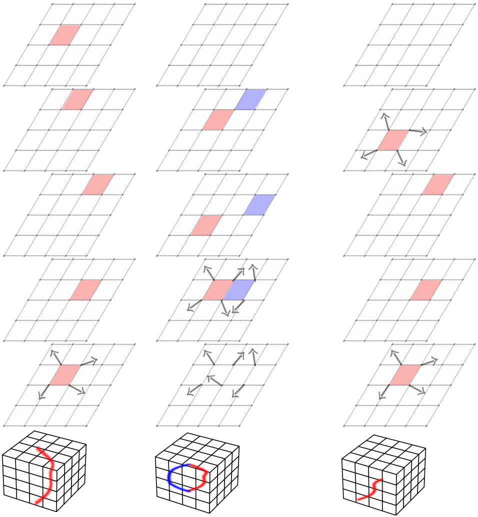
Goldstone bosons	$\rightleftarrows$	Photons
Vortex	$\rightleftarrows$	Charged scalar

The vortex as an infraparticle

$$\langle \chi_x^C \chi_y^{C\dagger} \rangle \underset{\text{back}}{\overset{\text{Dualize}}{\longrightarrow}} \frac{Z_v}{Z}$$



Contribution to Z Contribution to Z_v

Similar to: T. Banks, R. Myerson and J. Kogut, Nucl. Phys. B129 (1977) 493

The infraparticle

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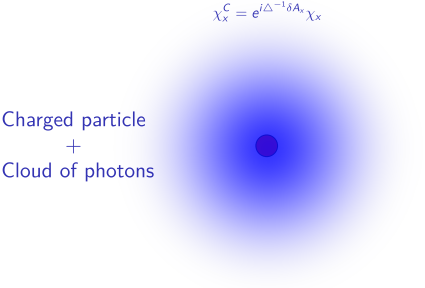
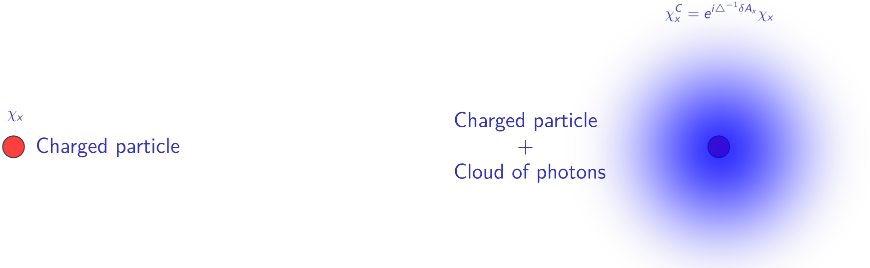


- Vortex correlation function: $\langle \chi_x^C \chi_y^{C\dagger} \rangle$

P. A. M. Dirac, Canad. J. Phys. 33 (1955) 650
J. Fröhlich, P. A. Marchetti, Euro. Phys. Lett. 2 (1986) 933

The vortex as an infraparticle



$O(2)$ model in the broken phase:

- Massless Goldstone boson;
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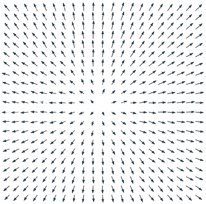
Scalar QED in the Coulomb phase:

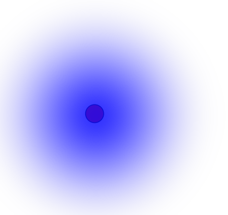
- Massless photon;
- Charged particle constitutes a non-local excitation formed by a cloud of photons.

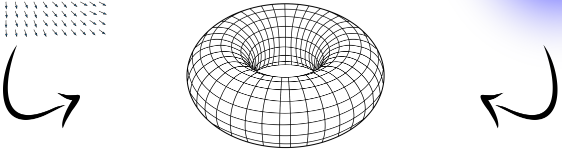
The charged particle at finite volume

Vortex
+
Cloud of Goldstone bosons



Charged particle
+
Cloud of photons





- No net charge on the torus;

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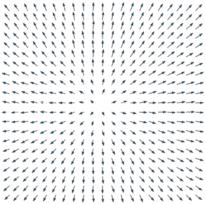

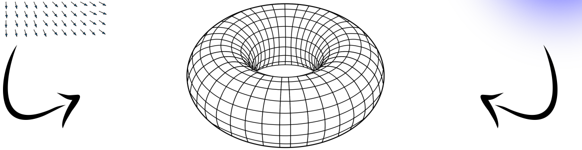

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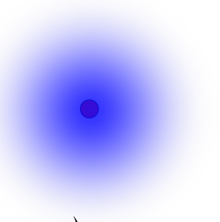
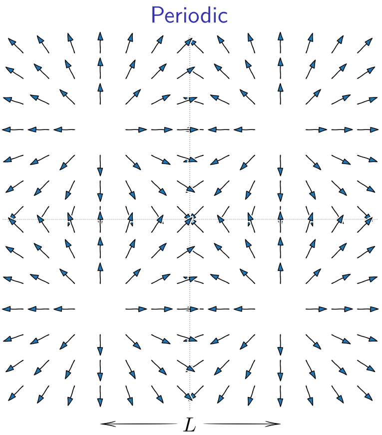


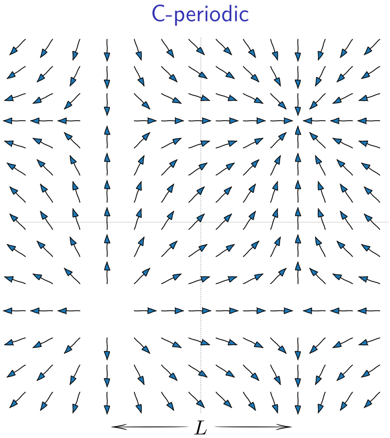
- No net charge on the torus;
- C-periodic boundary conditions (C* boundary conditions):

$$A_\mu\left(x+L\hat{i}\right) = -A_\mu\left(x\right) - \partial_\mu\varphi_i\left(x\right)$$
$$\chi\left(x+L\hat{i}\right) = \chi\left(x\right)^* e^{i\varphi_i(x)}$$

U.-J. Wiese, Nucl. Phys. B375 (1992) 45
B. Lucini, A. Patella, A. Ramos, N. Tantalo, JHEP 1602 (2016) 076C
I. Campos, P. Fritzsch, M. Hansen, M. K. Marinkovic, A. Patella, A. Ramos, N. Tantalo, Eur. Phys. J. C (2020) 80:195

The C-periodic vortex: mass and charge computation





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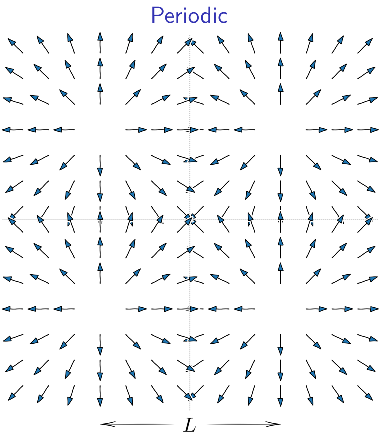

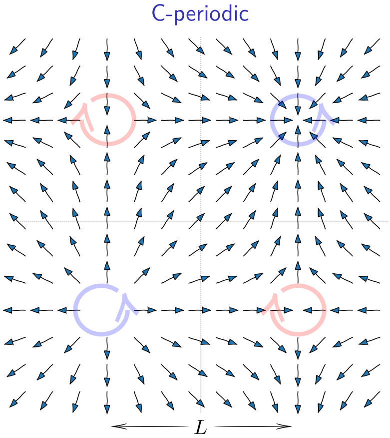

Vortices interact with their C-periodic copy

$$m = \frac{e^2}{4\pi} \log\left(\frac{L}{r_0}\right) \rightarrow \text{Determine the charge}$$

The C-periodic vortex

- Vortex operator in unitary gauge $\chi_x^C = e^{i\triangle^{-1}\delta A_x} \Rightarrow \chi^C_{x+L\hat{i}} = \left(\chi_x^C\right)^*$
  - Real part: periodic
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$$\chi^+(p_1, p_2, \tau) = \sum_{x_1, x_2} \mathrm{Re}\left[\chi^C(x_1, x_2, \tau)\right] e^{i(p_1 x_1 + p_2 x_2)} \qquad p_1, p_2 \in \frac{2\pi}{L}\mathbb{Z}$$

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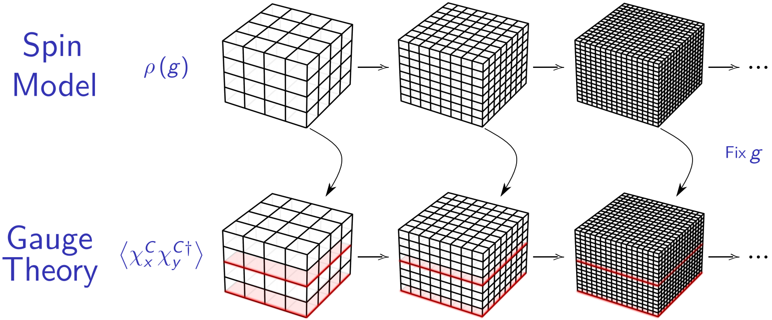
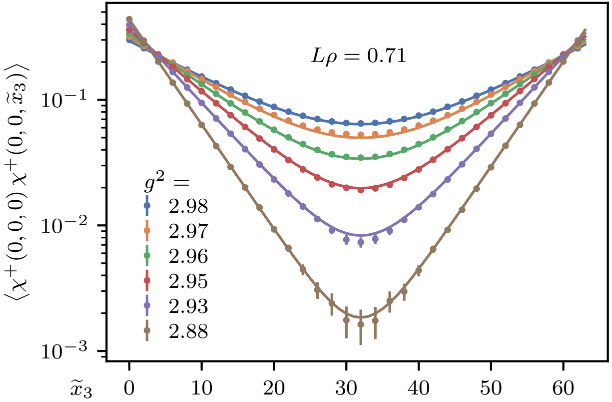
- large τ:

$$\langle \chi^+(0,0,0)\chi^+(0,0,\tau)\rangle \to e^{-m\tau}$$
$$\langle \chi^-(q_1,q_2,0)\chi^-(q_1,q_2,\tau)\rangle \to e^{-E\tau} \qquad q_i = \pm\frac{\pi}{L} \text{ (minimal momentum)}$$

Approach the continuum limit at a finite volume characterized by $\rho_0 = \rho L$



Results: approaching the continuum limit



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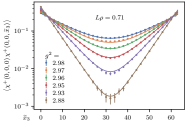

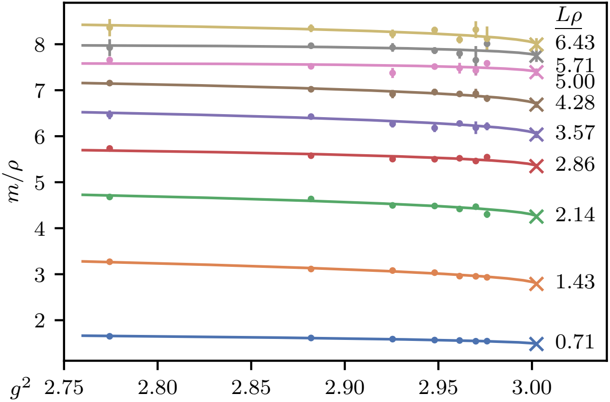

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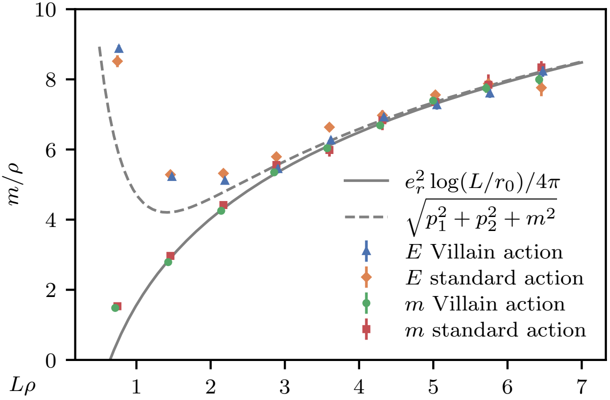

- Log divergent mass
- Universal vortex charge: $e_r^2 = 3.58(8) \times (4\pi\rho)$
- Breaking of Lorentz invariance? $E = m + \frac{p^2}{2m_k} \Rightarrow \frac{m_k}{m} = 0.71(3)$ (for $L\rho = 1.43(2)$).

Overview

M. Hornung, JCPB, and U-J. Wiese arXiv:2106.16191 (2021).

M. Hornung, JCPB, and U-J. Wiese arXiv:2106.16191 (2021).

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M. Hornung, JCPB, and U-J. Wiese arXiv:2106.16191 (2021).

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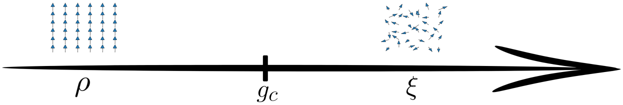
- Experimentally relevant.



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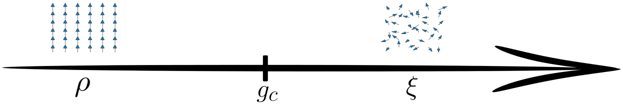
    - Symmetric phase is dual to the Higgs phase;



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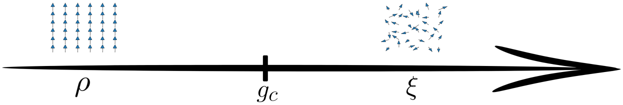

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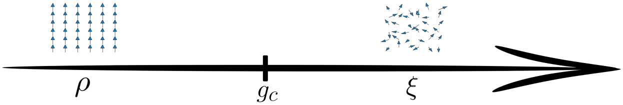
Outlook



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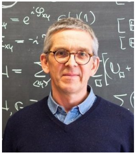
  - The non-Abelian infraparticle.

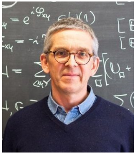
Acknowledgment



Uwe-Jens Wiese

Manes Hornung

Acknowledgment

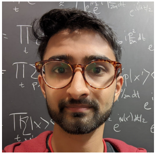


Uwe-Jens Wiese

Manes Hornung

Alessandro Mariani

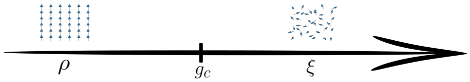
Gurtej Kanwar



- Extract **universal** vortex properties at **finite volume**

Computational strategy

- Extract **universal** vortex properties at **finite volume**



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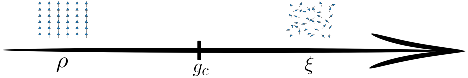
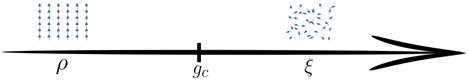


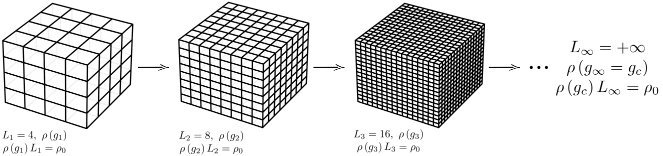
- Characterizing the distance to the critical point by the spin stiffness $(\rho(g_c) = 0)$

$$\rho(g) = -\frac{1}{L} \left. \frac{\partial^2 \log Z(\alpha)}{\partial \alpha^2} \right|_{\alpha=0}$$

with $Z(\alpha)$ defined as the partition function under the twisted boundary conditions: $\theta_{x+\hat{\mu}L_\mu} = \theta_x + \alpha_\mu$.

- Approach to the continuum limit:
  - $g \to g_c^-$;
  - $L \to \infty$;
  - $\rho \to 0$;
  - $\rho L = \rho_0$ constant.





Extracting the spin stiffness

$$\rho(t) = at^{\nu}\left(1 + bt^{\theta} + ct + \ldots\right)$$

$t = \frac{g_c - g}{g_c}$, $\nu = 0.67169(7)$, $\theta = 0.530(3)$.

M. Hasenbusch, Phys. Rev. B 100, 224517 (2019)