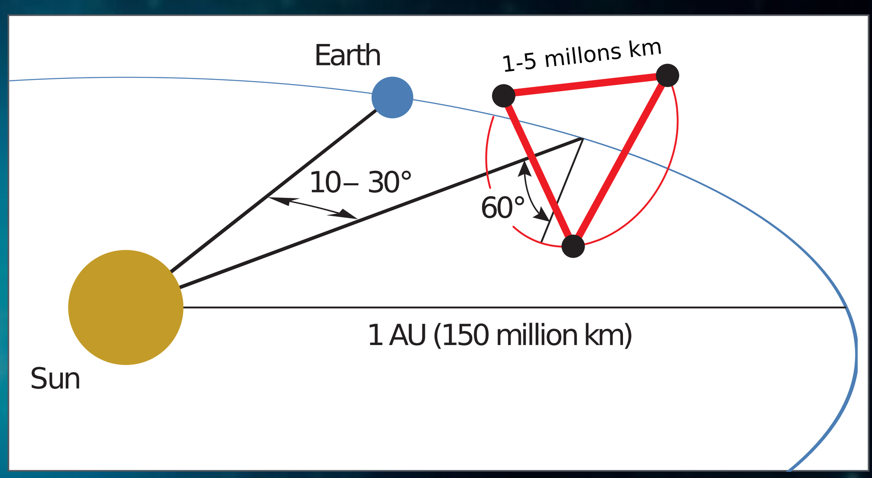
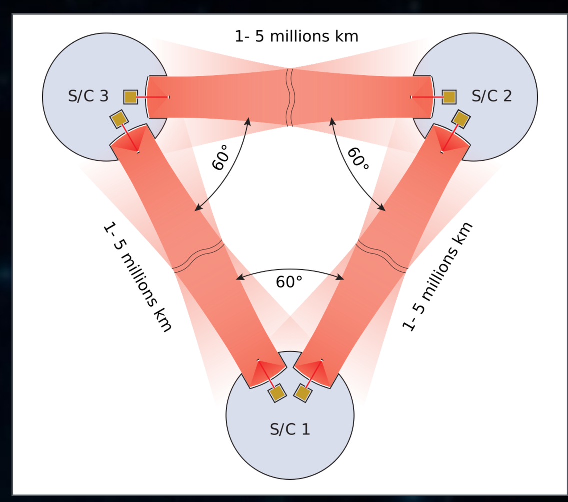
Optimizing orbits for (e)LISA

Hubert Halloin / APC - Université Paris Diderot, 10 rue Alice Domon et Léonie Duquet, Paris
hubert.halloin@apc.univ-paris7.fr

I - Introduction

The three, free-falling, satellites of (e)LISA form an (quasi) equilateral triangle trailing the Earth by about 20°.





While orbiting around the Sun, the distances between the S/C evolve, leading to 'breathing' and 'flexing' effects, which should be accommodated by the satellite design, e.g telescope steering (or laser in-field pointing capability) and wide photodetector bandwidth (because of the Doppler frequency shifts).

These fluctuations can be efficiently minimized analytically for pure keplerian orbits [1]. However, the constellation is also perturbed by the gravitational influence of the Earth and, to a lesser extent, Jupiter and Mars. Numerical optimization is required to find the 'optimal' initial orbital parameters and, possibly, velocity increments for orbit control during the mission lifetime.

The optimization process is subject to two other constraints :

- The data link budget requires a maximum distance to Earth of ≈70 10^6 km [2]
- The wet mass (i.e. including propellant for orbital maneuvers) of the constellation should be compatible with the launcher capability.

II - Optimization method

The numerical optimization process combines an orbit propagator and the minimization of a cost function:

- VEFRL (Velocity Extended Forest-Ruth Like) integrator [3] : Constant step size, quasi-6th order symplectic integrator, optimized for a weak perturbation of a central force potential
- active Covariance Matrix Adaptation Evolution Strategy (aCMAES) [4] : developed for non-linear non-convex black-box optimisation, especially used for continuous cost functions with sharp edges, local minima, outliers, etc.

The cost function is expressed as a linear combination of various penalties:

$$C(\mathcal{P}) = \alpha_A \cdot C_A(\mathcal{P}) + \alpha_B \cdot C_B(\mathcal{P}) + \alpha_F \cdot C_F(\mathcal{P}) + \alpha_M \cdot C_M(\mathcal{P}) + \alpha_D \cdot C_D(\mathcal{P})$$

$$C_A(\mathcal{P}) = \frac{1}{3}\sum_{i=1}^{3}\left\langle\left(\frac{L_i(t) - L_{ref}}{L_{ref}}\right)^2\right\rangle_t \quad : \text{ Armlength penalization } (L_{ref} : \text{ desired arm length})$$

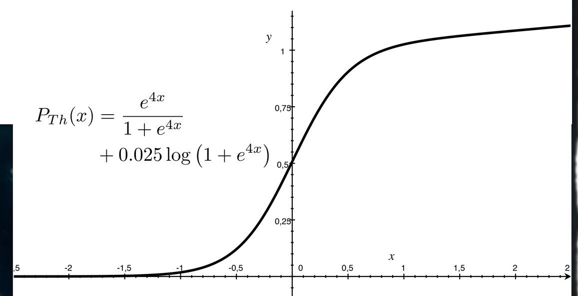
$$C_B(\mathcal{P}) = \max_{i\in[1:3]} \frac{\max_t \theta_i(t) - \min_t \theta_i(t)}{60°} \quad : \text{ Breathing penalization } (\theta_i(t) : \text{ inner angle at vertex i and time t})$$

$$C_F(\mathcal{P}) = \max_{i\in[1:3]} \frac{\max_t \Delta F_i(t) - \min_t \Delta F_i(t)}{1 \text{ MHz}} \quad : \text{Frequency shift penalization } (\Delta F_i : \text{ shift on arm i, at time t})$$

$$C_M(\mathcal{P}) = P_{Th}\left(\frac{\sum_{i=1}^{3} M_i - M_{max}}{M_{spread}}\right) \quad : \text{Mass penalization } (M_{max} : \text{ max allowed mass for the constellation})$$

$$C_D(\mathcal{P}) = P_{Th}\left(\frac{\max_{i,t} D_i(t) - D_{max}}{D_{spread}}\right) \quad : \text{Distance penalization } (D_{max} : \text{ maximum allowed distance to Earth})$$

$$P_{Th}(x) = \frac{e^{4x}}{1+e^{4x}} + 0.025\log\left(1+e^{4x}\right) \quad : \text{Threshold penalization function}$$



The hyper-parameters α are used to balance the weight of the different penalizations.

The estimation of the propellant mass is done using the following scenario :

- Launch on a Geostationary Transfer Orbit (200 km x 36 000 km)
- Escape manœuvre on the ecliptic plane for a cruise time of 15 months
- 6 to 12 months Lambert transfer between the ascending or descending node of the targeted orbital plane and the initial position of the science mission.
- Velocity correction to reach the desired initial orbital parameters at the beginning of the science mission.
- Regular velocity increments for orbit control during mission lifetime (if any)

III - Optimization results

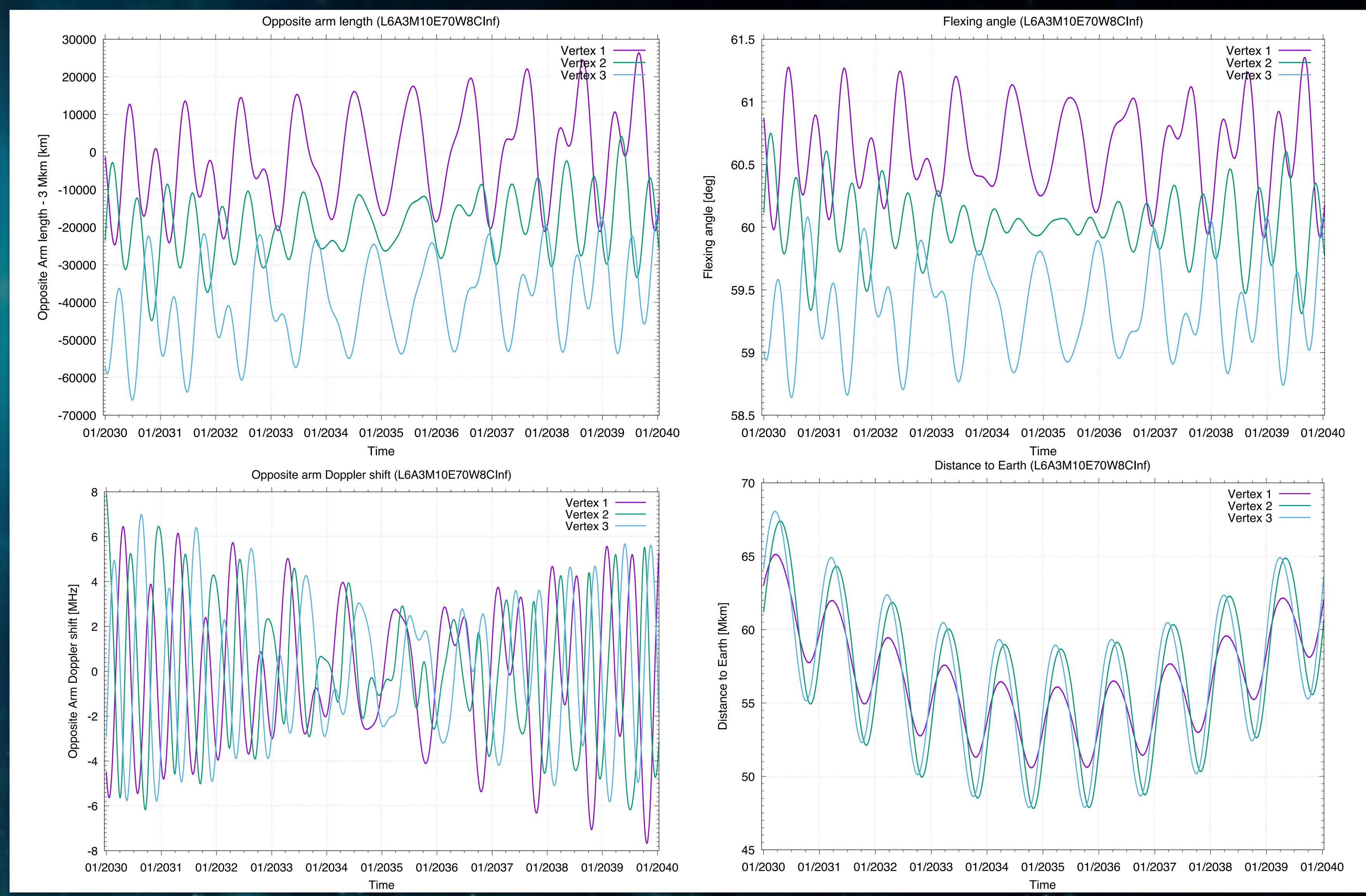
As a example, the above optimization scheme have been applied with the following configuration :

- 3 10^6 km armlength (L_{ref})
- 10 years mission lifetime (2030 - 2040)
- 8 t maximum launch mass (compatible with Ariane 6 A64 GTO capability)
- 70 10^6 km maximum distance to Earth
- 850 kg dry mass for each satellite, thrusters with 300 s specific impulse
- Perturbing bodies : Earth, Moon, Mars and Jupiter

IIIa. No orbit control maneuvers

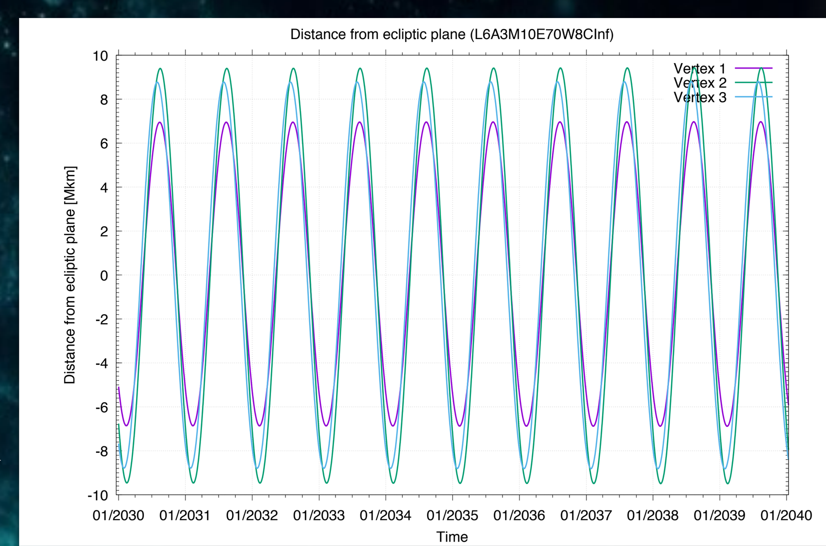
In a first optimization run, maneuvers are only used to position the S/C on their initial orbits, with no further orbit control maneuver during the duration of the mission.

The following figures represent the resulting time evolution of flexing angle, armlength, frequency shift and distance to Earth :



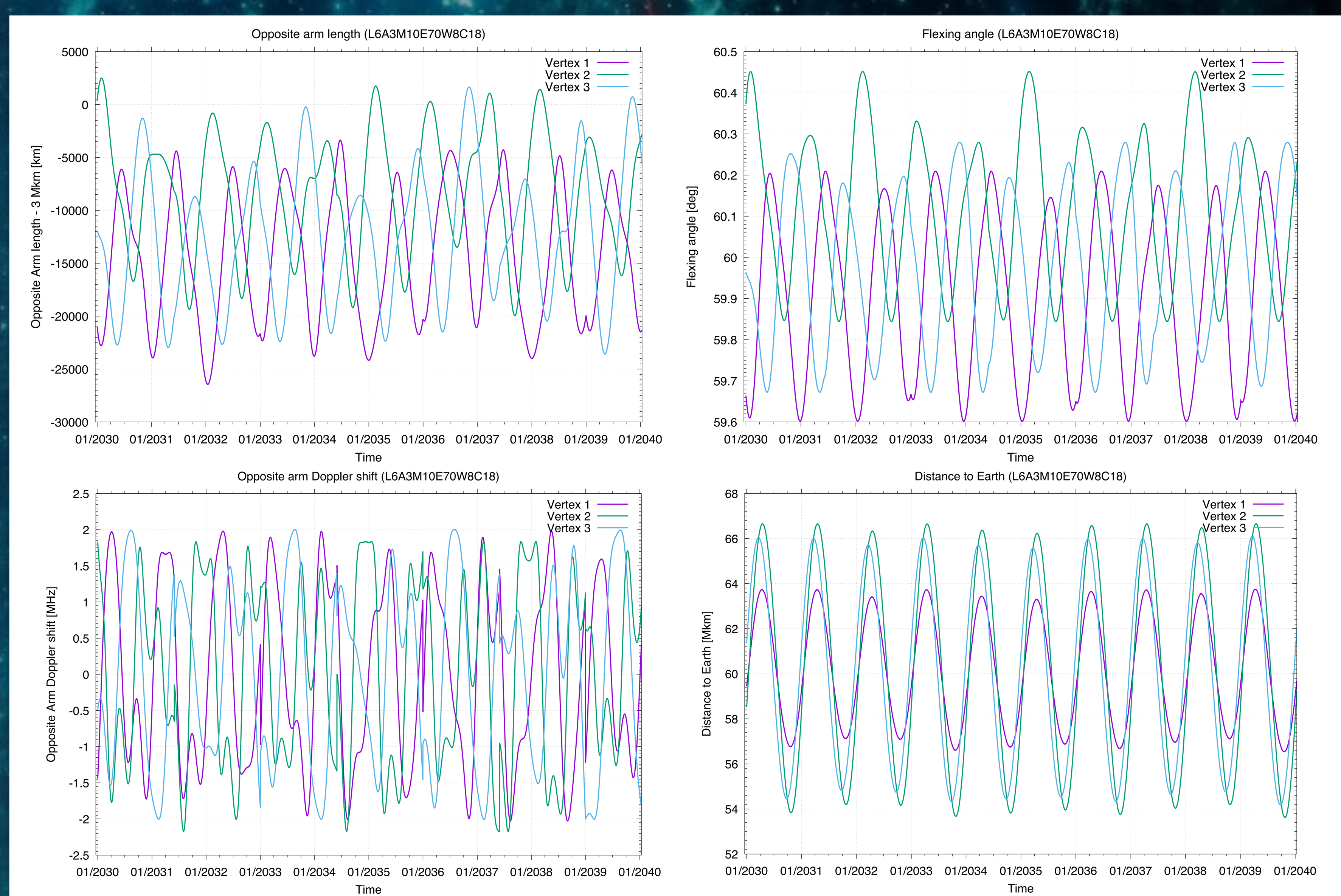
- Max vertex angle fluctuations (flexing): 1.5 deg peak-to-peak for each vertex
- Max Doppler shift : 14 MHz peak-to-peak
- Total wet mass : 7835 kg

Noticeably, the geometrical center of the constellation is oscillating by ±8 10^6 km above and below the ecliptic plane. This movement mitigates the pull of the Earth on the S/C (and therefore reduces its perturbing effect).



IIIb. With orbit control maneuvers

A second run was performed including orbital maneuvers (i.e. velocity increments) every 18 months (i.e. 6 maneuvers for each S/C in the mission lifetime) :



- Max vertex angle fluctuations (flexing): 0.6 deg peak-to-peak for each vertex
- Max Doppler shift : 4 MHz peak-to-peak
- Total wet mass : 7431 kg
- ΔV amplitude for each orbit control maneuvers : 70 to 140 m/s

Clearly, a few orbit control maneuvers can significantly reduce the beam pointing constraint and Doppler effect, with no mass penalty. These maneuvers would also be useful to compensate for previous maneuvers uncertainties and unmodeled perturbations.

As drawbacks, orbit control maneuvers would require to suspend the science acquisition, grab the test masses and, probably, re-acquire the laser links after the maneuvers. They will also induce significant changes in the mass distribution (hence the local gravitational field) of the S/C.

References

[1] Nayak et al. On the minimum flexing of LISA's arms. Classical and Quantum Gravity (2006) vol. 23 (5) pp. 1763-1778

[2] NGO Assesment Study Report, ESA/SRE(2011)19,

[3] Omelyan et al. Optimized Forest–Ruth-and Suzuki-like algorithms for integration of motion in many-body systems. Computer Physics Communications (2002) vol. 146 (5) pp. 188-202

[4] Jastrebski and Arnold. Improving evolution strategies through active covariance matrix adaptation. IEEE World Congress on Computational Intelligence (2006), Proceedings, pp. 9719-9726 and https://github.com/beniz/libcmaes