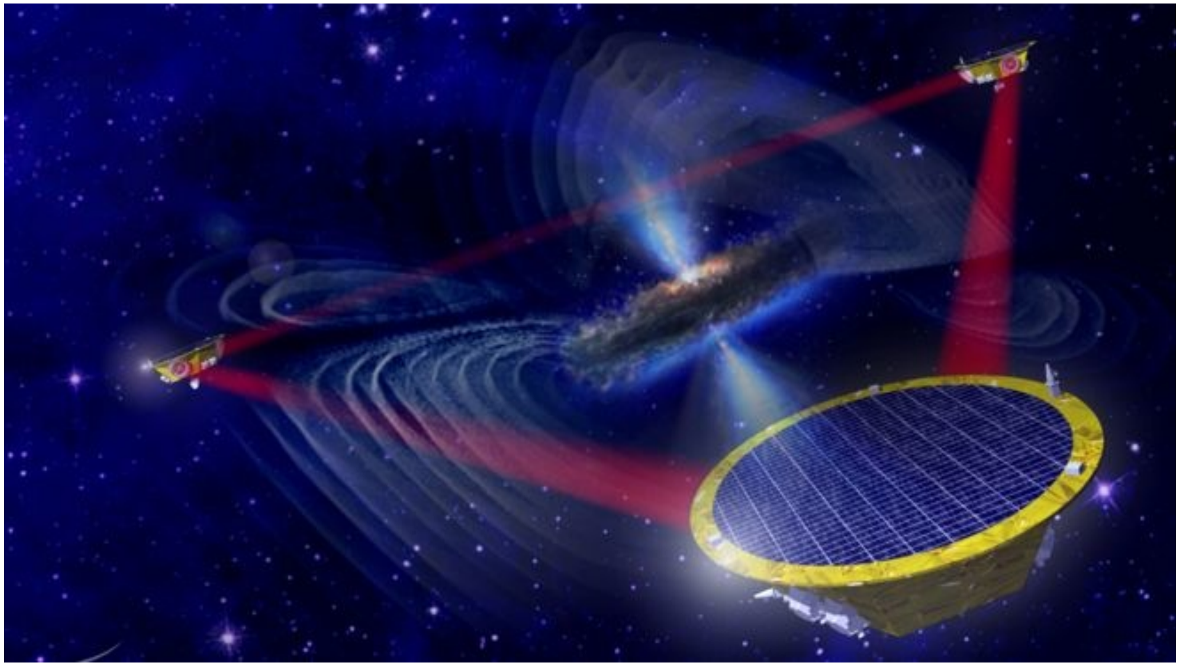
Measuring EMRIs: A reality check



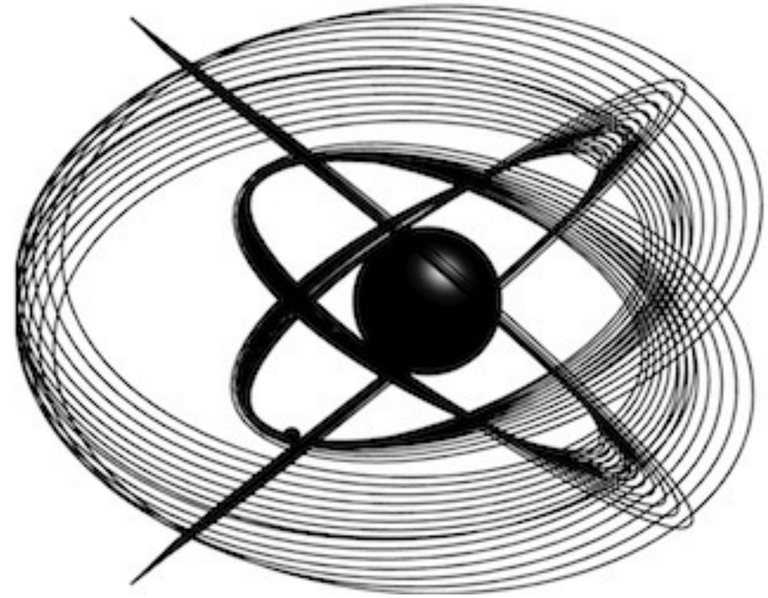


Image: Steve Drasco, CalPoly

What modeling and data analysis work must be done in order to achieve the science that has been promised for extreme mass ratio inspiral measurements?

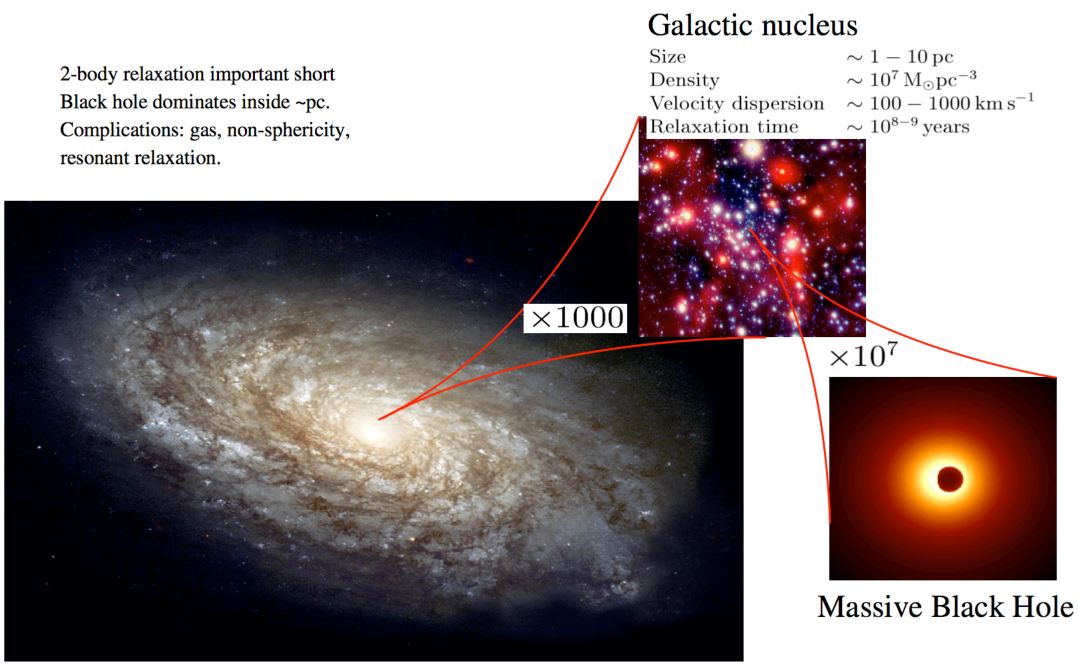
Outline and goal

Submitted title while revisiting EMRI measurement problem after about 5 years away … working on talk has made me confident we will solve this.

* Very very brief overview of EMRI astrophysics
* The science payoff of measuring EMRI waves
* How to model EMRI waves: Self forces and black hole perturbation theory
* Using those models to guide the development of EMRI data analysis methods
* What we need to do to ensure we can measure these waves once LISA flies.

EMRI science I

Major driver over the past decade and a half: The promise of extreme mass ratio inspirals as a source for low-frequency GW antennae.



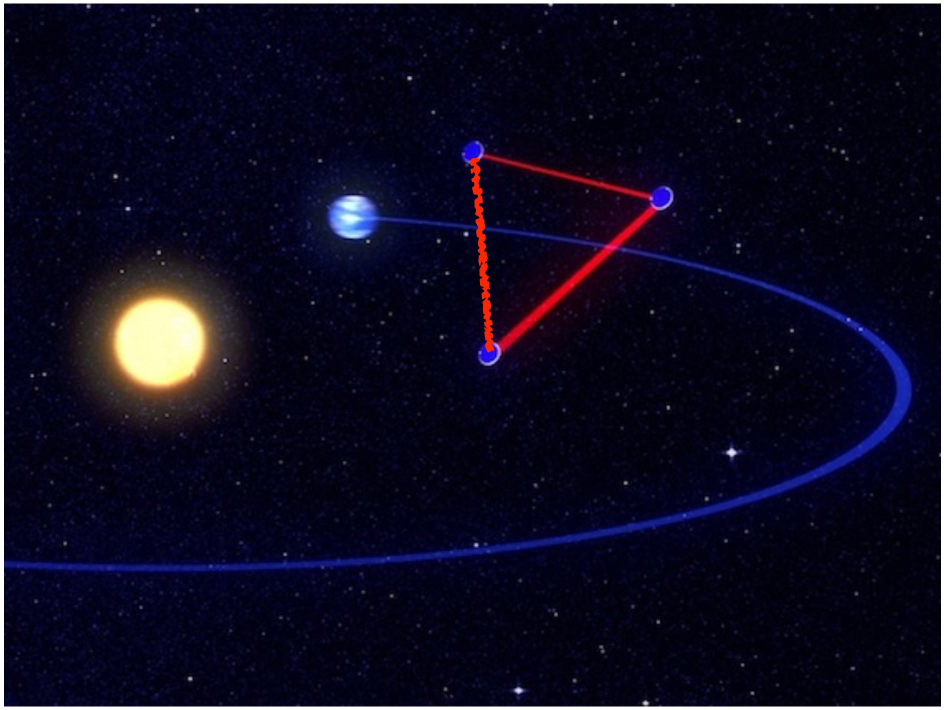
Graphic courtesy of Marc Freitag

Multi-body scattering in centers of galaxies puts compact object (long taken to be ~10 Msun black hole) onto an orbit that evolves into a strong-field, GW-driven inspiral…

EMRI science I

Major driver over the past decade and a half: The promise of extreme mass ratio inspirals as a source for low-frequency GW antennae.

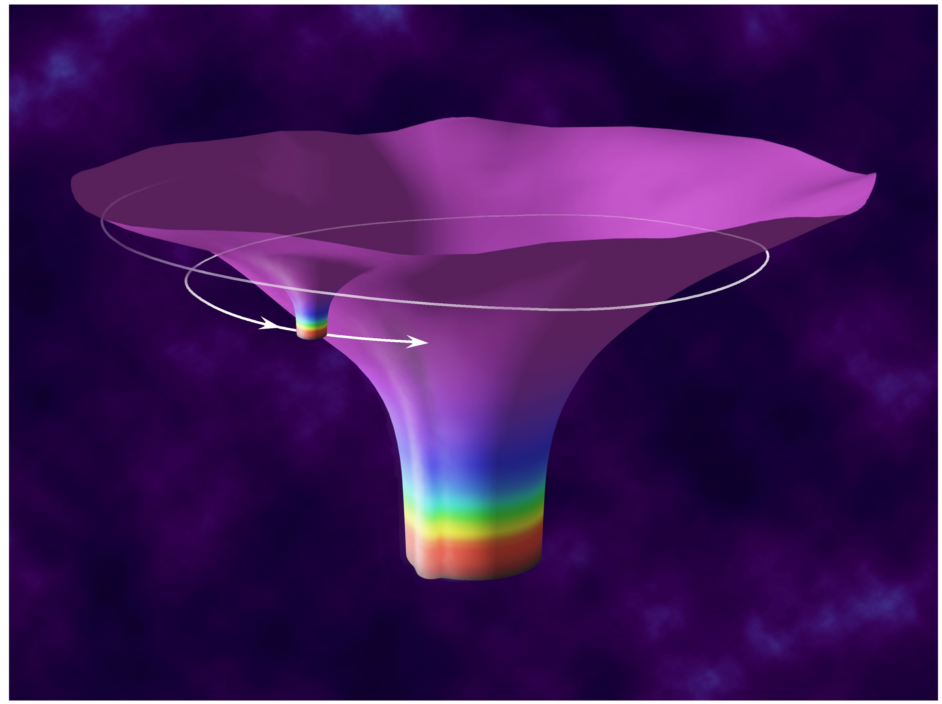
Events are rare per galaxy, but GWs they generate can be heard to z ~ 0.5—1 … or even farther if the black hole is larger than we once thought. Can expect dozens to hundreds of events per year in the centihertz band.



EMRI science II

EMRIs are ***almost*** a test particle probe of black hole spacetimes:

System is dominated by the properties of the big black hole … the motion of the small body and the waves that it generates are ***mostly*** determined by the big black hole's properties.



Measuring these waves measures the big black hole: precision probe of its properties.

EMRI science II

EMRIs are ***almost*** a test particle probe of black hole spacetimes:

Mass, spin, mass ratio:

$\delta M/M$, δa, $\delta\eta \sim 10^{-4} - 10^{-2}$

Orbit geometry:

$\delta e_0 \sim 10^{-3} - 10^{-2}$

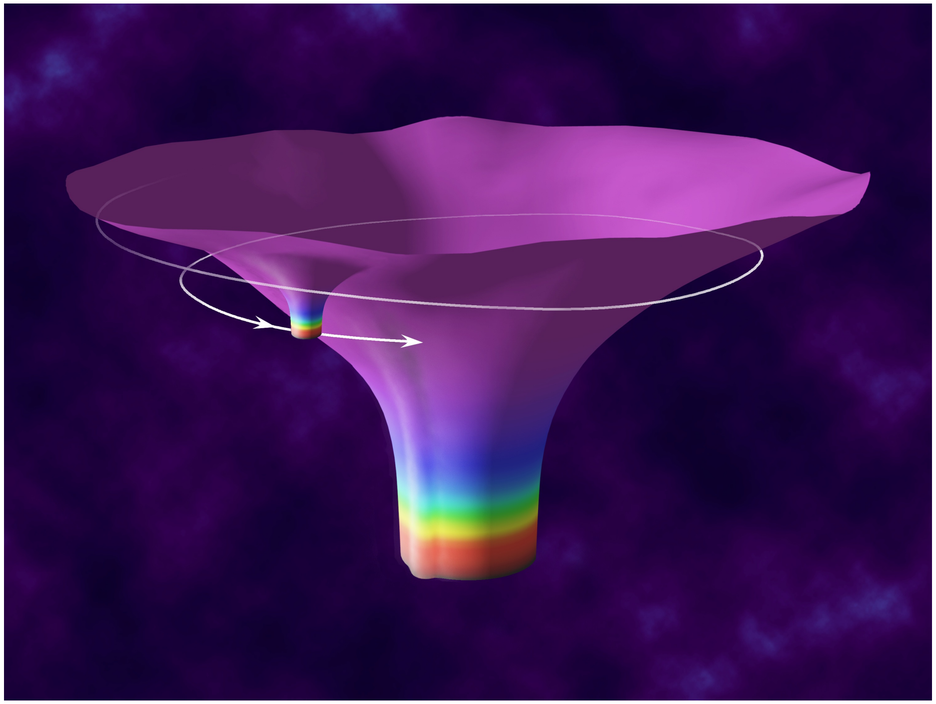
δ(spin direction) ~ a few deg^2

δ(orbit plane) ~ 10 deg^2

Distance to binary:

$\delta D/D \sim 0.03 - 0.1$

Barack & Cutler PRD **69**, 082005 (2004)



Measuring these waves measures the big black hole: precision probe of its properties.

EMRI science II

EMRIs are ***almost*** a test particle probe of black hole spacetimes:

Multipoles describing large body's spacetime:

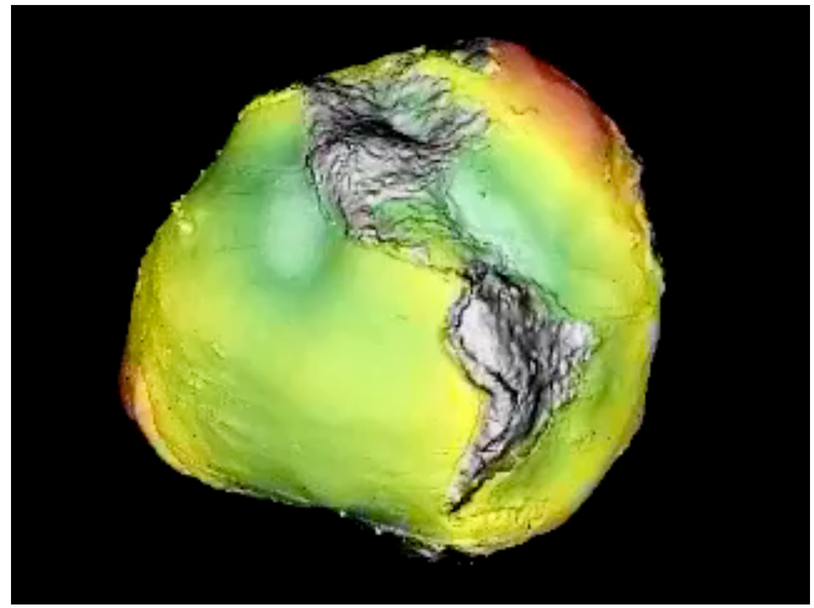
δ(mass quadrupole) ~ 10^{-3}

[precision degrades by about half an order of magnitude per multipole order]

Ryan PRD **56**, 1845 (1997)
Barack & Cutler PRD **75**, 042003 (2007)

Kerr expectation: $M_l + iS_l = iM(a)^l$

Geodesy for black holes?



GRACE gravity model

Measuring these waves measures the big black hole: precision probe of its properties.

Modeling EMRI waves

Modeling these binaries is in one way far easier than modeling comparable mass binaries: Large mass ratio means we can use perturbation theory.

$$g_{\alpha\beta} = g^{\rm Kerr}_{\alpha\beta} + \epsilon h^{(1)}_{\alpha\beta} + \epsilon^2 h^{(2)}_{\alpha\beta} + \ldots \qquad \epsilon \equiv \frac{m}{M}$$

HOWEVER**:** Number of cycles in band scales inversely with mass ratio. For EMRI systems, expect $10^4 - 10^6$ orbits … need **exquisitely** precise models to hold phase with data.

For this talk: Call this a "relativistic" EMRI model.

Motion in this model

Motion of small body looks like a geodesic of the larger black hole plus a correction arising from the smaller body's contribution to the spacetime:

$$\frac{d^2x^\alpha}{d\tau^2} + \Gamma^\alpha{}_{\beta\gamma}\frac{dx^\beta}{d\tau}\frac{dx^\gamma}{d\tau} = f^\gamma$$

"Self force" — $O(m/M)$ correction to geodesic black hole orbits.

Two contributions to the self force:

* ***Dissipative***: Takes energy and angular momentum from binary, drives long-time evolution of orbit.
* ***Conservative***: Conserves energy and angular momentum, but shifts motion from geodesic (e.g., changing orbital frequencies).

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"Self force" — O(m/M) correction to geodesic black hole orbits.

Comment: Neglecting influence of small body's spin.

Small body's spin causes precession and couples to curvature, leading to a non-geodesic force. Especially if the small body is not so small, this could be an important influence beyond the self-force only model.

How waveform phase accumulates

These effects contribute to the phase accumulated in-band at different orders in system's mass ratio:

Phase accumulated from t_1 to t_2:

$$\Phi(t_1, t_2) = \int_{t_1}^{t_2} \omega(t)\, dt$$

O(M/m): Evolving geodesic frequency [O($1/M$)] integrated over inspiral [O(M^2/m)]

$$= \Phi_{\mathrm{diss}-1}$$

O(1): Conservative correction to frequency [O(m/M^2)] integrated over inspiral

$$+ \Phi_{\mathrm{cons}-1}$$

O(1): Geodesic frequency integrated against next correction to inspiral [O(M)]

$$+ \Phi_{\mathrm{diss}-2}$$

O(m/M): Next correction to frequency [O(m^2/M^3)] integrated against inspiral [O(M^2/m)]

$$+ \Phi_{\mathrm{cons}-2}$$

How waveform phase accumulates

Conventional wisdom: Measuring EMRIs requires models accurate to O(1) in phase.

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O(m/M): Next correction to frequency [O(m^2/M^3)] integrated against inspiral [O(M^2/m)]

Conventional wisdom: What we need for sufficiently accurate EMRI models

Phase accuracy that appears to be needed is

$$\Phi_{\rm needed} = \Phi_{\rm diss-1} + \Phi_{\rm cons-1} + \Phi_{\rm diss-2}$$

From 1st order averaged dissipative self force.
Totally understood.

From 1st order averaged cons. & 1st order oscillatory diss. self force.
Understood for Schwarzschild … great progress for Kerr, expect results very soon.

From 2nd order averaged diss. & 1st order oscillatory cons. self force:
Current frontier of self force research.

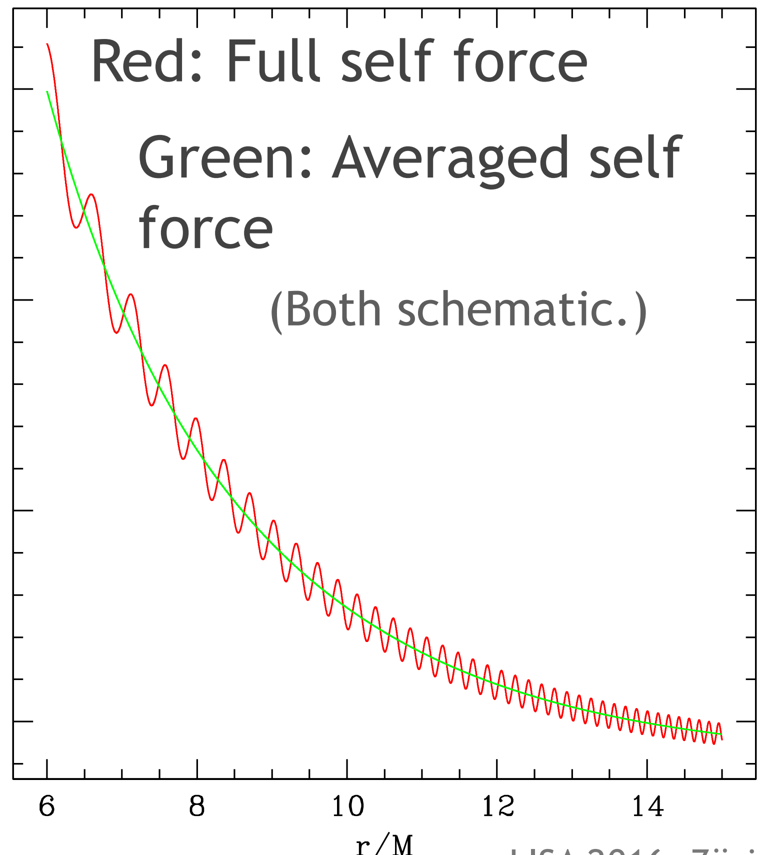
Surprise! Conventional wisdom breaks down: Averaging is not so simple.

While this picture was developing, Flanagan and Hinderer found that Kerr black hole orbits can "break" the averaging underlying this analysis.

Self force can be split into "average" and "oscillatory" pieces:

$$f^\gamma = \sum_{kn} (f^\gamma)_{kn} e^{-i(k\Omega_\theta + n\Omega_r)t}$$

$$= (f^\gamma)_{00} + \sum_{\substack{kn \\ k\neq 0\; n\neq 0}} (f^\gamma)_{kn} e^{-i(k\Omega_\theta + n\Omega_r)t}$$



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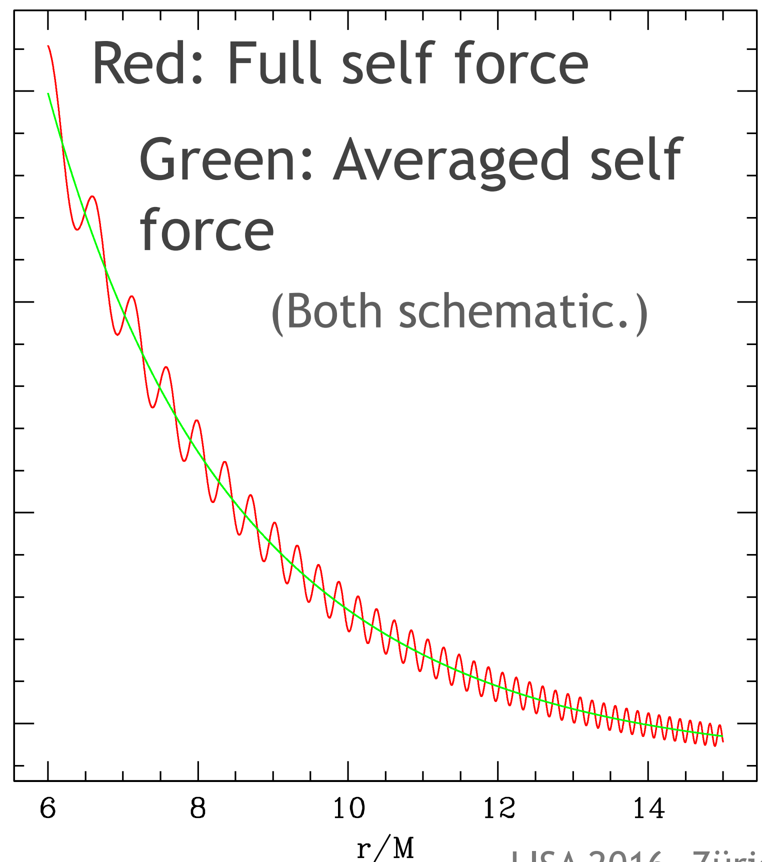
For "most" orbits, the oscillatory contribution is much less important than the average … can neglect at leading order, use only the average components.



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BUT: There exist "resonant" orbits for which "oscillatory" piece doesn't oscillate.

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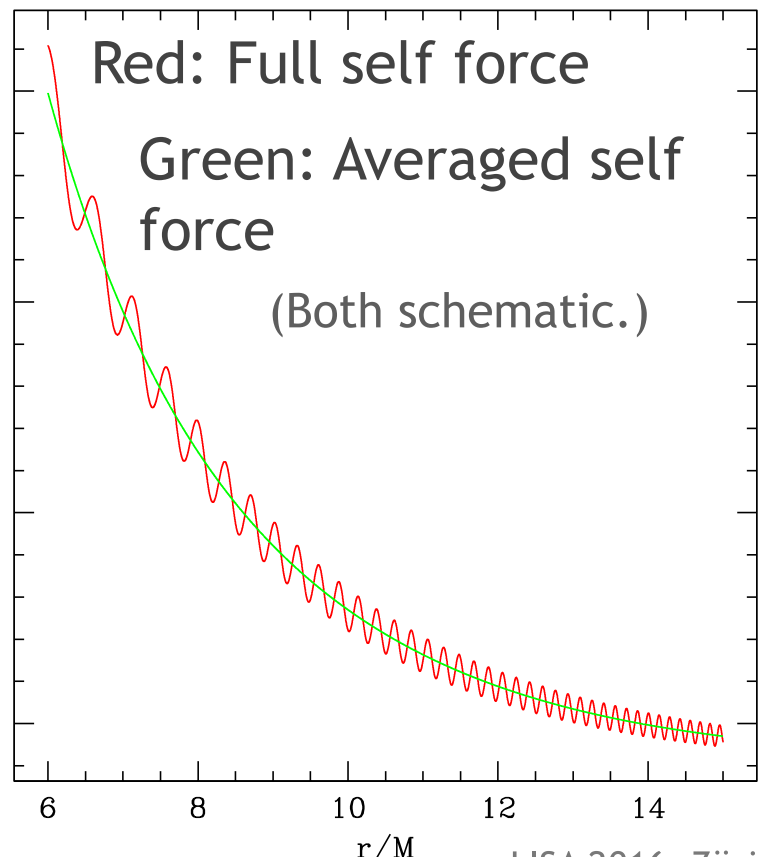

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Example: When $\Omega_\theta = 2\Omega_r$, the "oscillating" term is constant for all terms in the sum in which $n = -2k$.

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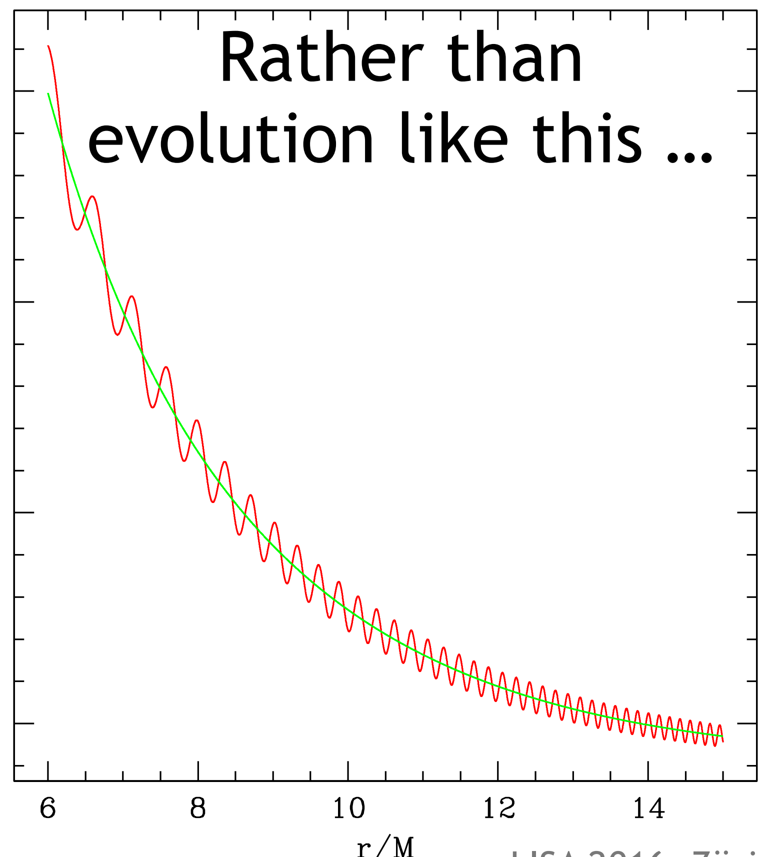

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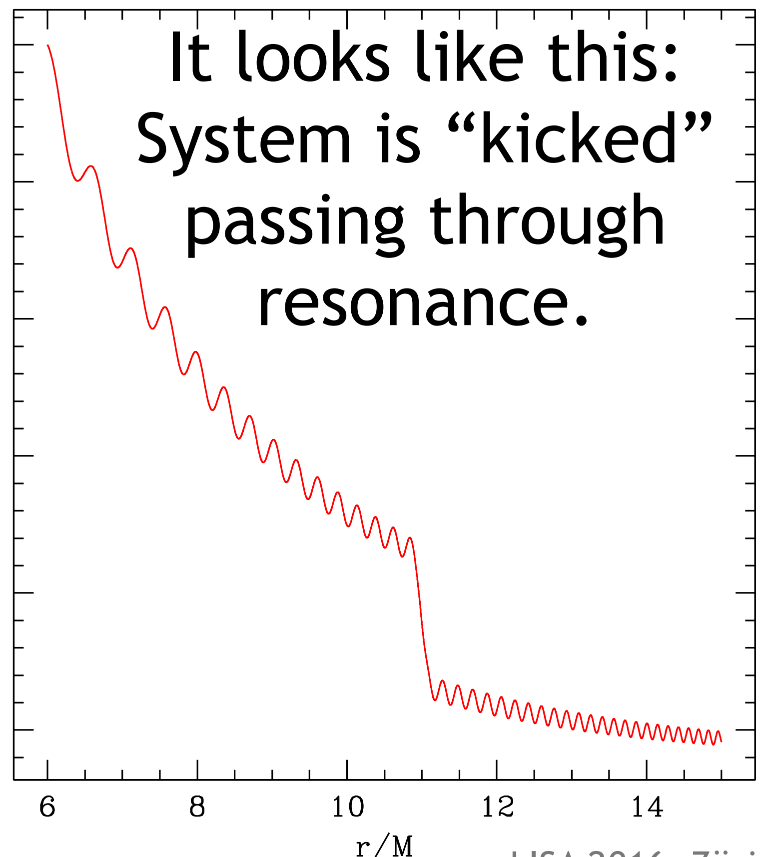
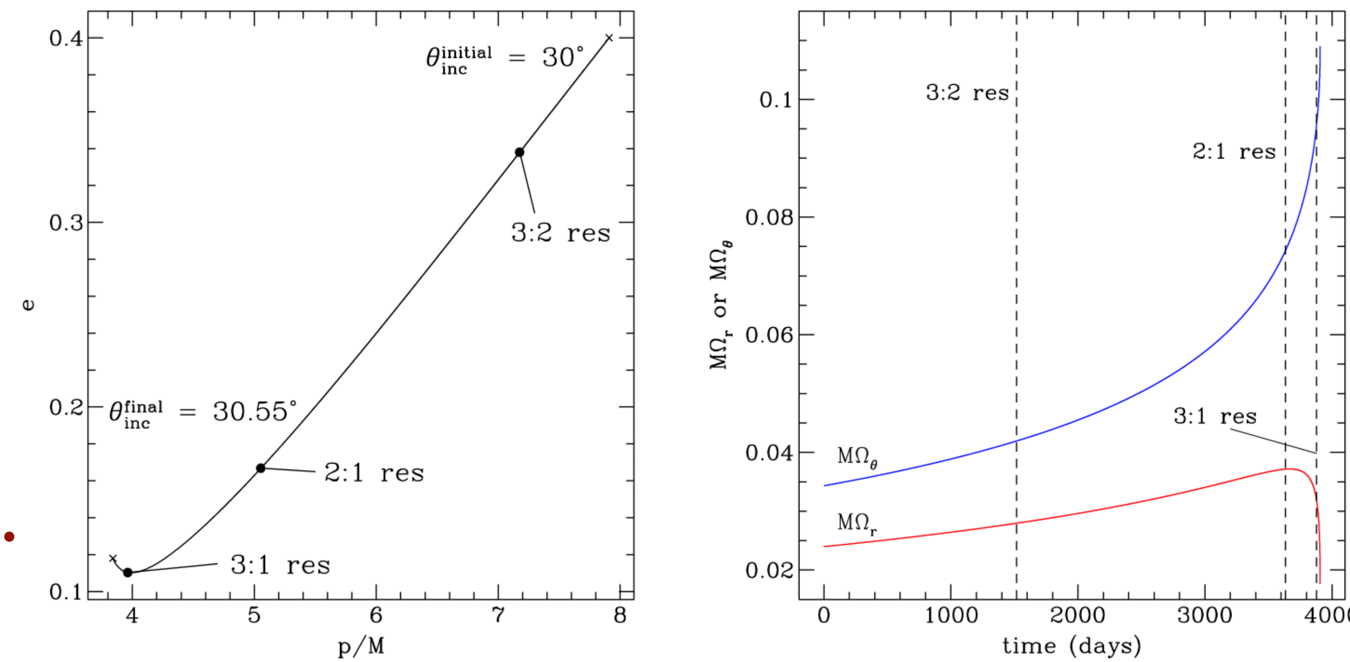


Surprise! Conventional wisdom breaks down: Averaging is not so simple.

This behavior is **generic:** *Every* EMRI encounters at least one resonance as it spirals through the strong-field. Many encounter two; a few encounter three.

Example: An inspiral that encounters three resonances in its last ~8 years of inspiral … two of them in final 250 days before plunge.



[1 M_{sun} spiraling into 10^6 M_{sun} with spin a/M = 0.7. From Ruangsri and Hughes, PRD **89**, 084036 (2014).]

Updated wisdom: What we need for sufficiently accurate EMRI models

Phase accuracy that appears to be needed is

$$\Phi_{\rm needed} = \Phi_{\rm diss-1} + \Phi_{\rm res} + \Phi_{\rm cons-1} + \Phi_{\rm diss-2}$$

Contribution to the phase from resonance crossings scales as sqrt(M/m) … dominating over **all terms except the leading dissipative one.**

Where do we stand on modeling?

We know what calculations we need to do:

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$\Phi_{\text{diss-1}}$: O(M/m); dominates. Techniques to compute this term totally under control.

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$\Phi_{\rm res}$: O[sqrt(M/m)]; first subleading term. Discovered only recently, has not yet been studied as carefully.

arXiv.org > gr-qc > arXiv:1608.08951

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General Relativity and Quantum Cosmology

The importance of transient resonances in extreme-mass-ratio inspirals

Christopher P. L. Berry, Robert H. Cole, Priscilla Cañizares, Jonathan R. Gair

(Submitted on 31 Aug 2016)

The inspiral of stellar-mass compact objects, like neutron stars or stellar-mass black holes, into supermassive black holes provides a wealth of information about the strong gravitational-field regime via the emission of gravitational waves. In order to detect and analyse these signals, accurate waveform templates which include the effects of the compact object's gravitational self-force are required. For computational efficiency, adiabatic templates are often used. These accurately reproduce orbit-averaged trajectories arising from the first-order self-force, but neglect other effects, such as transient resonances, where the radial and poloidal fundamental frequencies become commensurate. During such resonances the flux of gravitational waves can be diminished or enhanced, leading to a shift in the compact object's trajectory and the phase of the waveform. We present an evolution scheme for studying the effects of transient resonances and apply this to an astrophysically motivated population. We find that a large proportion of systems encounter a low-order resonance in the later stages of inspiral; however, the resulting effect on signal-to-noise recovery is small as a consequence of the low eccentricity of the inspirals. Neglecting the effects of transient resonances leads to a loss of 4% of detectable signals.

Comments: 24 pages, 12 figures, 2 appendices

Where do we stand on modeling?

We know what calculations we need to do:

$$\Phi_{\rm needed} = \Phi_{\rm diss-1} + \Phi_{\rm res} + \Phi_{\rm cons-1} + \Phi_{\rm diss-2}$$

$\Phi_{\text{diss-1}}$: O(M/m); dominates. Techniques to compute this term totally under control.

Φ_{res}: O[sqrt(M/m)]; first subleading term. Discovered only recently, has not yet been studied as carefully.

$\Phi_{\text{cons-1}}$: O(1); just big enough to influence measurement. Formal techniques to compute this are nearly in hand for Kerr, studies now underway.

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We know what calculations we need to do:

$$\Phi_{\rm needed} = \Phi_{\rm diss-1} + \Phi_{\rm res} + \Phi_{\rm cons-1} + \Phi_{\rm diss-2}$$

$\Phi_{\text{diss-1}}$: O(*M*/*m*); dominates. Techniques to compute this term totally under control.

Φ_{res}: O[sqrt(*M*/*m*)]; first subleading term. Discovered only recently, has not yet been studied as carefully.

$\Phi_{\text{cons-1}}$: O(1); just big enough to influence measurement. Formal techniques to compute this are nearly in hand for Kerr, studies now underway.

$\Phi_{\text{diss-2}}$: O(1); same order as “cons-1” term. Studies of this are the frontier of self force research.

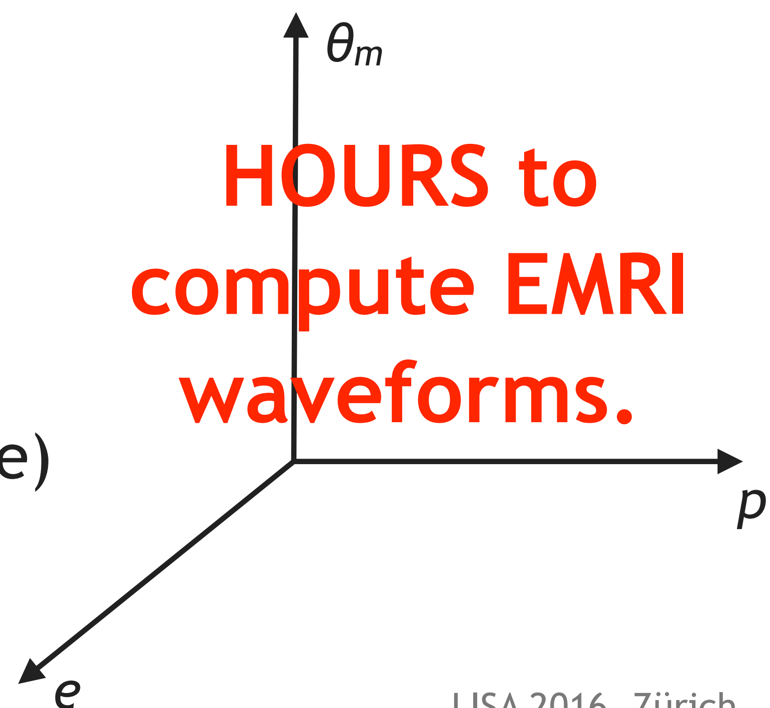
Computational cost

Rosy scenario: Suggests we just need to turn some cranks and we'll have a good set of models. As additional physics comes under control, we refine models. When LISA flies, we'll be in great shape.

Issue: These models are very expensive to compute!

Recipe for simplest model:

1. Lay out grid in orbit parameter space (10^3 — 10^4 points to cover strong field)
2. Solve linearized Einstein equation at each point (10^2 — 10^4 multipoles per point … about 0.01—0.1 CPU seconds per multipole)
3. Use data to evolve from orbit to orbit, build waveform.



HOURS to compute EMRI waveforms.

Kludges[1]: Approximate models for testing EMRI data analysis

Long known that relativistic wave models would be too expensive for EMRI data analysis studies … "kludges" developed as tools for this.

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[1] From Oxford English Dictionary:

kludge: A hastily improvised and poorly thought-out solution to a fault or 'bug.'

Kludges[1]: Approximate models for testing EMRI data analysis

Long known that relativistic wave models would be too expensive for EMRI data analysis studies … "kludges" developed as tools for this.

Analytic kludge of Barack and Cutler (2004): Analytic model based on post-Newtonian approximation to EMRIs. Has main qualitative features (3 orbital frequencies, strong precession).

Very fast, easy to implement. Extremely useful for studying time-frequency structure of EMRI waves in simulated data.

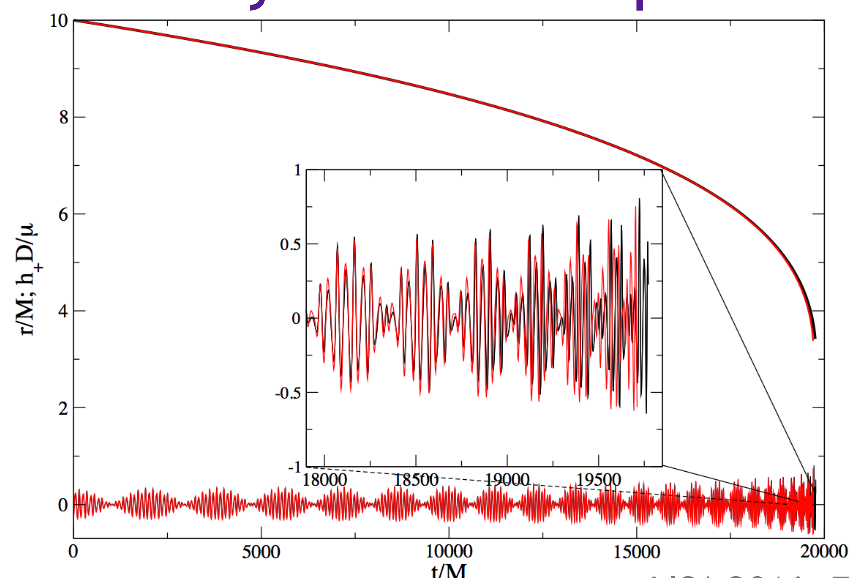
Does not remain phase locked with relativistic models for long time! Great tool for exploring algorithmics, but not accurate model of Nature's EMRIs.

Kludges[1]: Approximate models for testing EMRI data analysis

Long known that relativistic wave models would be too expensive for EMRI data analysis studies … "kludges" developed as tools for this.

Numerical kludge of Babak et al (2008): Use fits to relativistic data for the small body's inspiral; use a simple multipole formula to make GWs from a small body on that inspiral.

Much slower than the analytic kludge … but maintains high fidelity with relativistic models.



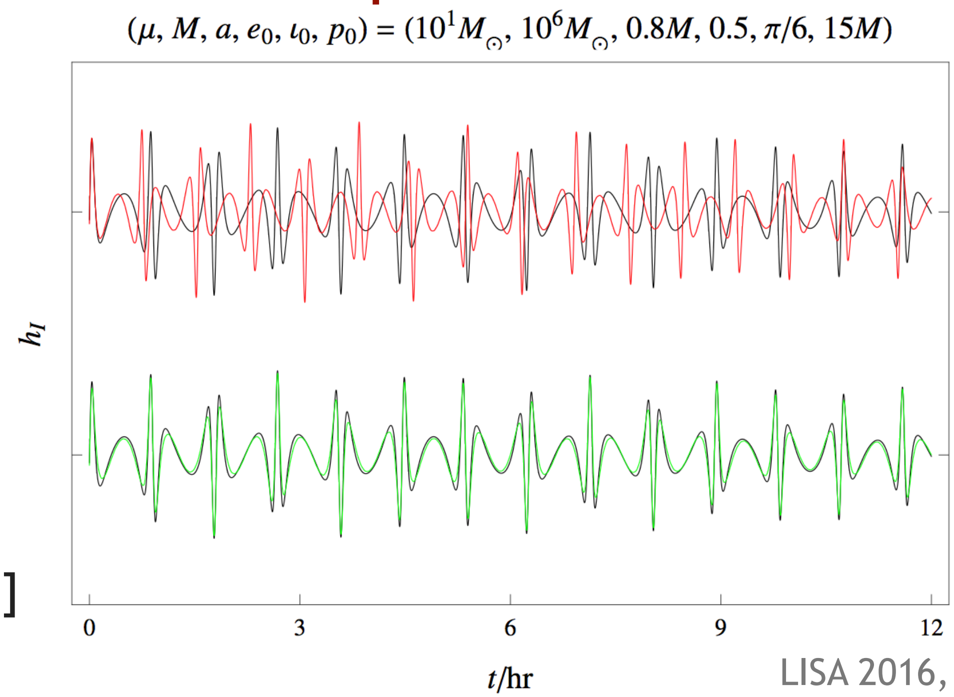
[From Babak et al, PRD **75**, 024005 (2007).]

Kludges[1]: Approximate models for testing EMRI data analysis

Long known that relativistic wave models would be too expensive for EMRI data analysis studies … "kludges" developed as tools for this.

A middle ground: Chua and Gair (2015) greatly improves analytic kludge with little extra computational cost!

Model follows Barack and Cutler recipe for inspiral, but uses *exact* Kerr frequencies at each moment to build waves.

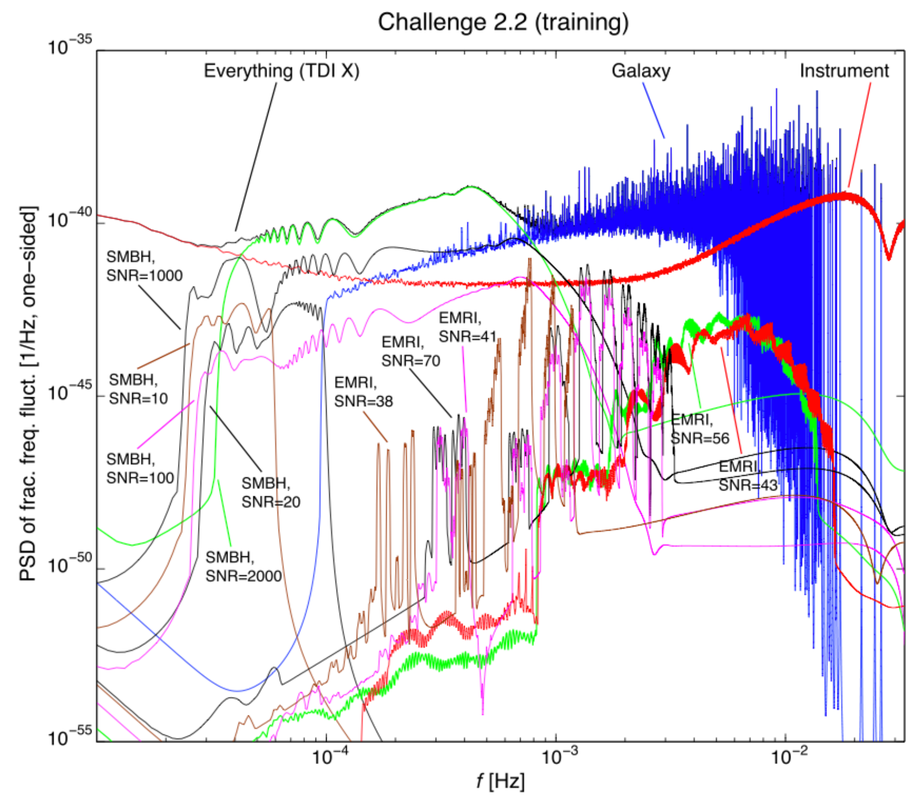


[From Chua and Gair, CQG **32**, 232002 (2015).]

EMRIs in MLDCs

EMRIs have been included in a few MLDCs … provided to be a challenging source to measure, but there were several successful entries.

Bear in mind: **MLDC EMRIs based on analytic kludge.**



[From Arnaud et al, CQG **24**, S551 (2007).]

Some of the most interesting EMRI behaviors are not captured by this kludge … will be important to make sure that we test our ability to capture these behaviors with our data analysis algorithms.

Where to go from here

We have a well defined research program, with concentrated effort needed on multiple fronts:

Make relativistic EMRI models

Waves using 1st order, dissipative self force, neglecting resonances, can be made ***today***. Extension to include resonances will be in hand quite soon.

1st order conservative self force is coming together … must be prepared to fold in the results of that program.

2nd order self forces will be a while longer, but we can be ready to update models as results are delivered.

Think about other physics we can include! (E.g, spin of small body.)

Make less kludgy kludges

Approximations that capture features of relativistic models have proven invaluable … can use them to test the influence of resonance kicks, impact of small secular forces, spin of the small body, …

Start up fresh MLDCs

MLDCs that use EMRI models that include the most interesting relativistic effects will demonstrate LISA's ability to deliver the EMRI astrophysics and gravity science.