Speeding up LISA analysis: Frequency domain waveforms for fully precessing systems

Neil J. Cornish
Montana State University
(with K. Chatziioannou, A. Klein, & N. Yunes)

Outline

- Motivation & History
- Analytic, fully precessing frequency domain waveforms
  - Reduction to Quadrature for Conservative Dynamics
  - Multiple Scale Analysis
  - Beyond the Stationary Phase Approximation
- Next Steps

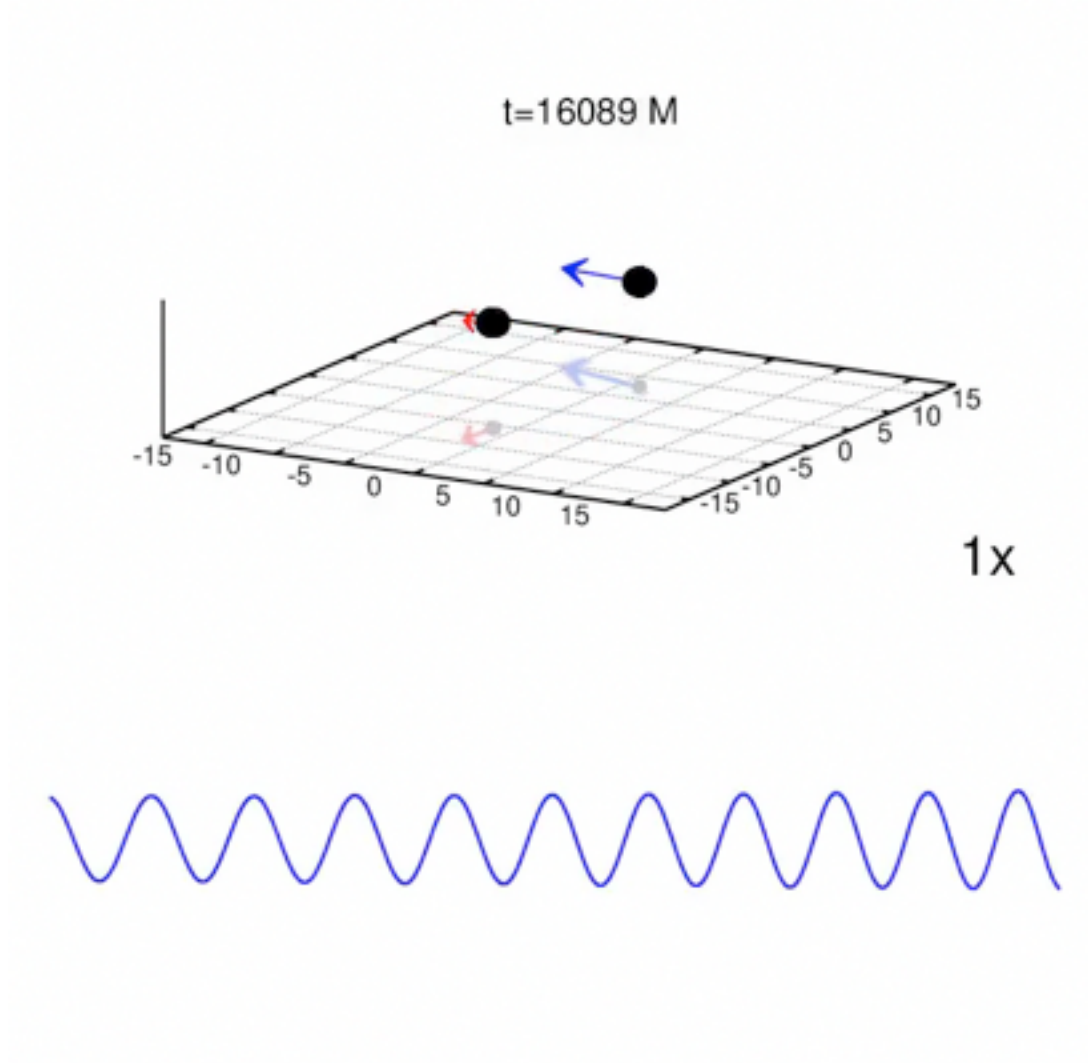
Why Spin?

Spin Precession

$$\dot{\hat{\boldsymbol{L}}} = \left\{\left(2+\frac{3}{2}q\right) - \frac{3}{2}\frac{v}{\eta}\left[(\boldsymbol{S}_2+q\boldsymbol{S}_1)\cdot\hat{\boldsymbol{L}}\right]\right\} v^6 \left(\boldsymbol{S}_1\times\hat{\boldsymbol{L}}\right)$$
$$+\left\{\left(2+\frac{3}{2q}\right) - \frac{3}{2}\frac{v}{\eta}\left[\left(\boldsymbol{S}_1+\frac{1}{q}\boldsymbol{S}_2\right)\cdot\hat{\boldsymbol{L}}\right]\right\} v^6 \left(\boldsymbol{S}_2\times\hat{\boldsymbol{L}}\right)$$
$$+\mathcal{O}(v^7), \qquad (1)$$

$$\dot{\boldsymbol{S}}_1 = \left\{\eta\left(2+\frac{3}{2}q\right) - \frac{3v}{2}\left[(q\boldsymbol{S}_1+\boldsymbol{S}_2)\cdot\hat{\boldsymbol{L}}\right]\right\} v^5 \left(\hat{\boldsymbol{L}}\times\boldsymbol{S}_1\right)$$
$$+\frac{v^6}{2}\boldsymbol{S}_2\times\boldsymbol{S}_1+\mathcal{O}(v^7), \qquad (2)$$

$$\dot{\boldsymbol{S}}_2 = \left\{\eta\left(2+\frac{3}{2q}\right) - \frac{3v}{2}\left[\left(\frac{1}{q}\boldsymbol{S}_2+\boldsymbol{S}_1\right)\cdot\hat{\boldsymbol{L}}\right]\right\} v^5 \left(\hat{\boldsymbol{L}}\times\boldsymbol{S}_2\right)$$
$$+\frac{v^6}{2}\boldsymbol{S}_1\times\boldsymbol{S}_2+\mathcal{O}(v^7). \qquad (3)$$



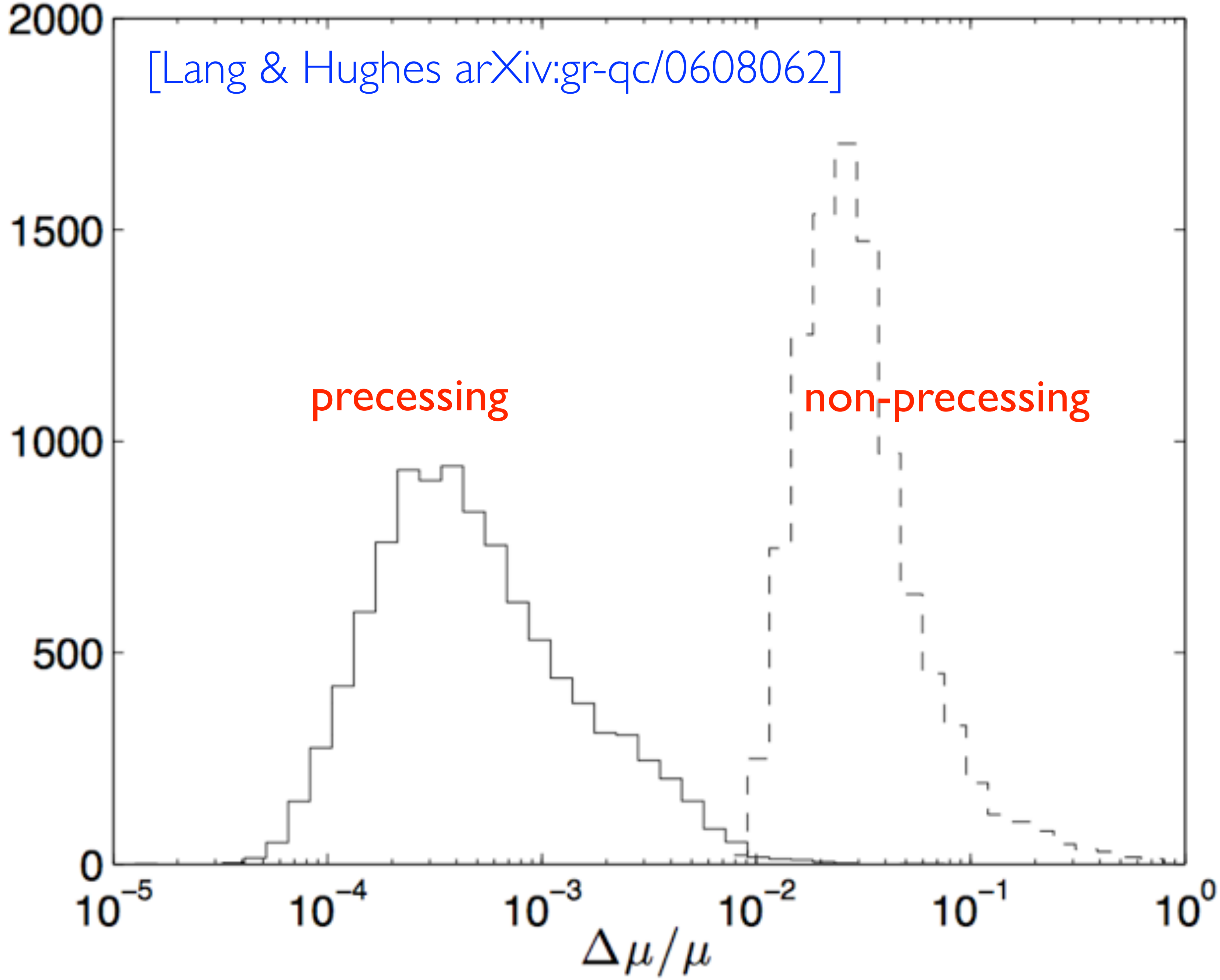
Post-Newtonian Waveforms

$$h(f) = \mathcal{A}_\ell(f) e^{i\Psi_\ell(f)} \qquad\qquad u = (\pi \mathcal{M} f)^{1/3} \sim v$$

$$\Psi_2(f) = 2\pi f t_c - \phi_c - \frac{\pi}{4} +$$

0 PN	$\frac{3}{128}\, u^{-5}$	Measure chirp mass
1PN	$\left(\frac{3715}{32256} + \eta \frac{55}{384}\right) \eta^{-2/5} u^{-3}$	Measure individual masses
1.5PN	$-\left(\frac{3\pi}{8} - \frac{1}{32}\left[113(1 \pm \sqrt{1-4\eta}) - 76\eta\right] \hat{L}\cdot\vec{\chi}_{1,2}\right) \eta^{-3/5}\, u^{-2}$	Measure spin combination
2PN	$\left(\frac{15293365}{21676032} + \frac{27145}{21504}\eta + \frac{3085}{3072}\eta^2 + \sigma(\hat{L}\cdot\vec{\chi}_{1,2},\, \vec{\chi_1}\cdot\vec{\chi_2},\, \chi^2_{1,2})\right) \eta^{-4/5}\, u^{-1}$	Measure individual spins

Spin Precession & Parameter Estimation



[Vecchio arXiv:astro-ph/0304051]

[Lang, Hughes & Cornish arXiv:1101.3591]

Slide from LISA Symposium IX, 2012

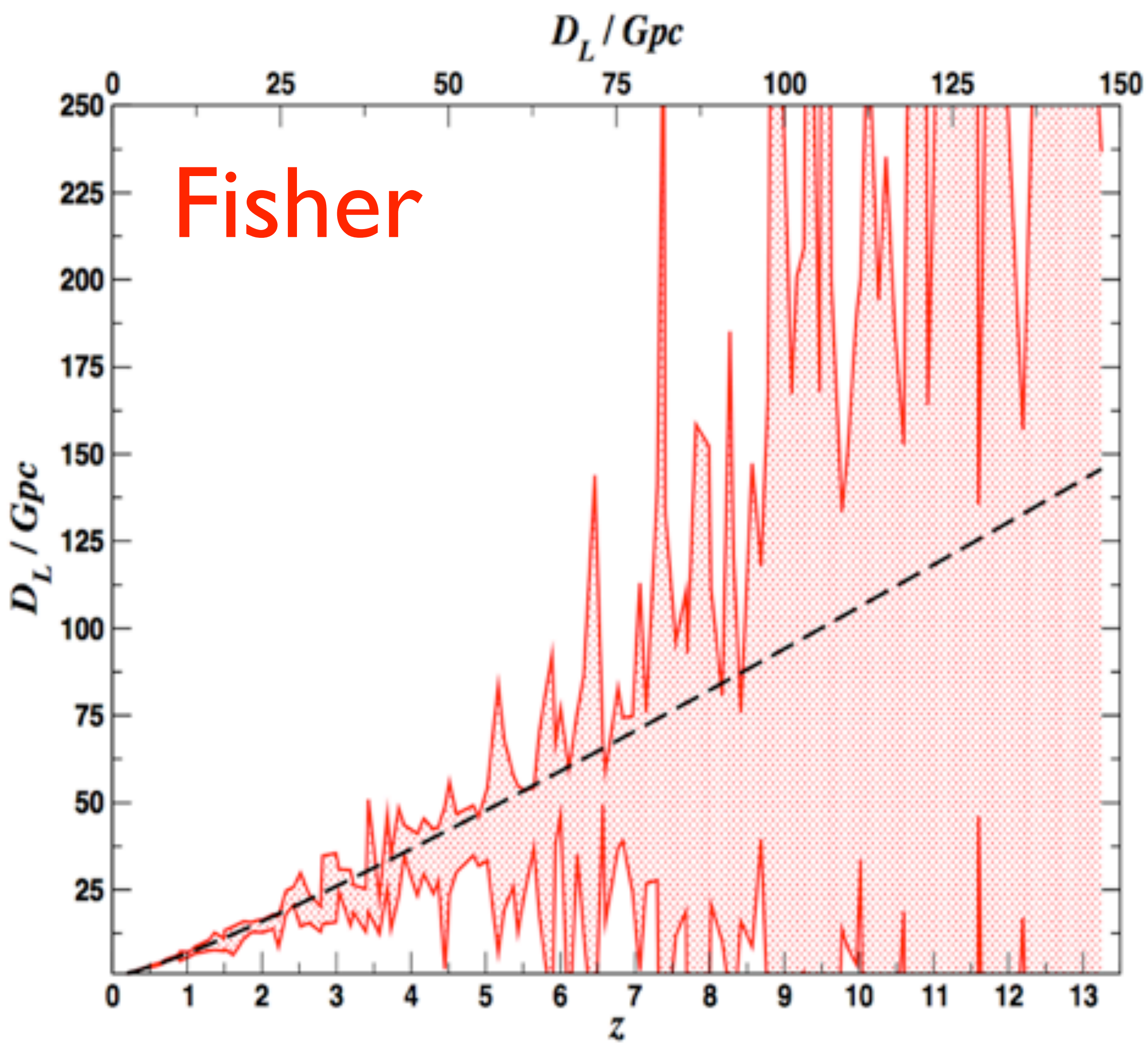
Authors	HH	MR	Spin	Prec	e	FR
Berti, Cardoso & Cavaglia 07	✓ (R)	✓ (R)	✓ (R)	✗	✗	✗
Arun, Iyer, Sathya & Siddhartha 07	✓	✗	✗	✗	✗	✗
Trias & Sintes 08	✓	✗	✗	✗	✗	✗
Porter & Cornish 08	✓	✗	✗	✗	✗	✗
Thorpe, McWilliams, Kelly, Fahey, Arnaud & Baker 09	✓	✓	✗	✗	✗	✓
LISA PE Taskforce 09	✓	✗	✓	✓	✗	✓
Klein, Jetzer & Sereno 09	✓	✗	✓	✓	✗	✗
Porter & Sessana 10	✗	✗	✗	✗	✓	✗
Key & Cornish 11	✓	✗	✓	✓	✓	✓
Lang, Hughes & Cornish 11	✓	✗	✓	✓	✗	✓
Cornish, Klein, Lang & Berti 12	✓ (I)	✓	✓	✓	✗	✓

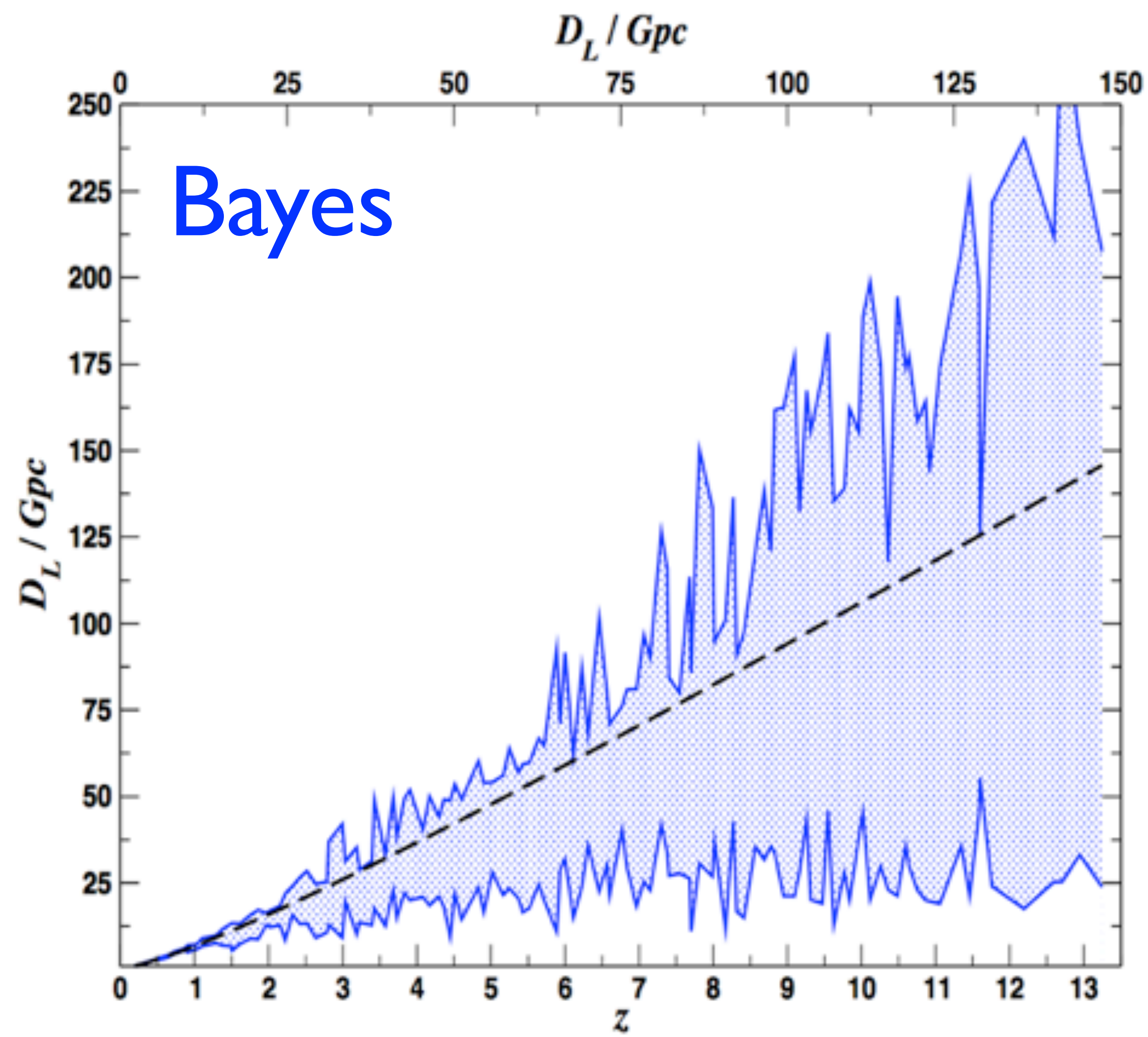
A brief history of LISA Black Hole parameter estimation

The need for speed

- Need to study many mission configurations
- Need to study wide range of BH systems
- Previous large-scale studies limited to Fisher matrix approximation

Fisher versus Bayes





[Porter & Cornish arXiv:1502.05735]

Note: Used simple non-spinning, inspiral-only waveform to make Bayesian analysis computationally feasible

How to Speed Up the Analysis?

- Reusing parts of the likelihood calculation

[Cornish arXiv:1007.4820, Pankow et al arXiv:1502.04370, Cornish arXiv:1606.00953]

- Reduced order models

[Cannon et al arXiv:1005.0012, Field et al arXiv:1101.3765, Blackman et al arXiv:1401.7038]

- Analytic, frequency domain waveforms

[Chatziioannou Klein, Cornish, Yunes arXiv:1606.03117]

Analytic, fully precessing frequency domain waveforms

[Chatziioannou Klein, Cornish, Yunes arXiv:1606.03117]

- Reduction to Quadrature for Conservative Dynamics

  [Kesden, Gerosa, O'Shaughnessy, Berti & Sperhake arXiv:1411.0674]

- Multiple Scale Analysis

  [Pound arXiv:1006.3903, Klein, Cornish, Yunes arXiv:1305.1932]

- Beyond the Stationary Phase Approximation

  [Klein, Cornish, Yunes arXiv:1408.5158]

Timescales

$$T_{\text{orbital}} \sim \frac{1}{v}$$

Fast

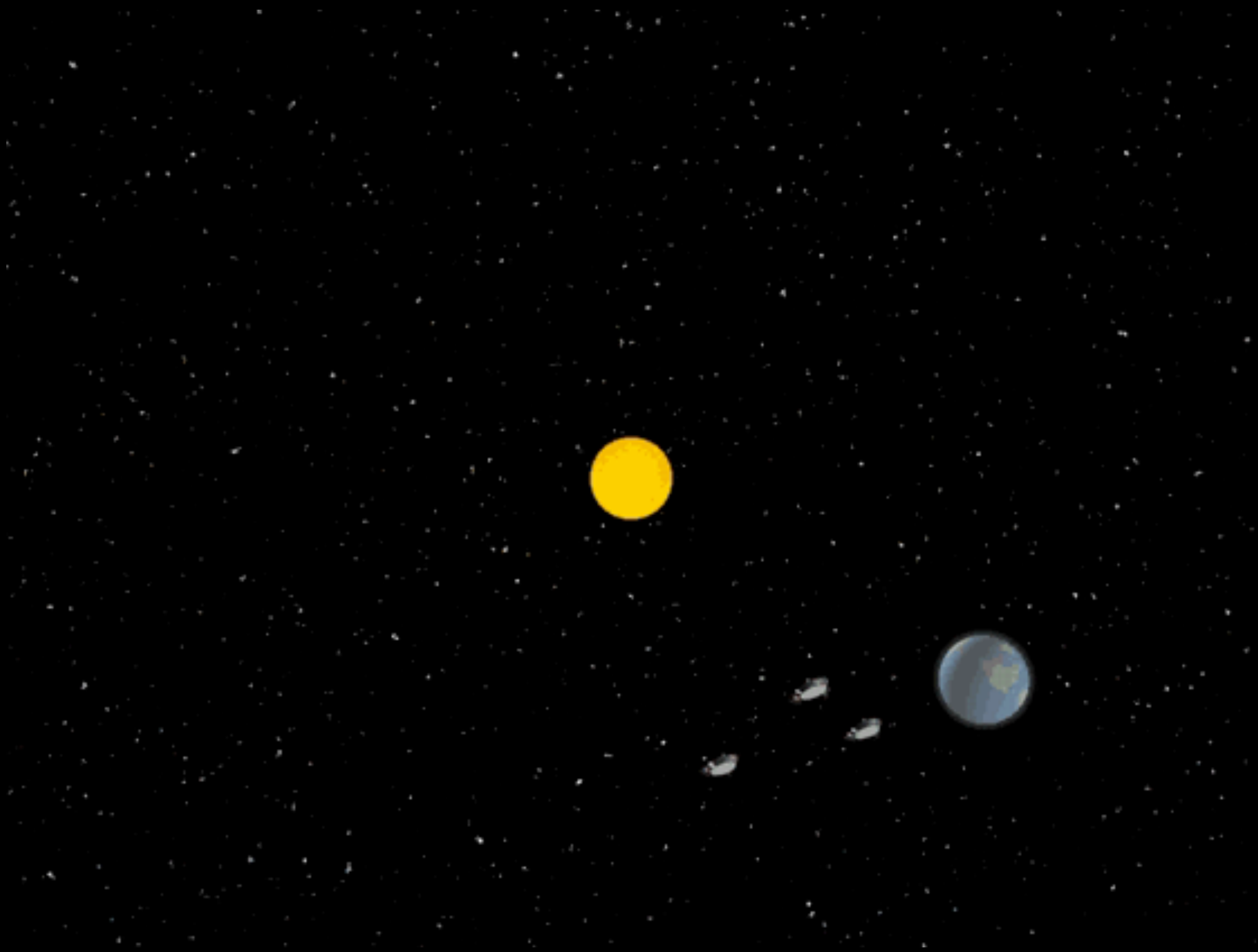
$$T_{\text{precession}} \sim \frac{1}{v^5}$$

$$T_{\text{LISA}} \sim \text{months}$$

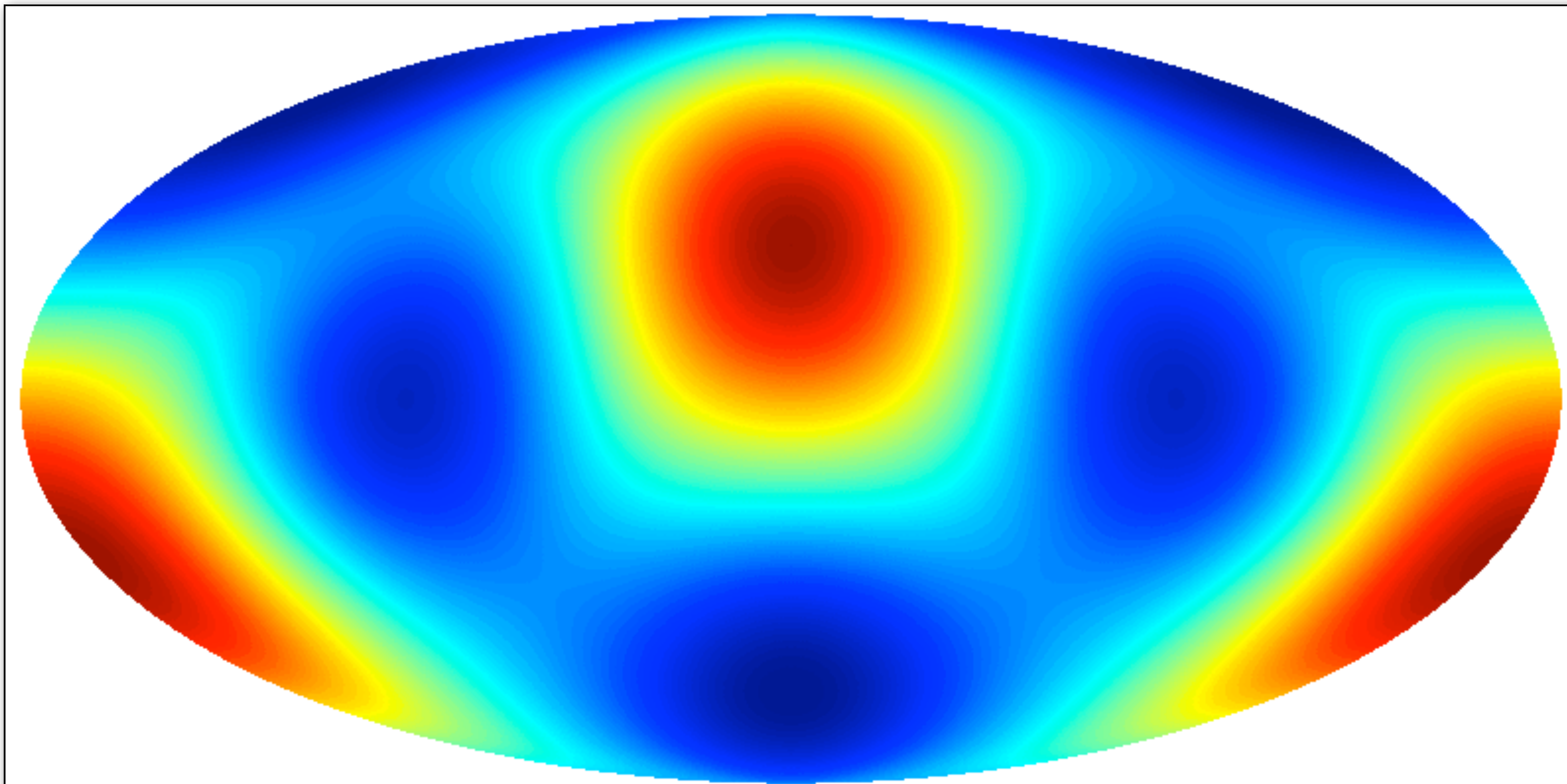
$$T_{\text{decay}} \sim \frac{1}{v^8}$$

Slow

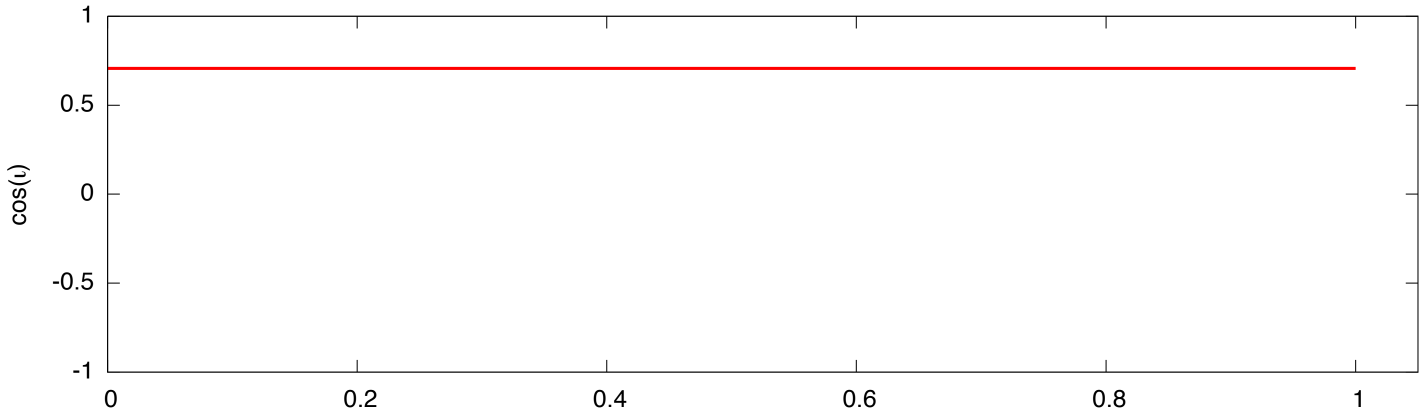
Orbital Modulation



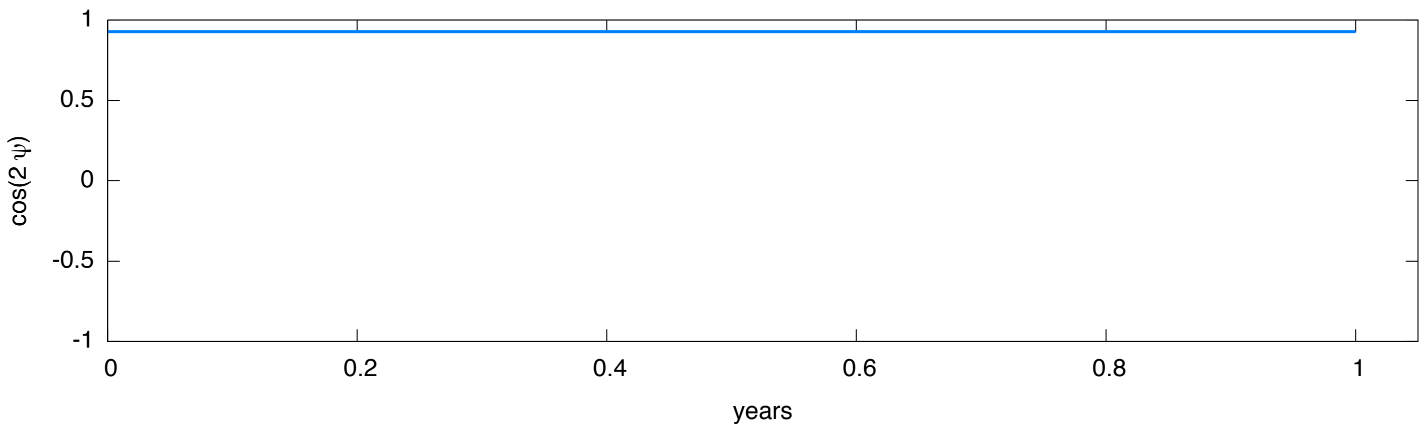
LISA Orbital Modulation



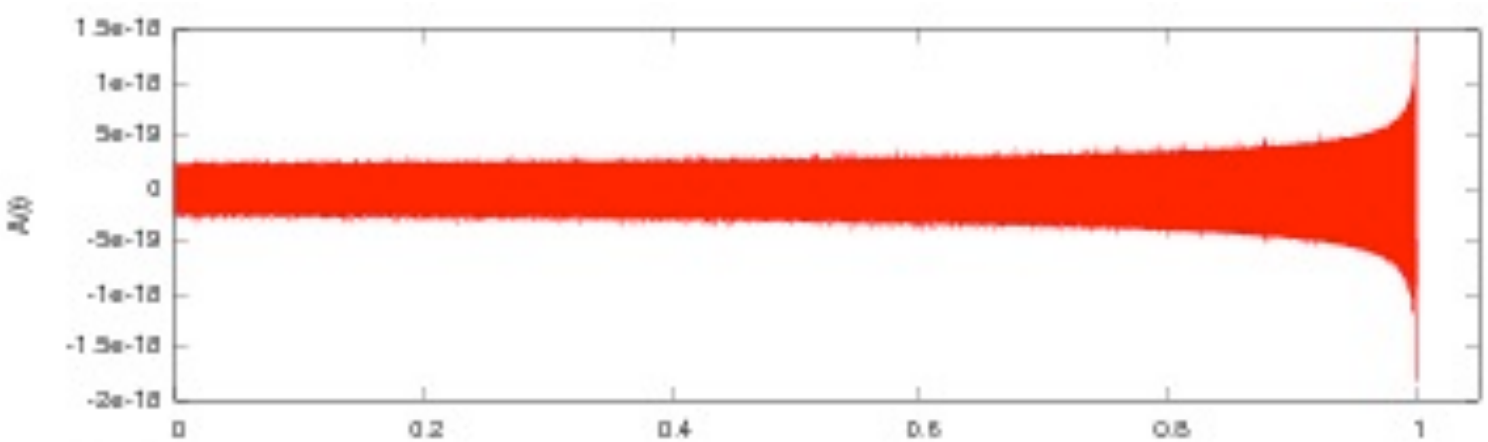
Static Detector, Non-Precessing Source in Ecliptic



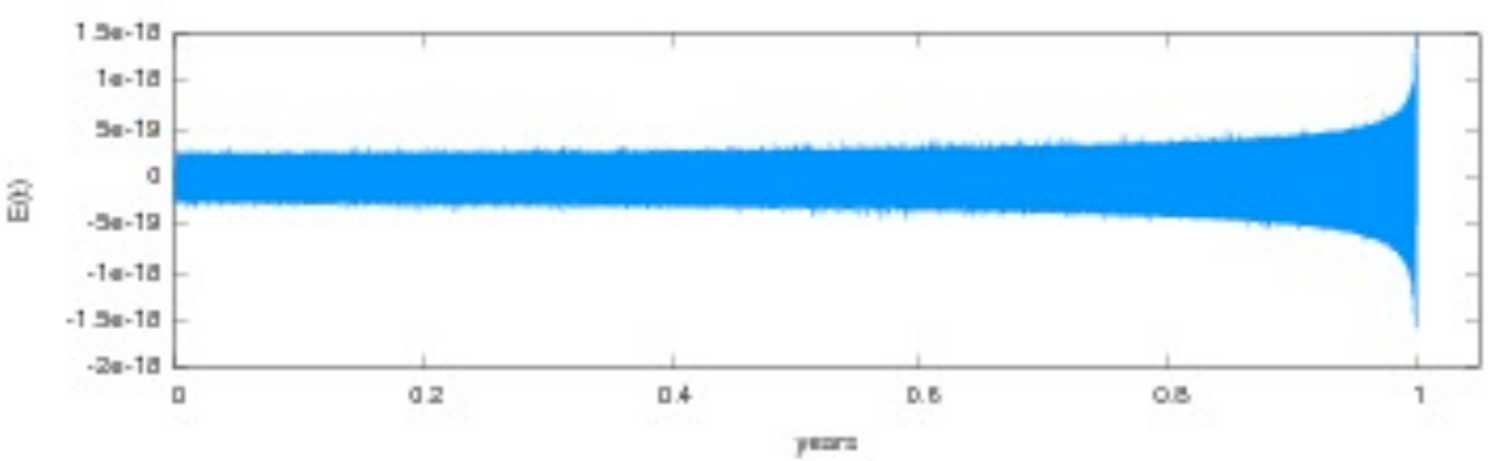
inclination



polarization

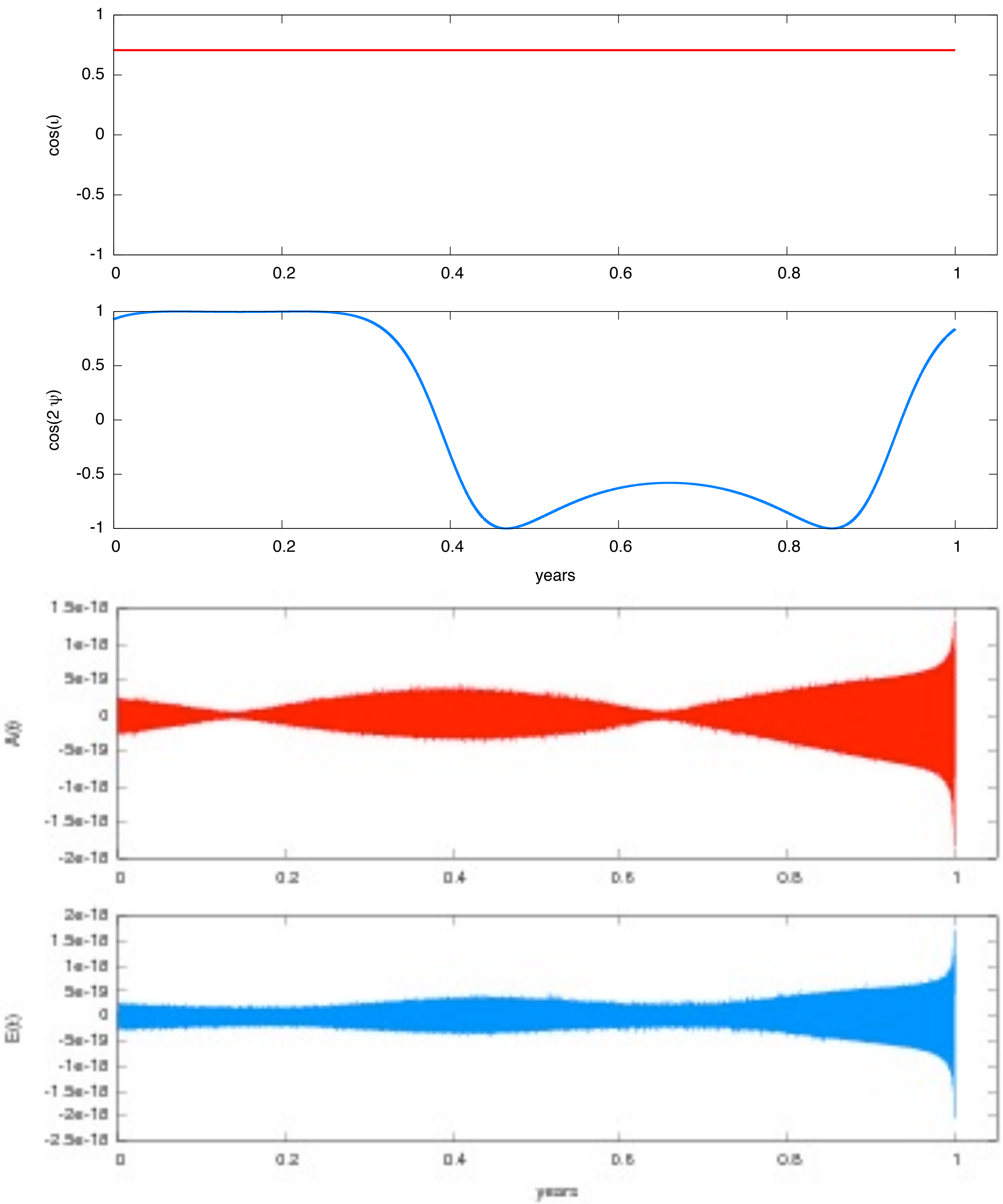


h_+



$h_\times$

Moving Detector, Non-Precessing Source in Ecliptic



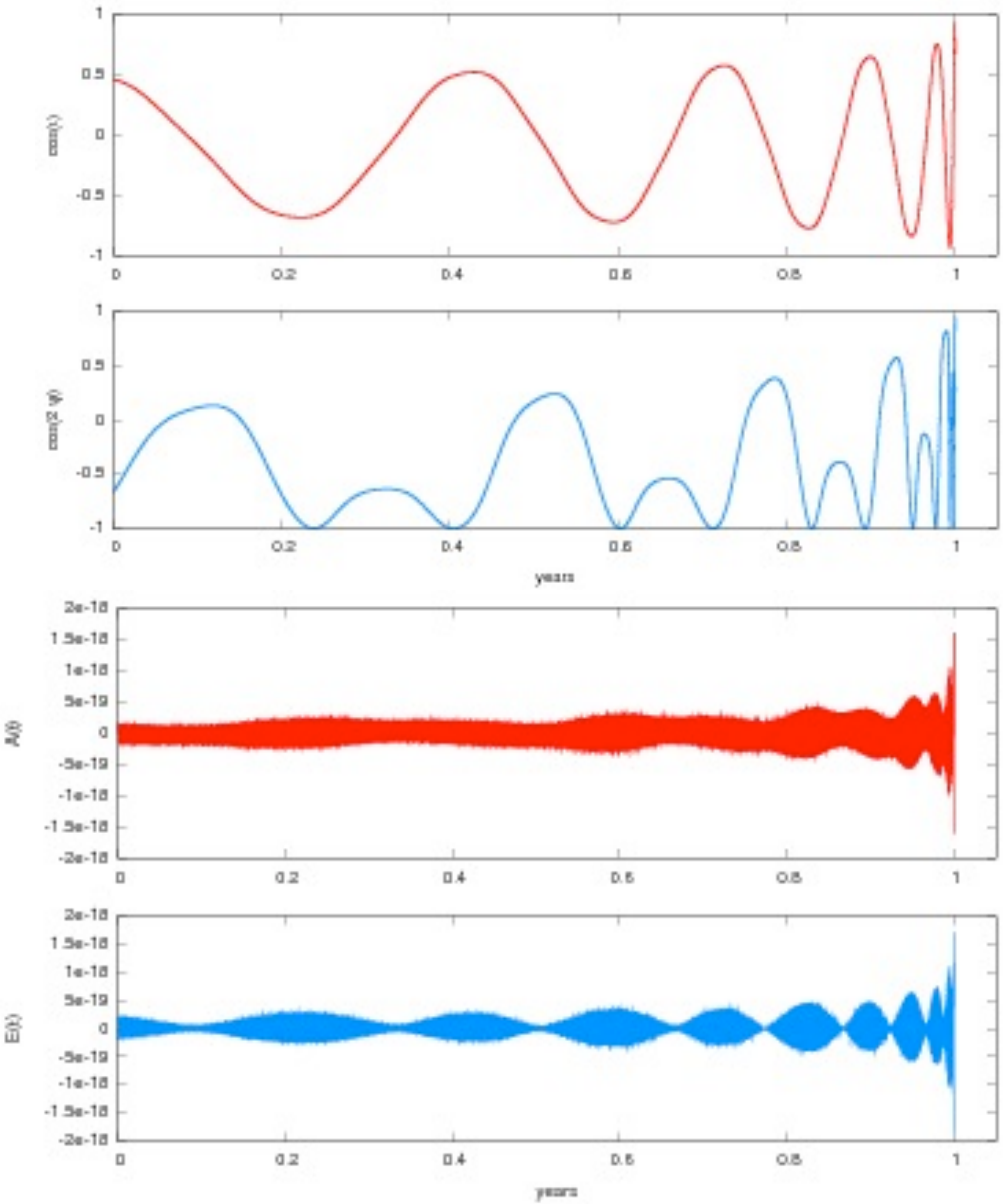
inclination

polarization

h_+

$h_\times$

Static Detector, Precessing Source in Ecliptic



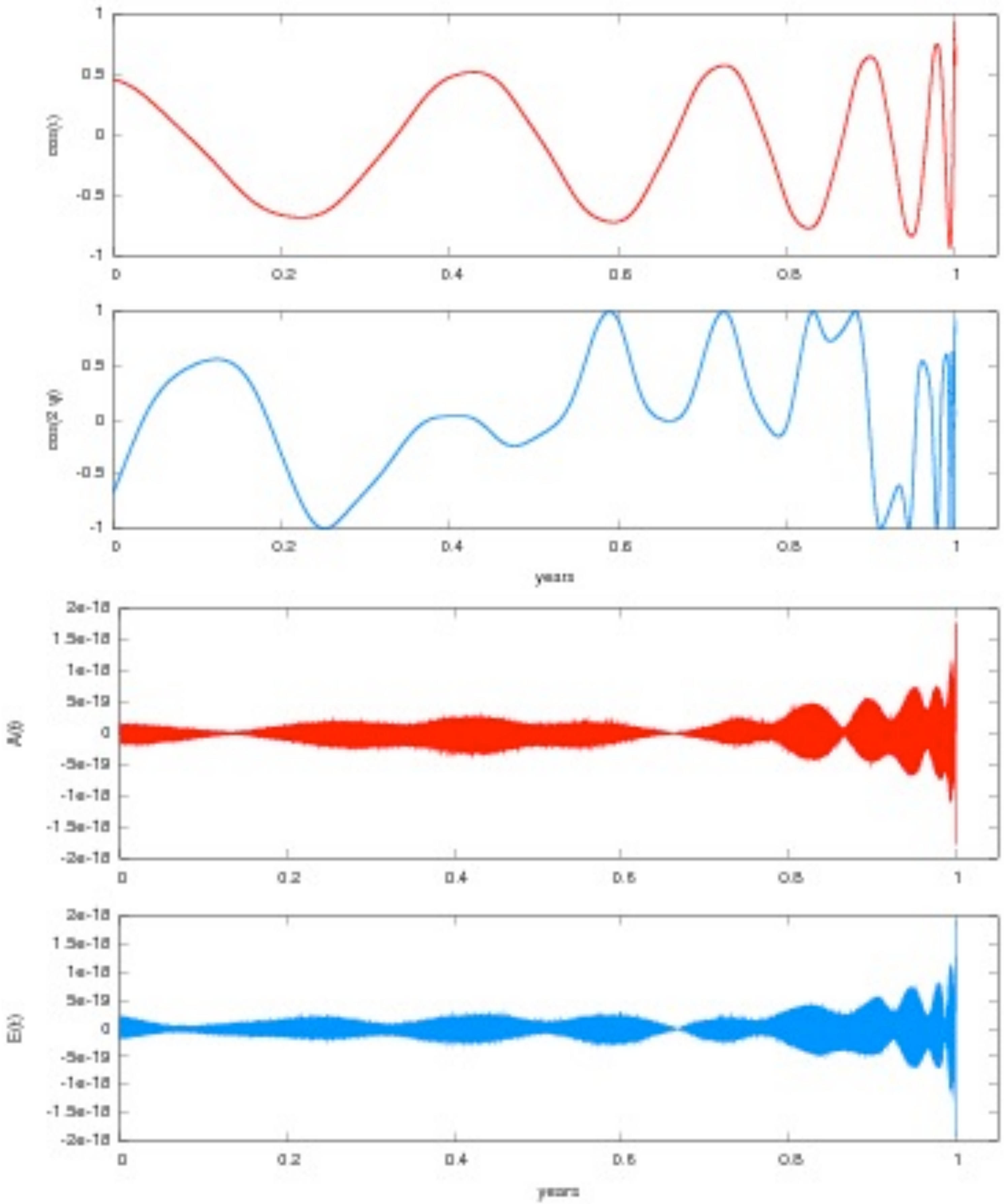
inclination

polarization

h_+

$h_\times$

Moving Detector, Precessing Source in Ecliptic



inclination

polarization

h_+

$h_\times$

Multiple-Scale Analysis

Example: Damped Harmonic Oscillator

$$\ddot{x} + 2\epsilon\dot{x} + x = 0$$

Exact Solution: $x = e^{-\epsilon t}\cos(\sqrt{1-\epsilon^2}\,t)$

Naive Solution: $x = x_0 + \epsilon x_1 + \dots$

$$x = \cos(t) - \epsilon\, t\cos(t) + \dots$$

Multiple-Scale Analysis

Example: Damped Harmonic Oscillator

$$\ddot{x} + 2\epsilon\dot{x} + x = 0$$

Exact Solution: $$x = e^{-\epsilon t}\cos(\sqrt{1-\epsilon^2}\,t)$$

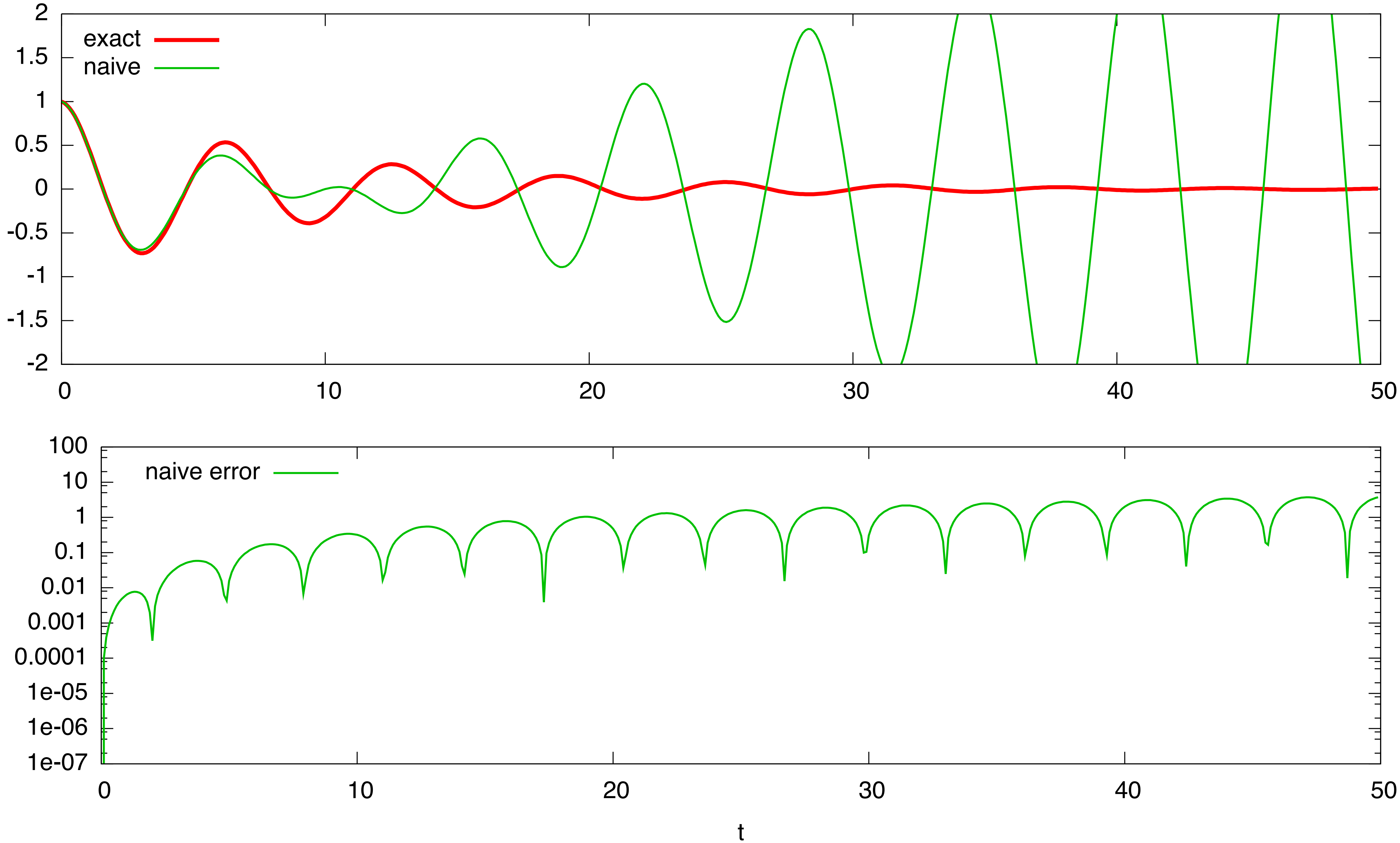
Multiple Scale Analysis: $$t\,, \quad \tau_1 = \epsilon\,t\,, \quad \tau_2 = \epsilon^2\,t\,, \quad \ldots$$

$$x_1 = e^{-\tau_1}\cos(t)$$

$$x_2 = e^{-\tau_1}\cos(t - \frac{1}{2}\tau_2)$$

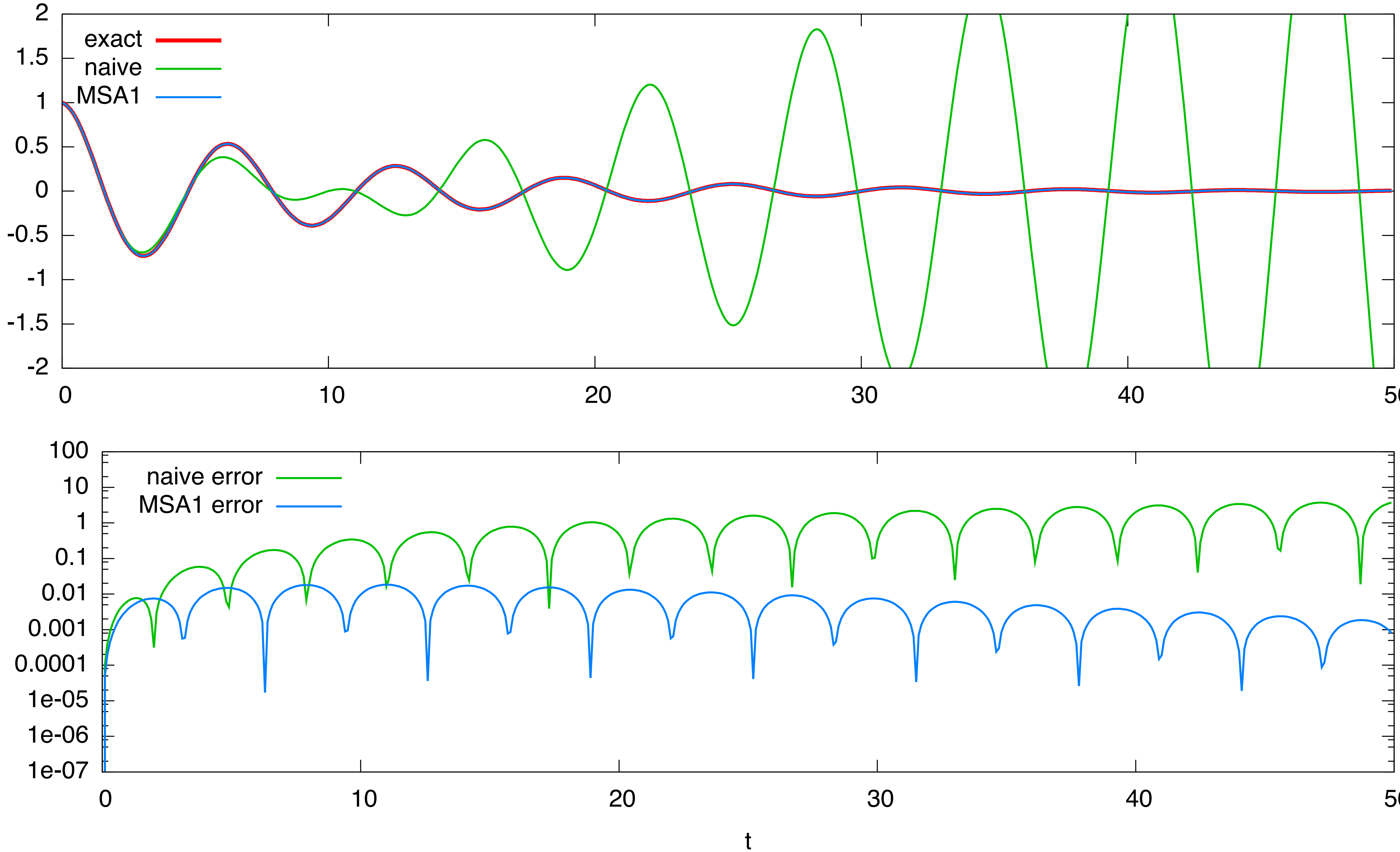
Multiple-Scale Analysis

Example: Damped Harmonic Oscillator



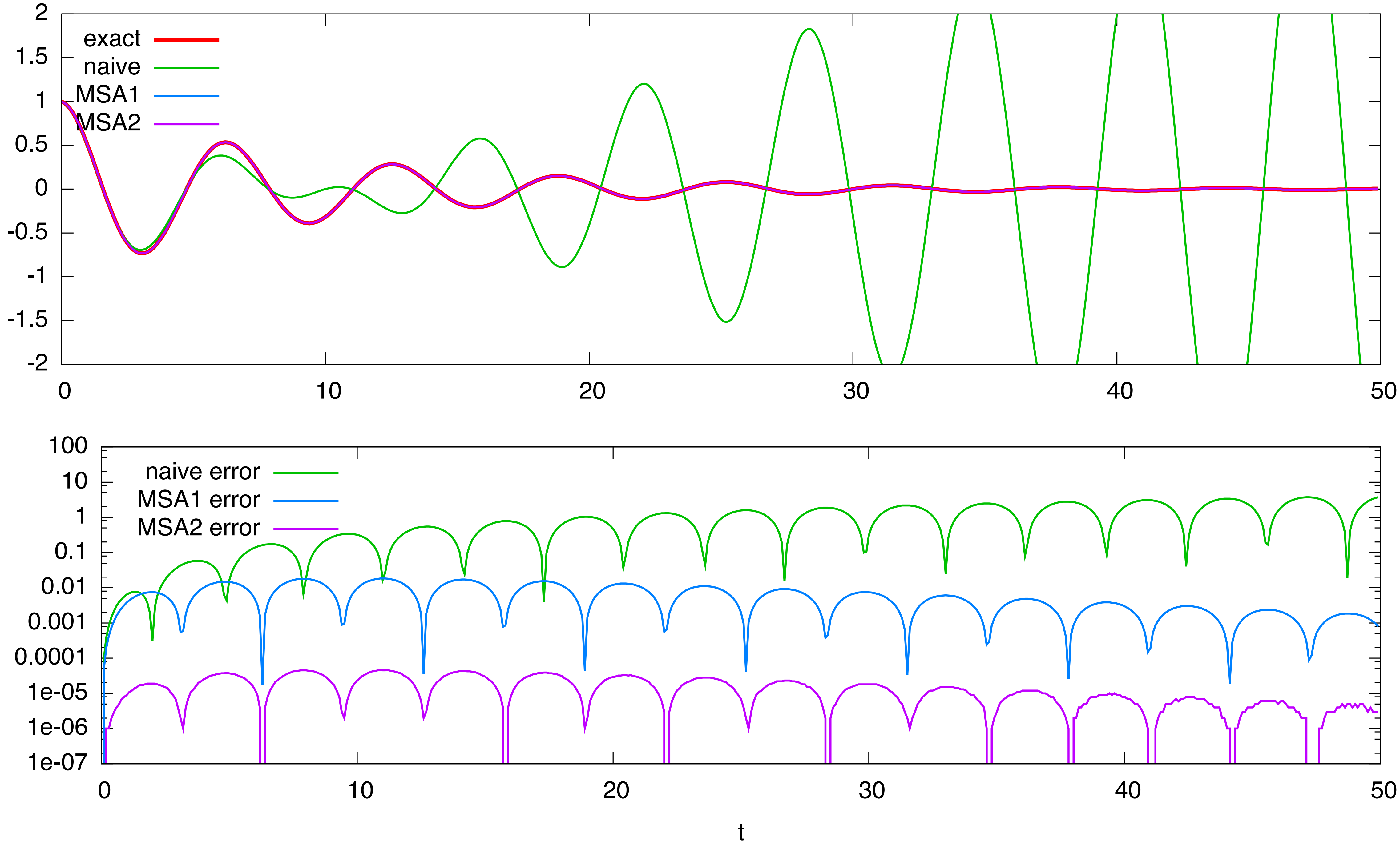
Multiple-Scale Analysis

Example: Damped Harmonic Oscillator



Multiple-Scale Analysis

Example: Damped Harmonic Oscillator



Beyond Stationary Phase

$$h(t) = A(t)\, \cos\phi(t)$$

SPA:

$$h(f) = A(t_f)\sqrt{\frac{2\pi}{\ddot{\phi}(t_f)}}\, e^{i(2\pi f t_0 - \phi(t_f) - \pi/4)}$$

$$\dot{\phi}(t_f) \equiv 2\pi f$$

Problem when second derivative of phase vanishes at stationary point

SPA fails for precessing systems

Beyond Stationary Phase

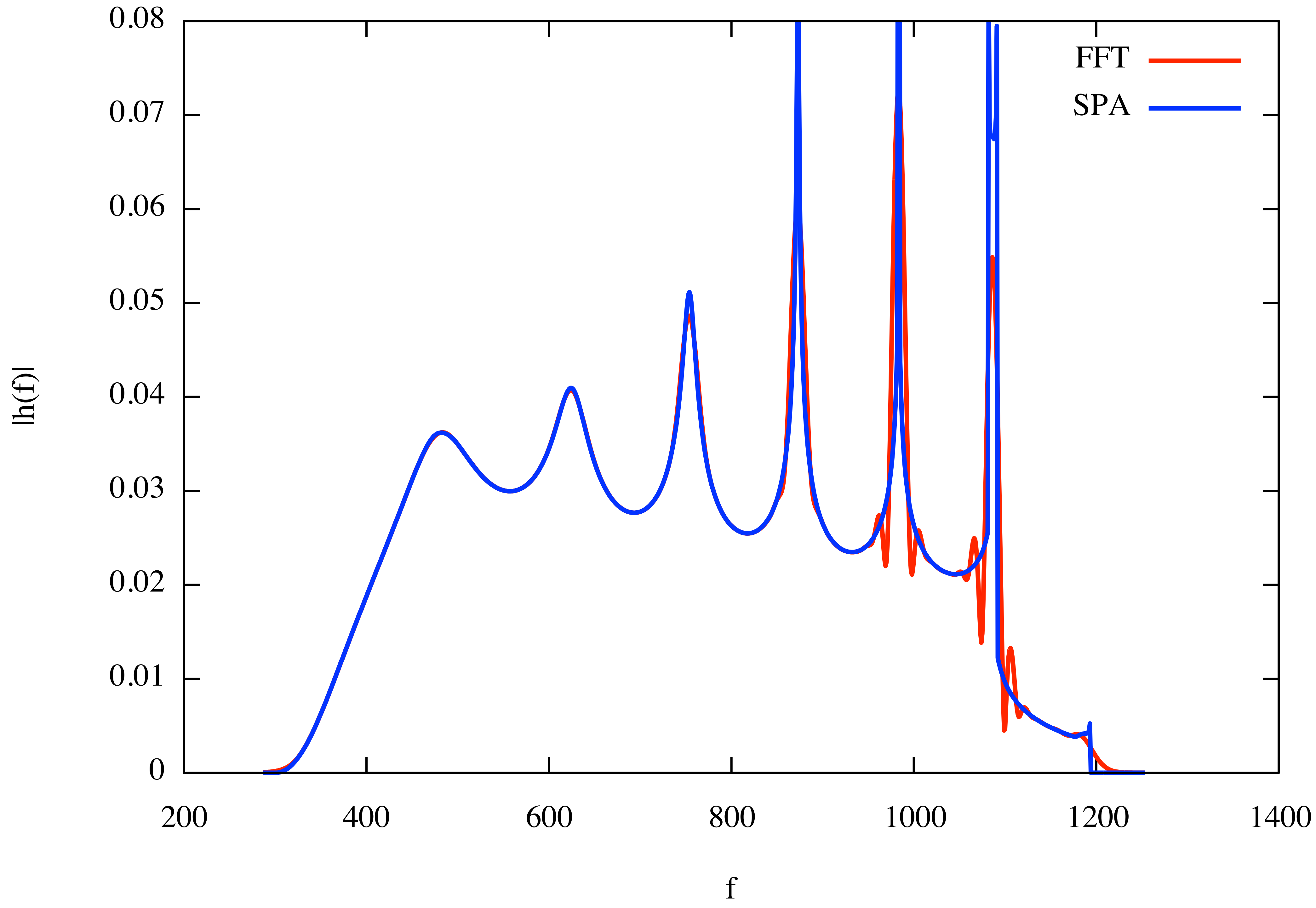
$$h(t) = A(t)\ \cos\phi(t)$$

SUA: $$\phi(t) = \phi_c(t) + \delta\phi(t) \qquad \dot{\phi}_c(t_f) \equiv 2\pi f$$

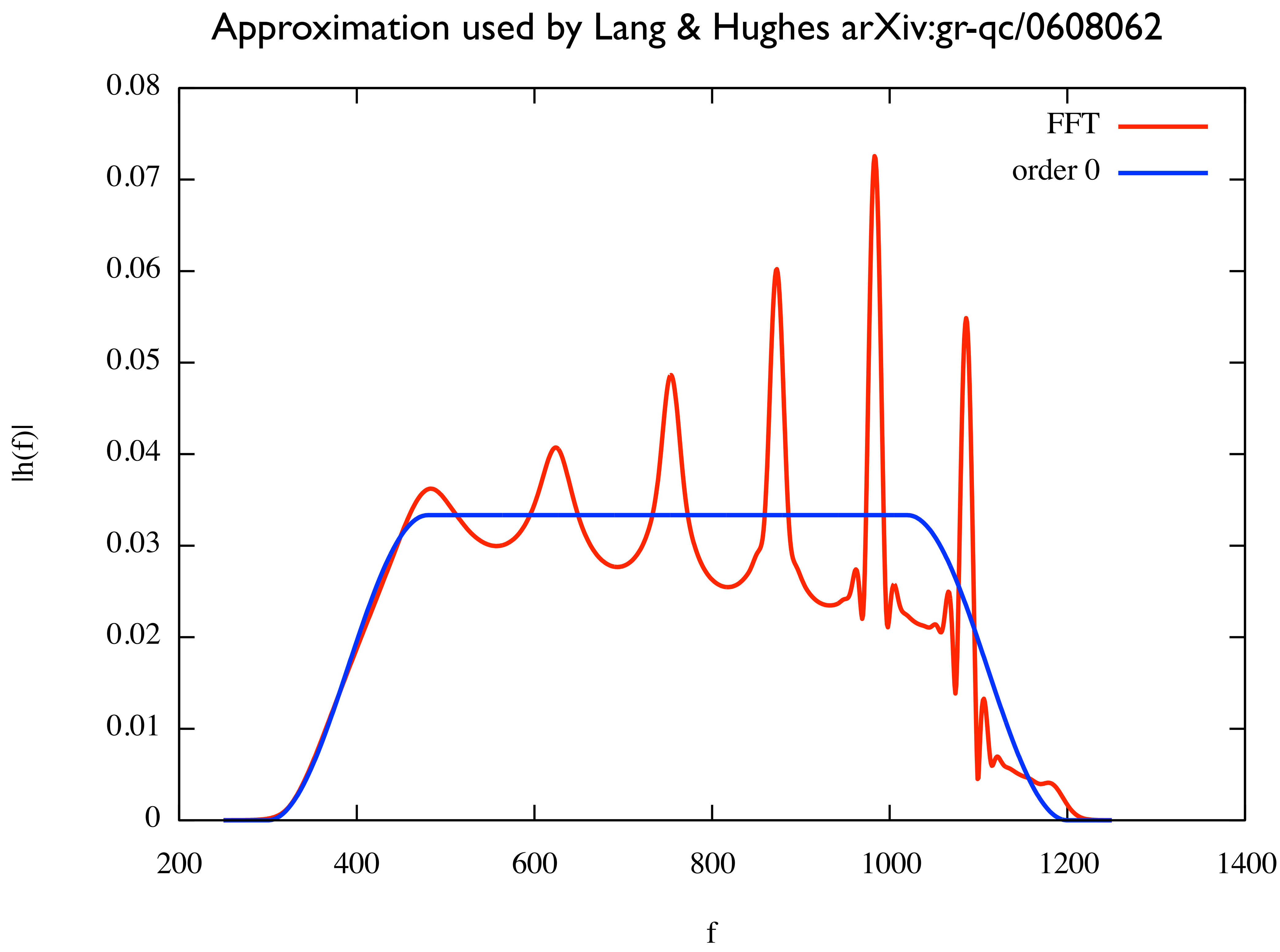
$$h(f) = \bar{A}(t_f)\sqrt{\frac{2\pi}{\ddot{\phi}_c(t_f)}}\, e^{i(2\pi f t_0 - \phi_c(t_f) - \pi/4)}$$

$$\bar{A}(t_f) = \sum_{k=0}^{K} \frac{a_{k,K}}{2}\left[A\left(t_f + \frac{k}{\sqrt{\ddot{\phi}_c(t_f)}}\right) + A\left(t_f - \frac{k}{\sqrt{\ddot{\phi}_c(t_f)}}\right)\right]$$

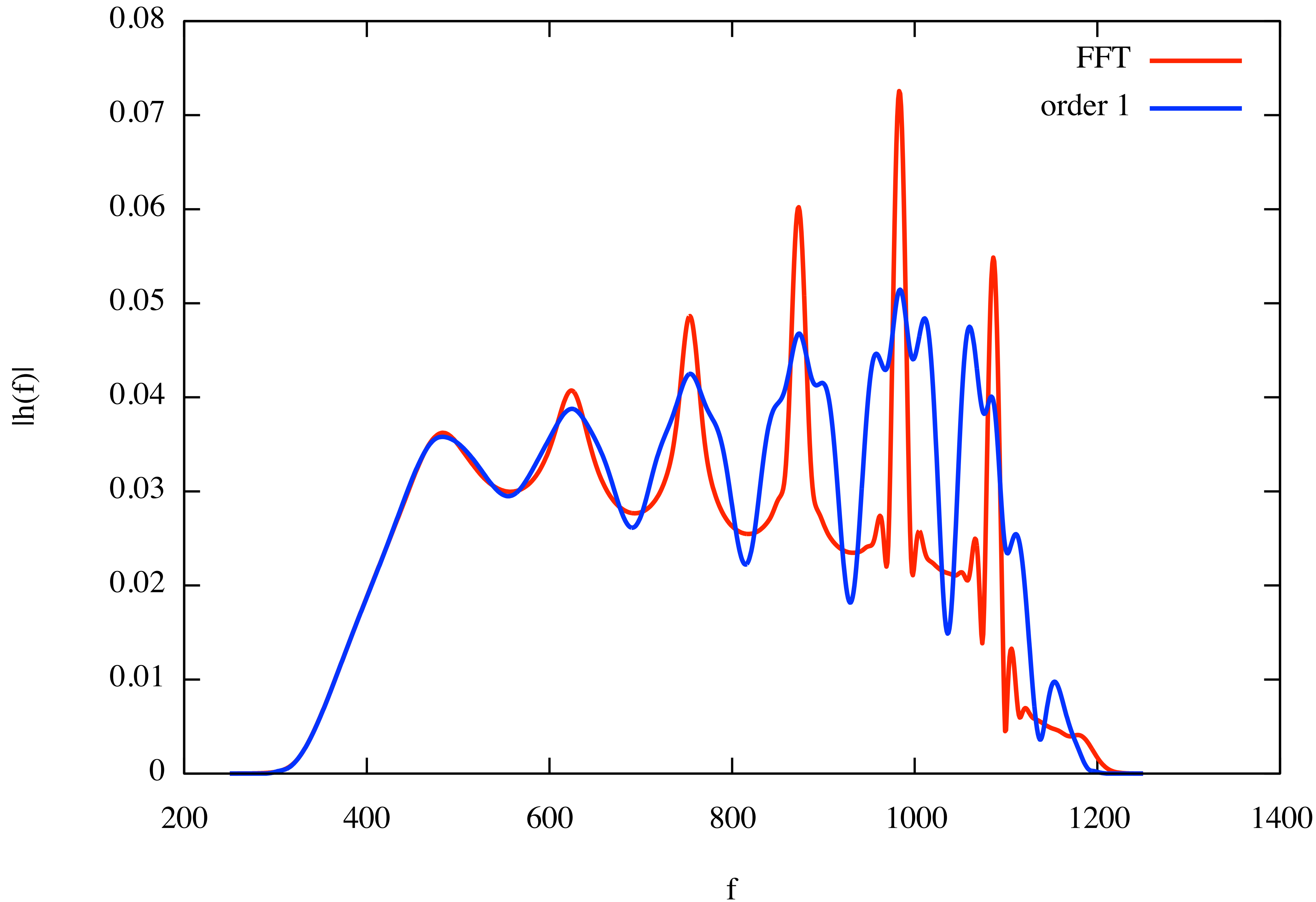
Beyond Stationary Phase



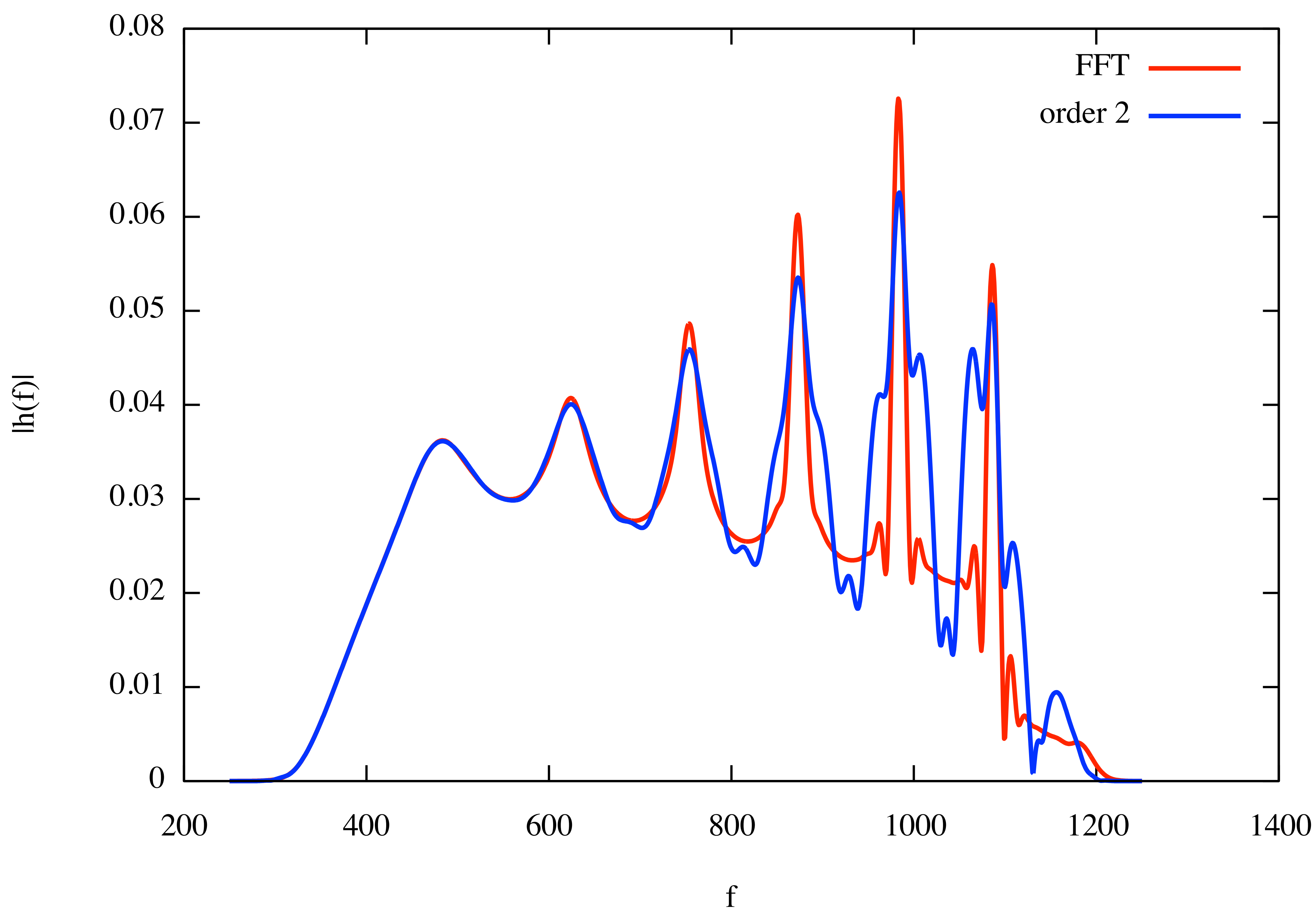
Beyond Stationary Phase



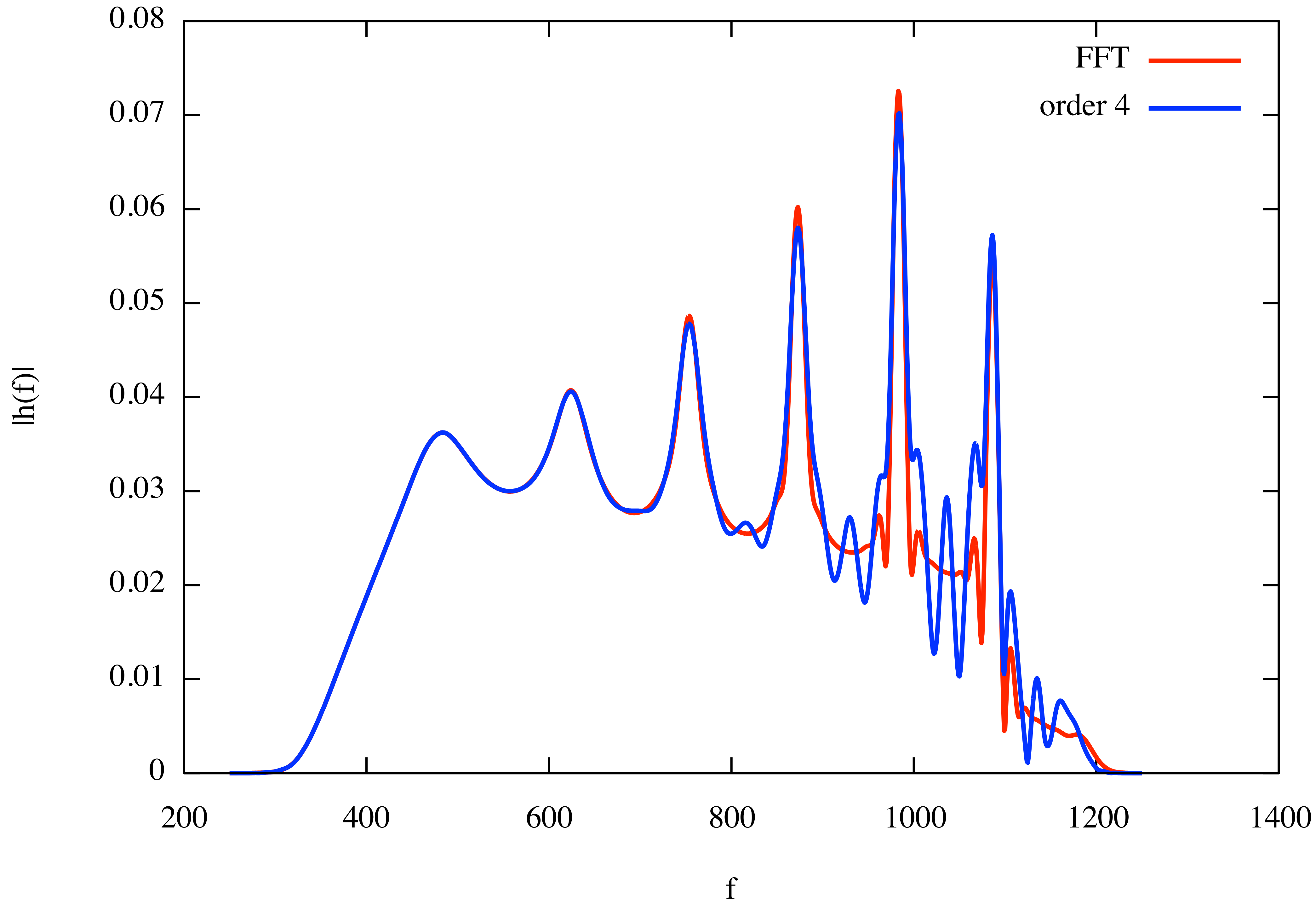
Beyond Stationary Phase



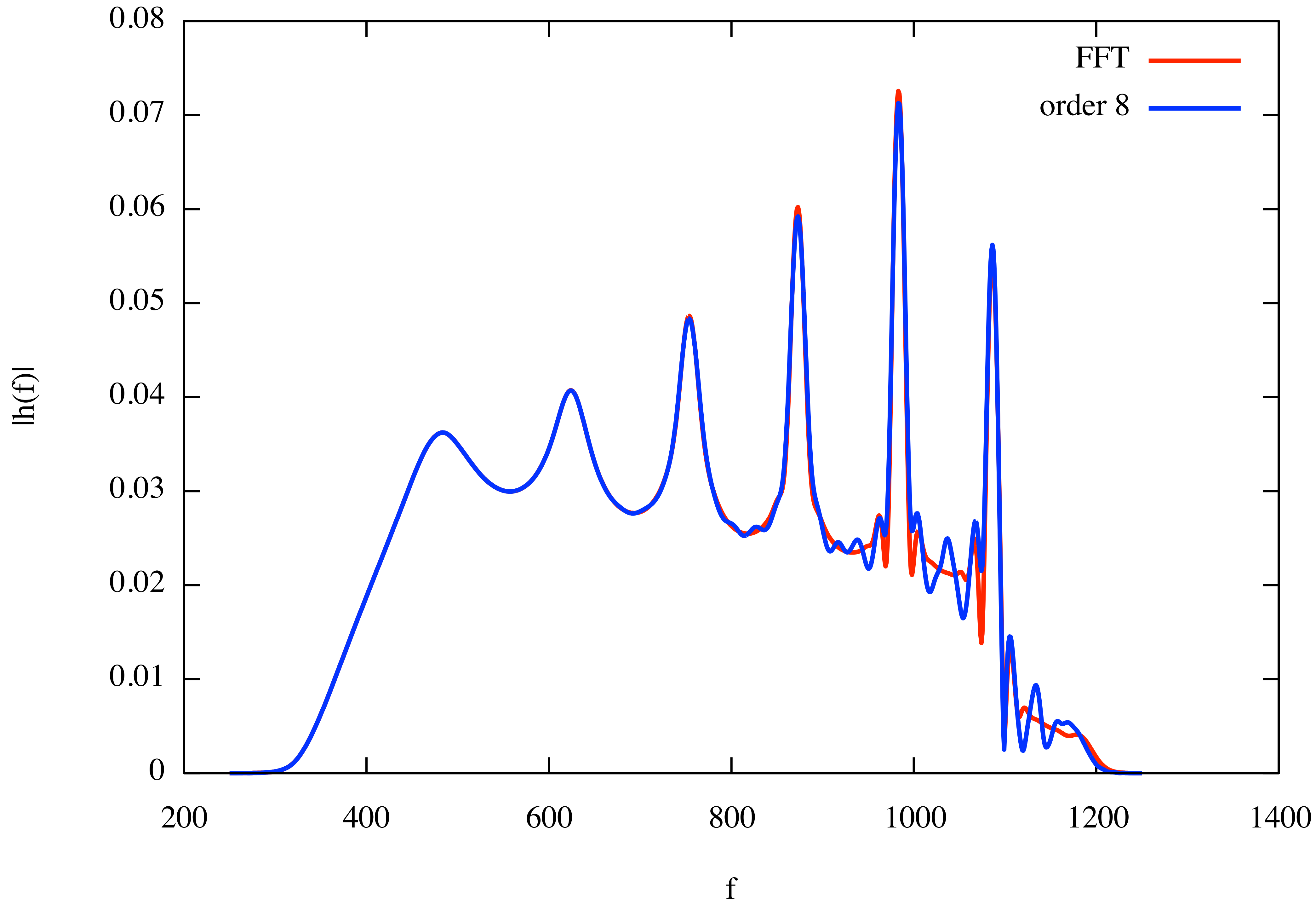
Beyond Stationary Phase



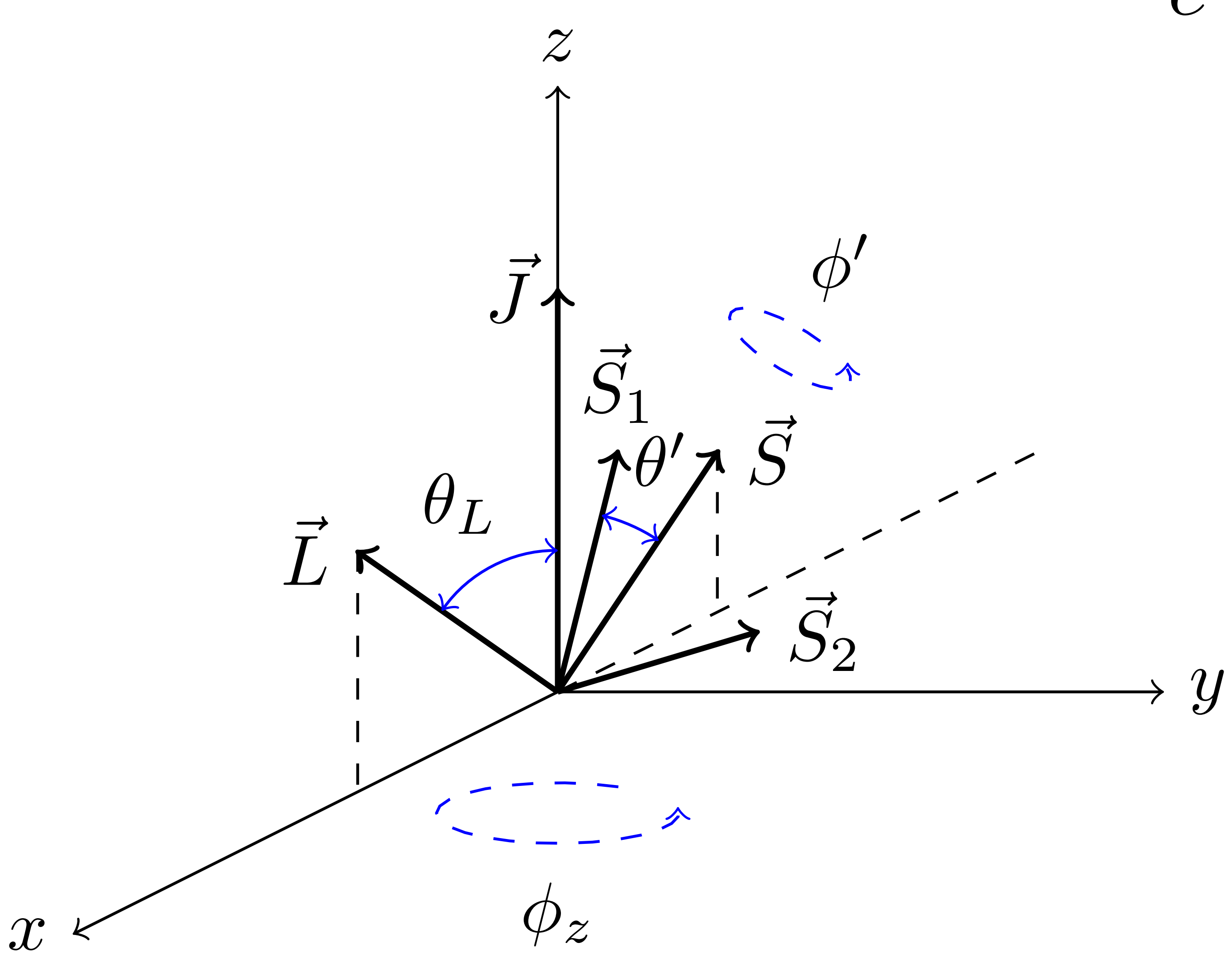
Beyond Stationary Phase



Beyond Stationary Phase



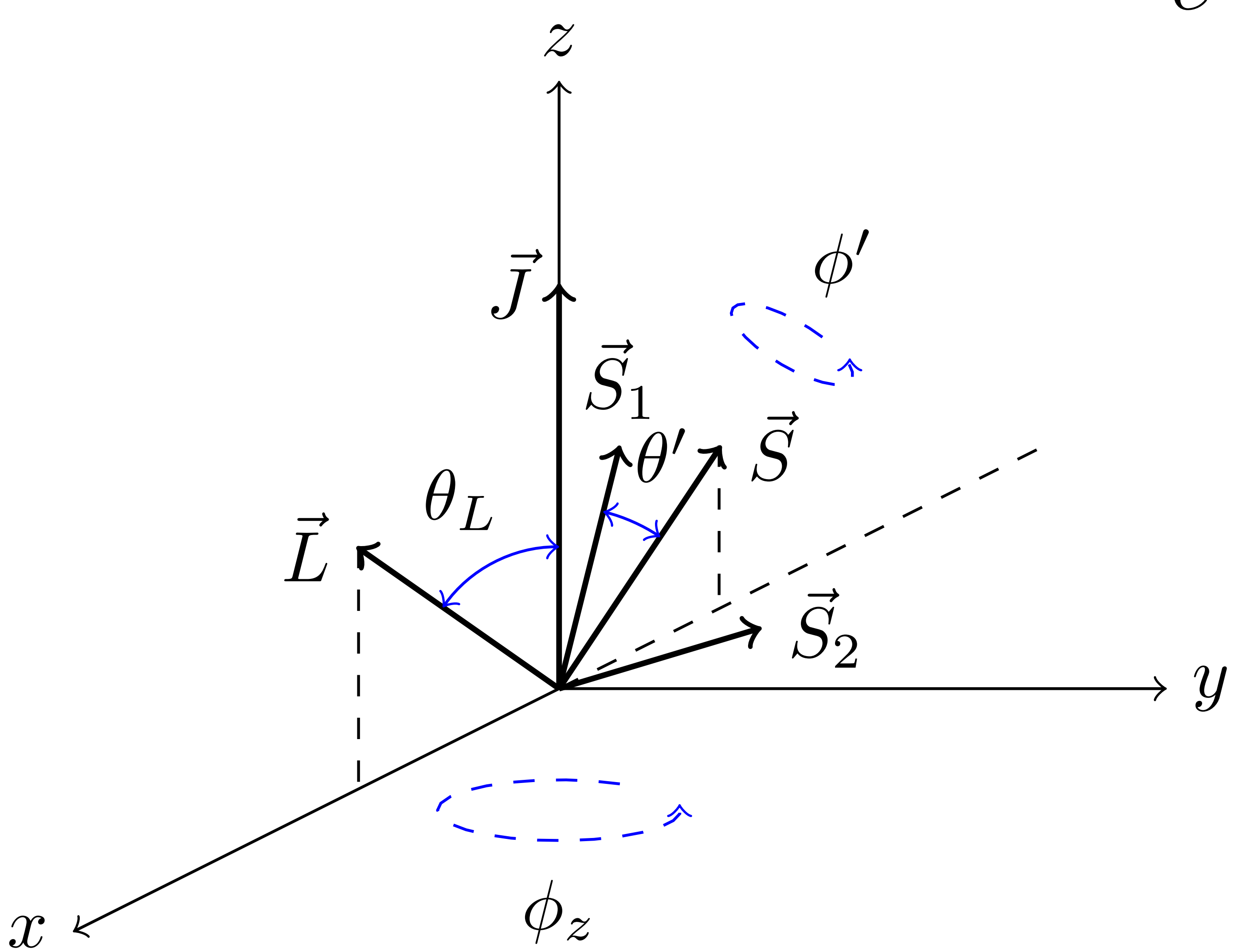
Conservative Dynamics



$$\vec{c} = (\vec{J}, L, S_1, S_2, S_{eff}) = \text{const}$$

[Kesden, Gerosa, O'Shaughnessy, Berti & Sperhake arXiv:1411.0674]

Conservative Dynamics



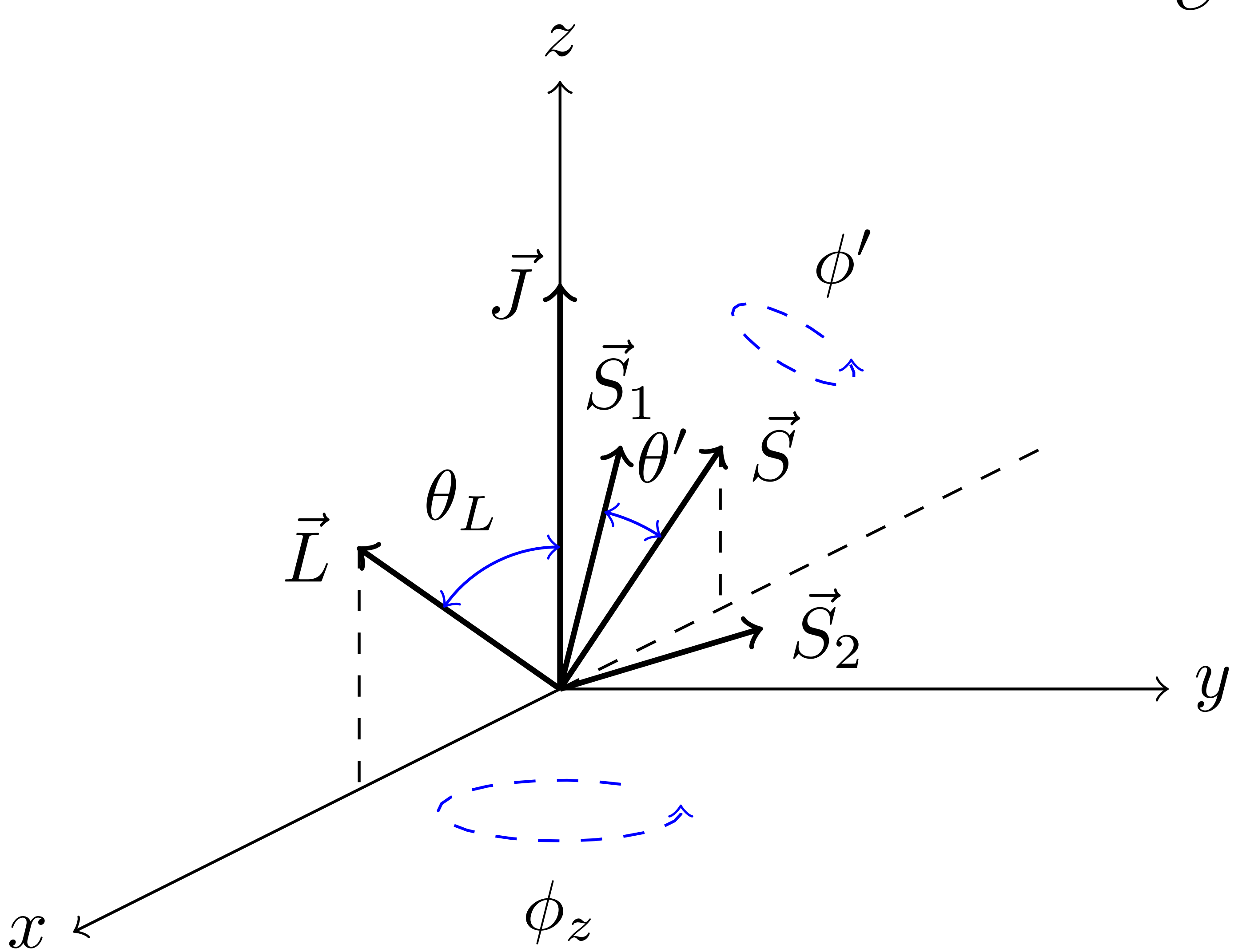
$$\vec{c} = (\vec{J}, L, S_1, S_2, S_{eff}) = \text{const}$$

$$\Downarrow$$

$$\vec{L}(S(t); \vec{c})$$

[Kesden, Gerosa, O'Shaughnessy, Berti & Sperhake arXiv:1411.0674]

Conservative Dynamics



$$\vec{c} = (\vec{J}, L, S_1, S_2, S_{eff}) = \text{const}$$

$$\Downarrow$$

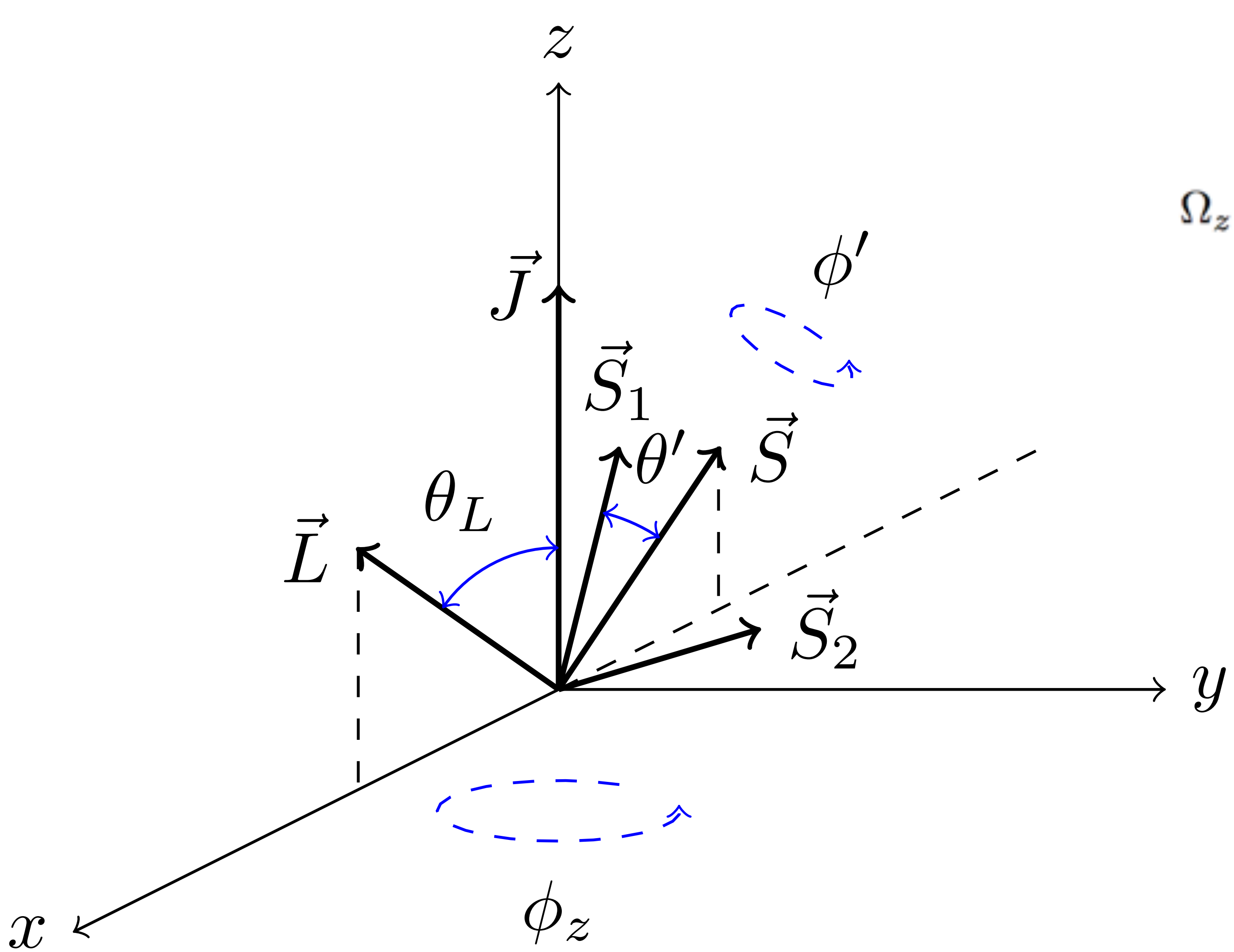
$$\vec{L}(S(t); \vec{c})$$

$$\Downarrow$$

$$R\left[\hat{z}, \phi_z(t)\right] \vec{L}(S(t), \vec{c})$$

[Kesden, Gerosa, O'Shaughnessy, Berti & Sperhake arXiv:1411.0674]

Reduction to Quadratures

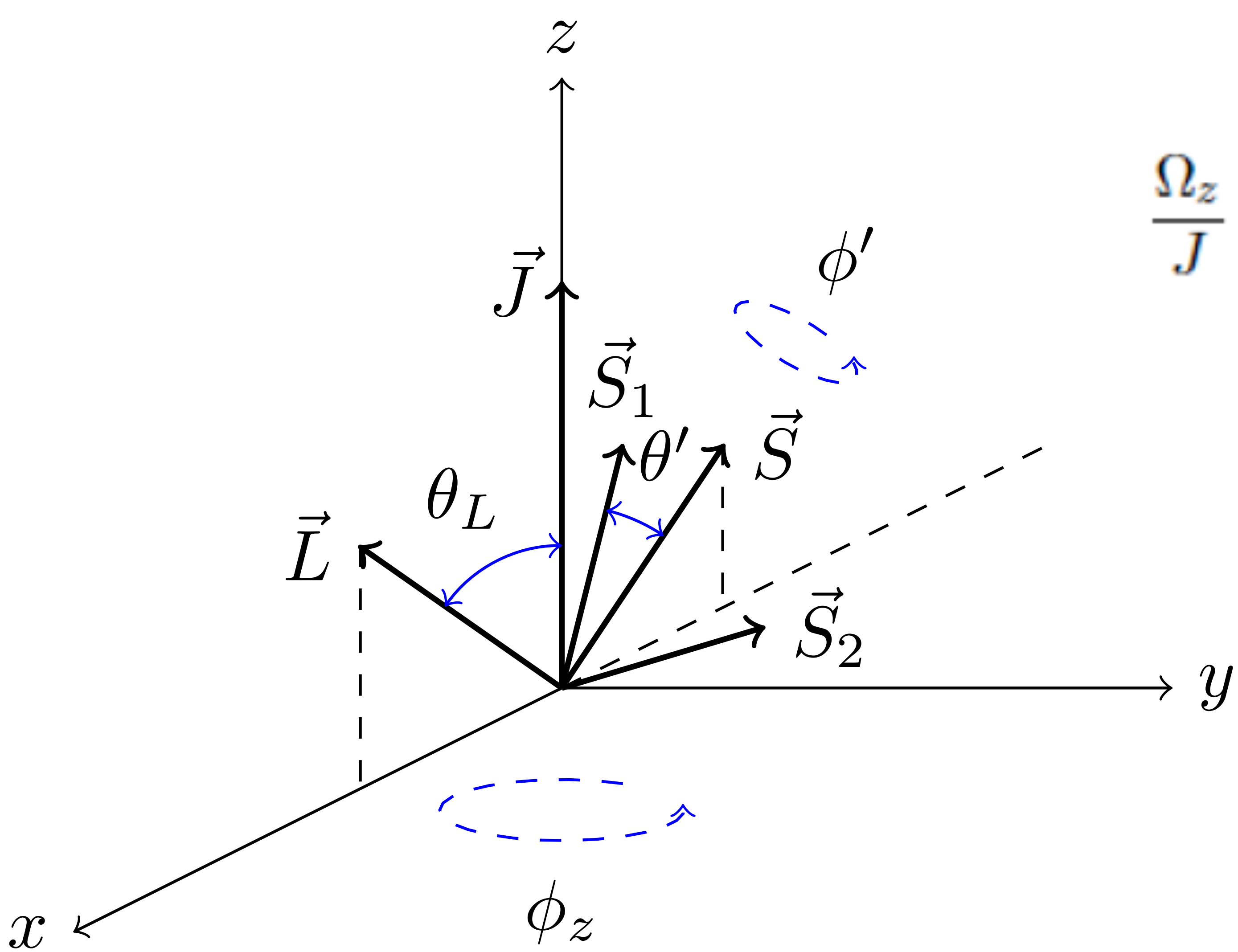


$$\frac{dS}{dt} = -\frac{3(1-q^2)}{2q}\frac{S_1 S_2}{S}\frac{(\eta^2 M^3)^3}{L^5}\left(1-\frac{\eta M^2 \xi}{L}\right) \times \sin\theta_1 \sin\theta_2 \sin\Delta\Phi$$

$$\Omega_z = \frac{J}{2}\left(\frac{\eta^2 M^3}{L^2}\right)^3 \left\{1+\frac{3}{2\eta}\left(1-\frac{\eta M^2\xi}{L}\right) - \frac{3(1+q)}{2qA_1^2A_2^2}\left(1-\frac{\eta M^2\xi}{L}\right)[4(1-q)L^2(S_1^2-S_2^2) - (1+q)(J^2-L^2-S^2)(J^2-L^2-S^2-4\eta M^2 L\xi)]\right\}$$

[Kesden, Gerosa, O'Shaughnessy, Berti & Sperhake arXiv:1411.0674]

Closed form solution



$$S^2 = S_+^2 + (S_-^2 - S_+^2)\,\mathrm{sn}^2(\psi, \mathrm{m})$$

$$\frac{\Omega_z}{J} = a + \frac{c_0 + c_2\,\mathrm{sn}^2(\psi, m) + c_4\,\mathrm{sn}^4(\psi, m)}{d_0 + d_2\,\mathrm{sn}^2(\psi, m) + d_4\,\mathrm{sn}^4(\psi, m)}$$

$$m = \frac{S_+^2 - S_-^2}{S_+^2 - S_3^2}$$

(using Jacobi elliptic functions)

[Chatziioannou Klein, Cornish, Yunes arXiv:1606.03117]

Including Radiation Reaction

How do the ‘constants’ change?

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$$\vec{c} = (\vec{J}, L, S_1, S_2, S_{eff})$$

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Including Radiation Reaction

How do the ‘constants’ change?

$$\vec{c} = (\vec{J}, L, S_1, S_2, S_{eff})$$

$$\int \frac{dL}{d\omega}\frac{d\omega}{dt}dt$$

PN integration

Including Radiation Reaction

How do the ‘constants’ change?

$$\vec{c} = (\vec{J}, L, S_1, S_2, S_{eff})$$

$$\int \frac{dL}{d\omega}\frac{d\omega}{dt}dt$$

PN integration

$$\langle \dot{\hat{J}} \rangle_{pr} = 0$$

Including Radiation Reaction

How do the 'constants' change?

$$\vec{c} = (\vec{J}, L, \underline{S_1}, \underline{S_2}, \underline{S_{eff}})$$

$$\int \frac{dL}{d\omega}\frac{d\omega}{dt}dt$$ PN integration

$$\langle \dot{\hat{J}} \rangle_{pr} = 0$$

$$\langle J(t) \rangle_{pr} = \sqrt{L^2 + \langle S^2 \rangle_{pr} + 2Lc_1}$$

Including Radiation Reaction

Finally, we have to compute $\phi_z(t) = \int \Omega_z(S(t), \vec{c}(t))dt$

[Pound arXiv:1006.3903]

Including Radiation Reaction

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t_{pr} t_{rr}

[Pound arXiv:1006.3903]

Including Radiation Reaction

Finally, we have to compute $\phi_z(t) = \int \Omega_z(S(t), \vec{c}(t))dt$

$S(t) \rightarrow t_{pr}$, $\vec{c}(t) \rightarrow t_{rr}$

Expand in the ratio of the timescales $\dfrac{t_{pr}}{t_{rr}}$

[Pound arXiv:1006.3903]

Including Radiation Reaction

Finally, we have to compute $\phi_z(t) = \int \Omega_z(S(t), \vec{c}(t))dt$

$$t_{pr} \qquad t_{rr}$$

Expand in the ratio of the timescales $\dfrac{t_{pr}}{t_{rr}}$

$$\int \langle \Omega_z(t_{rr}) \rangle_{pr} \, dt_{rr} + \int \Omega_z(S(t_{pr}), \vec{c}(t_{rr}))dt_{pr} - \int \langle \Omega_z(t_{rr}) \rangle_{pr} dt_{pr}$$

[Pound arXiv:1006.3903]

Including Radiation Reaction

Finally, we have to compute $\phi_z(t) = \int \Omega_z(S(t), \vec{c}(t))dt$

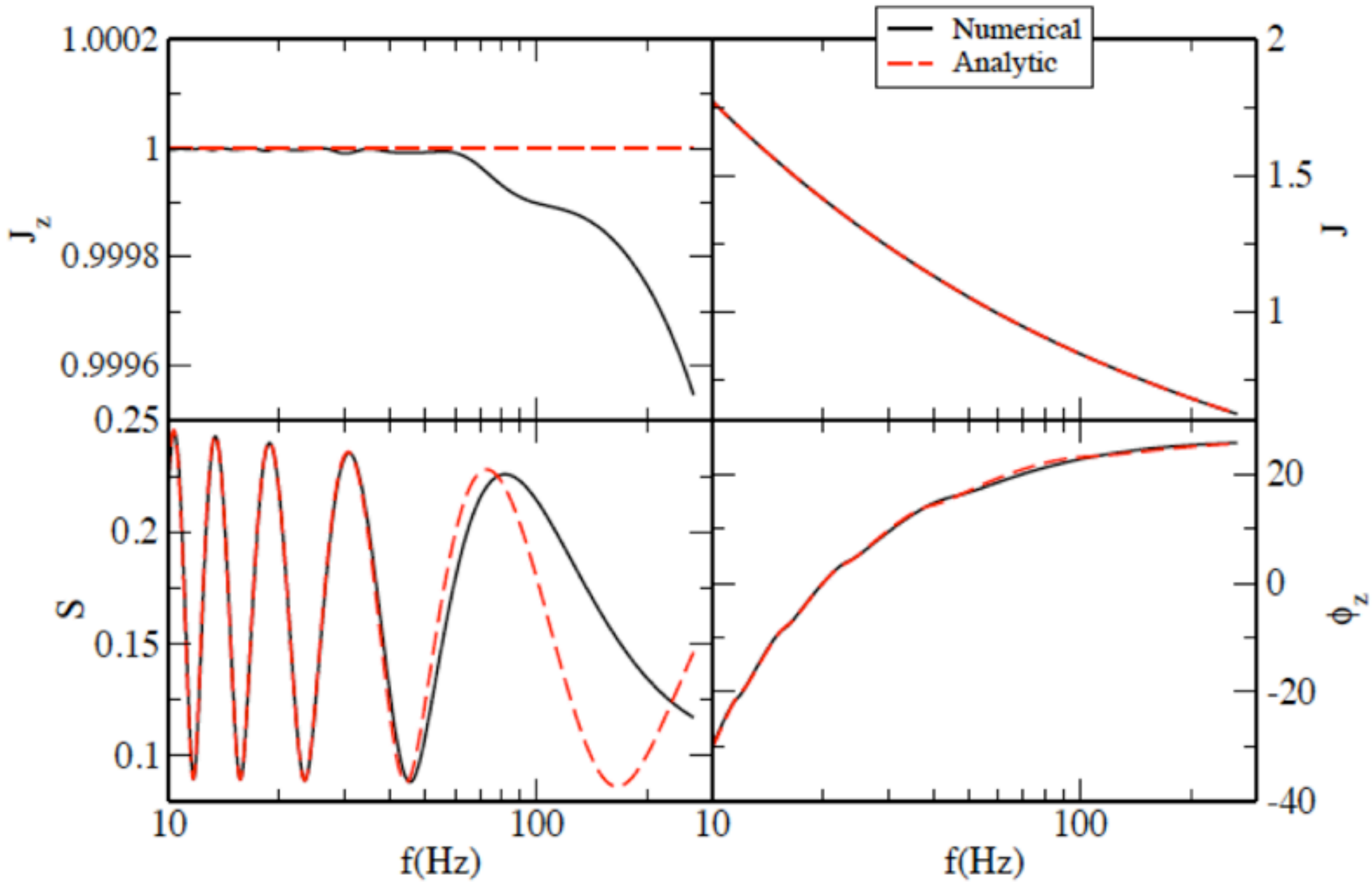
$S(t) \rightarrow t_{pr}$, $\vec{c}(t) \rightarrow t_{rr}$

Expand in the ratio of the timescales $\dfrac{t_{pr}}{t_{rr}}$

$$\underbrace{\int \langle \Omega_z(t_{rr}) \rangle_{pr} \, dt_{rr}}_{\mathcal{O}(t_{rr}/t_{pr})} + \underbrace{\int \Omega_z(S(t_{pr}), \vec{c}(t_{rr}))dt_{pr} - \int \langle \Omega_z(t_{rr}) \rangle_{pr} dt_{pr}}_{\mathcal{O}(1)}$$

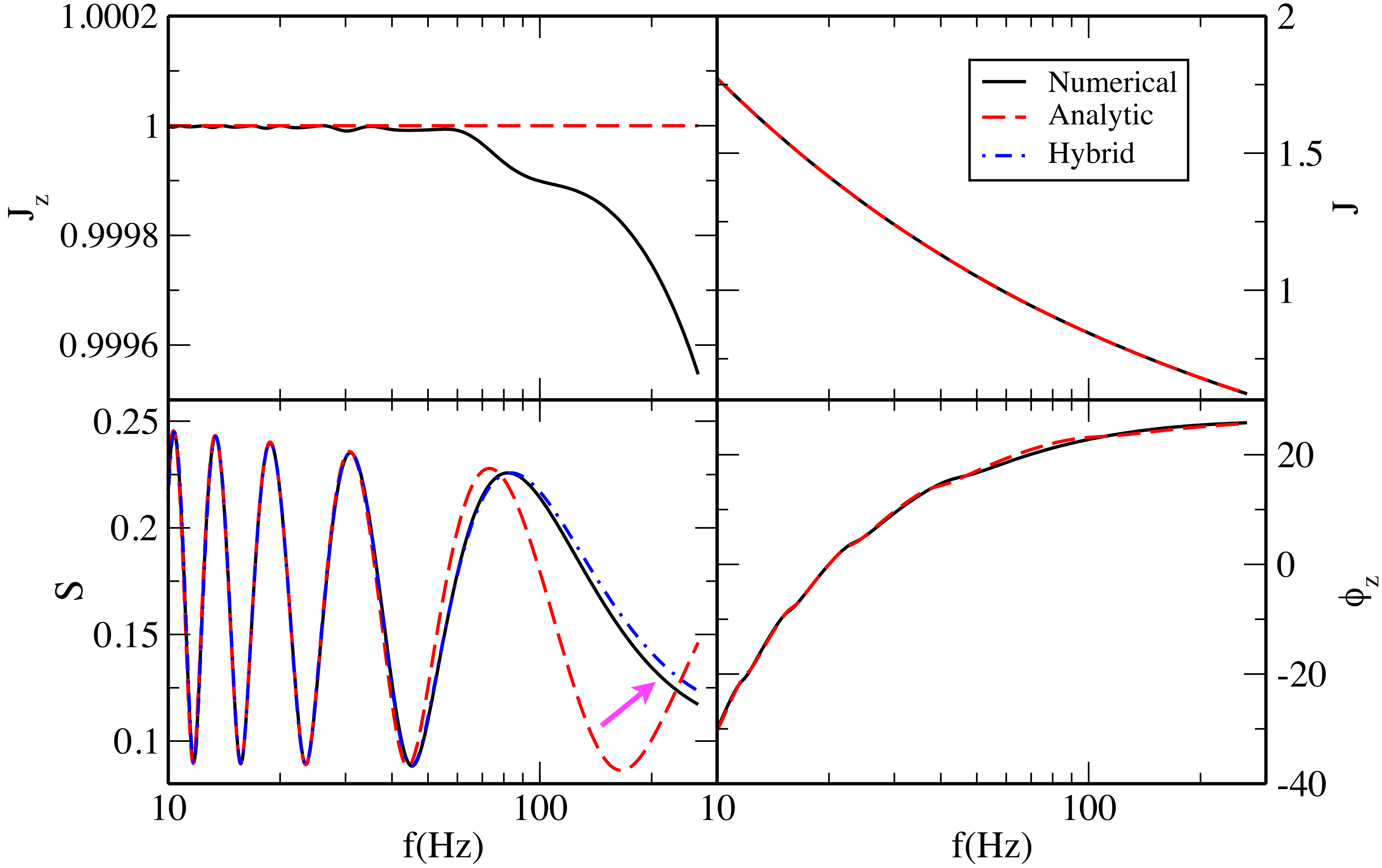
[Pound arXiv:1006.3903]

Including Radiation Reaction



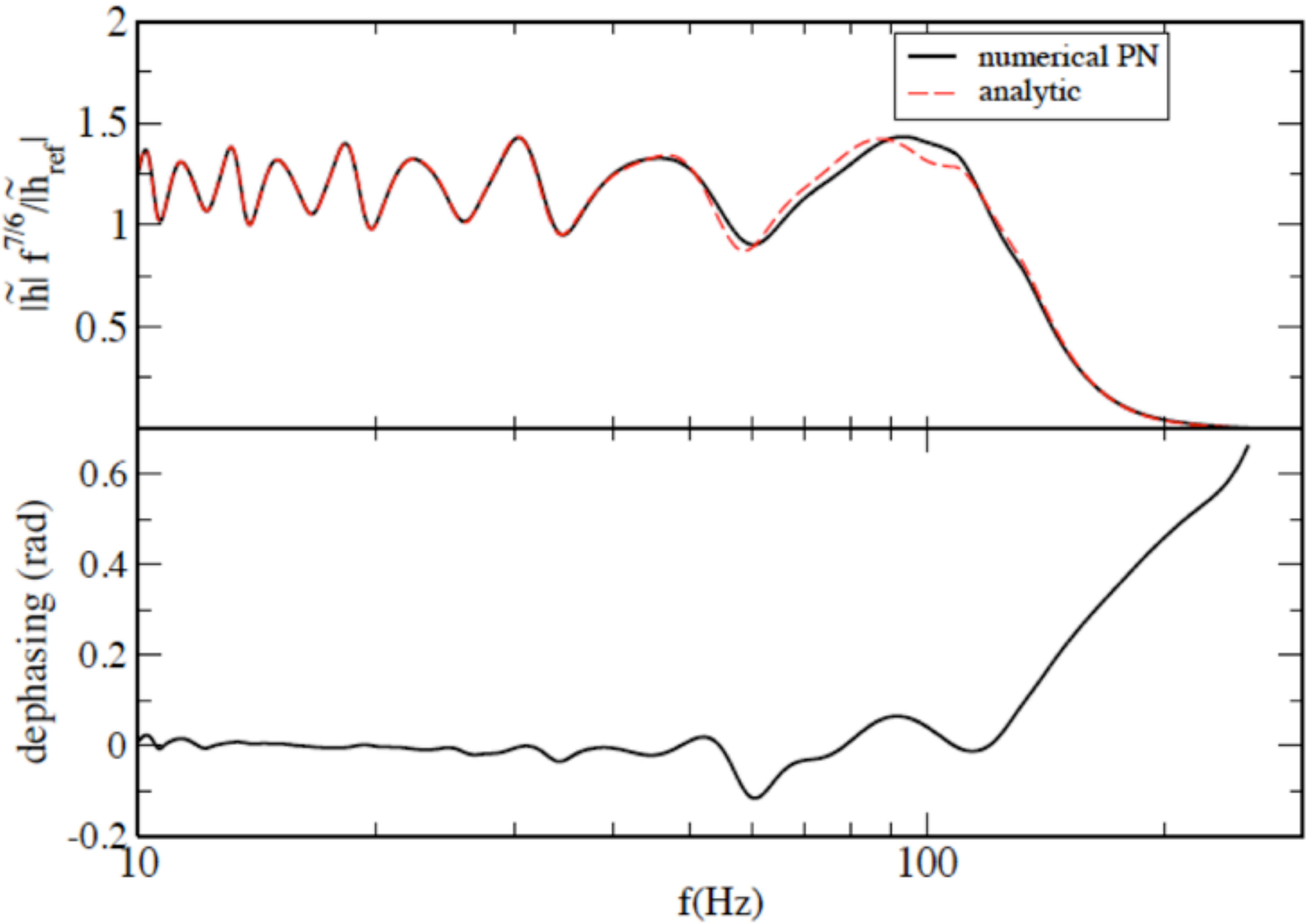
e.g. LIGO BH-BH system

Including Radiation Reaction



Extending PN integration to higher order

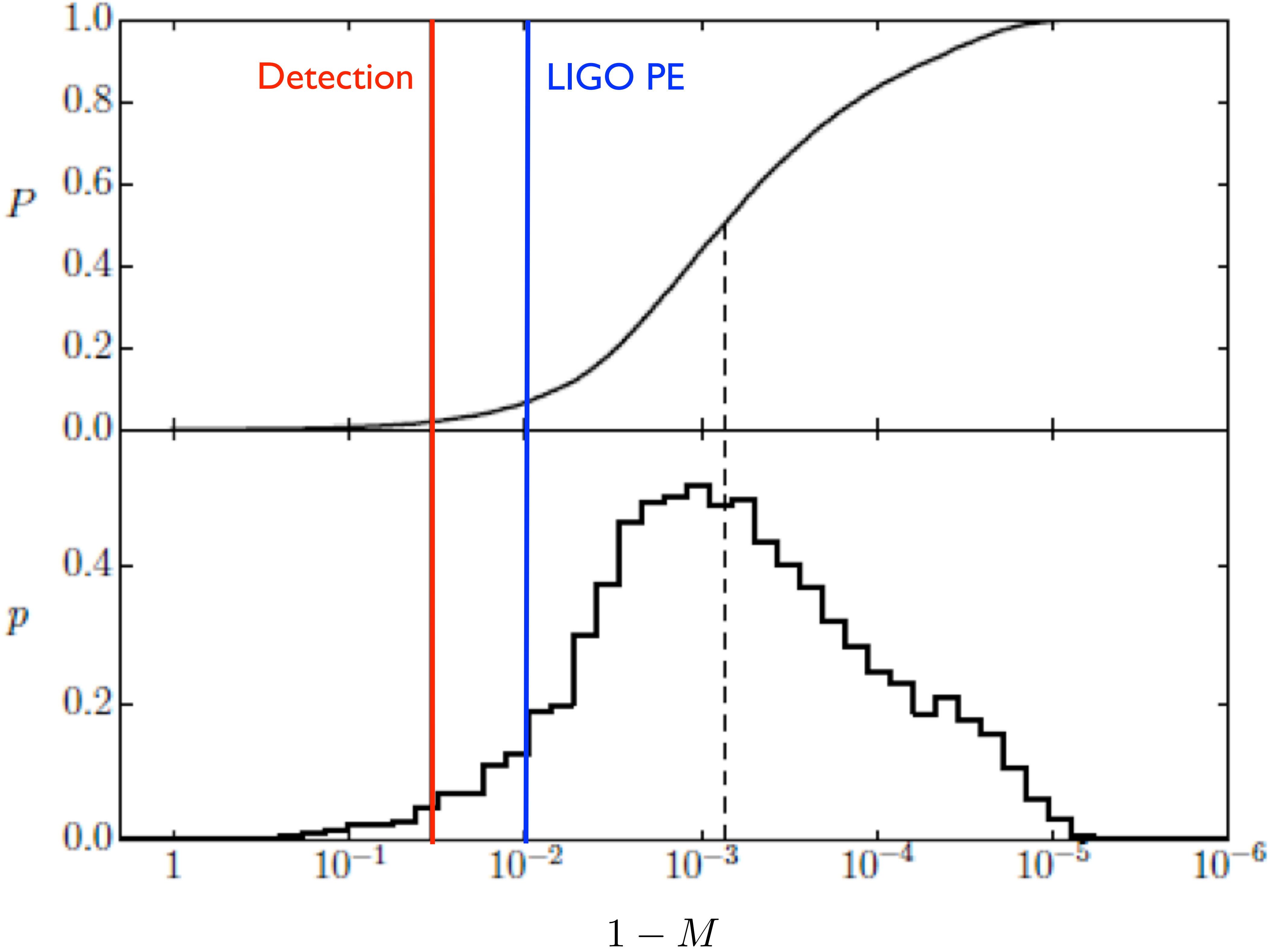
Full Waveform



e.g. LIGO BH-BH system

Mis-Match

$$E[1 - M] = \frac{D}{2\,\mathrm{SNR}^2}$$

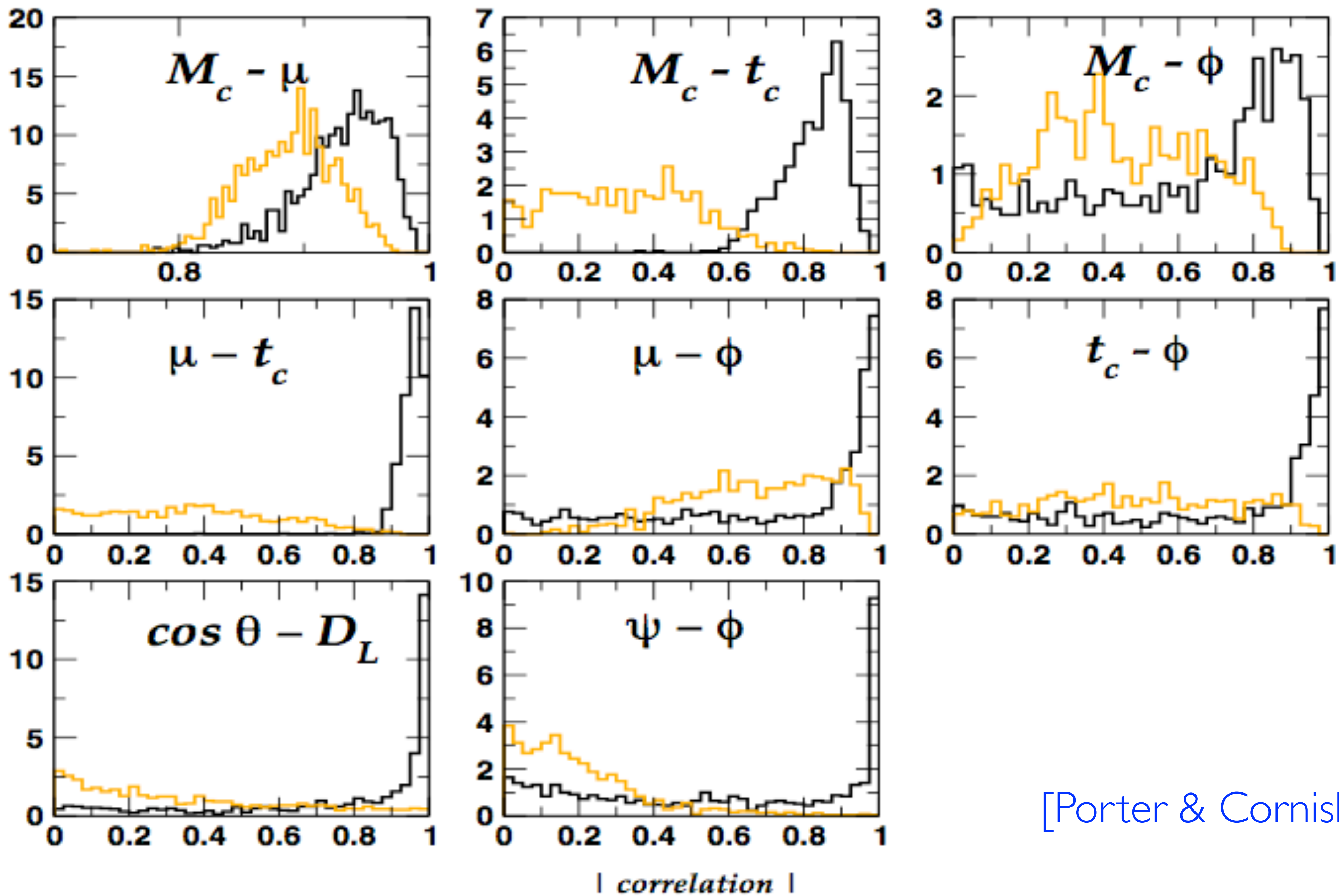


Next Steps

- Improve match by going to next order
- Include merger and ringdown using PhenomP
- Perform large-scale Bayesian analysis of eLISA systems

Extra Slides

Importance of Higher-Harmonics



[Porter & Cornish 2008]