Physical model of the LISA Pathfinder differential acceleration measurement and its application to LISA

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for the LPF science collaboration

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LISA Pathfinder acceleration noise budget

- Evidence for main noise sources
  - actuation and brownian noise

- «Known» noise budget and open questions for known noise sources

- Remaining challenges
  - Actuation calibration error
  - Low frequency noise
  - Glitches

- Application to LISA
LPF: Testing jump from pico-g/Hz^{1/2} to sub-femto-g/Hz^{1/2}:

Geodesy in low earth orbit
\((\mu m/s^2)\)

LPF at L1
\(nm/s^2\)

Are surface forces low enough to allow this jump?

- Heavy TM, 2 kg Au-Pt
- 3-4 mm gaps
- no contacts (no discharge wire)
- AC-carrier force actuation
- Vent to space (< 10 \(\mu Pa\))

- tough caging
- UV discharge
- need IFO

LPF small force test: pre-launch data

Torsion pendulum ground testing

- Surface forces: factor 2 of LPF goal at 1 mHz
- Individual noise sources even better
- Not sensitive to all forces
- No 6+ DOF control
- Not final environment
- Not end-to-end
LPF differential acceleration ($\Delta g$) noise floor

- Noise non-stationary ... improving!
- How much do we understand? Can we reproduce?
- Things could get worse
LPF differential acceleration ($\Delta g$) noise floor

- Actuation calibration errors (we think...)
- Actuation gain fluctuations
- Brownian motion (residual gas)
- IFO (+ coupling to SC motion)

Frequency [Hz]

$[m \cdot s^{-2} \cdot Hz^{-1/2}]$
Evolution of noise and «quasi-DC» $\Delta g$

LPF Noise: gravitational balance and actuation gain fluctuations

«accelerometer dynamic range» problem

Noise in “DC” force applied to compensate local $\Delta g$

\[ F \propto V_{ACT}^2 \quad \rightarrow \quad S_{a}^{1/2} \approx 2 \Delta g \cdot S_{\delta V/V}^{1/2} \]

- **FEE actuators:**
  Measured stability 3-8 ppm/Hz$^{1/2}$

- **LPF designed for $\Delta g$ 650 pm/s$^{2}$**
  - **Budget:** 10 fm/s$^{2}$/Hz$^{1/2}$ at 1 mHz

- **LPF in-flight $\Delta g < 50$ pm/s$^{2}$**
  \[ \rightarrow \text{Excellent gravitational balancing} \]

See: talk Trenkel and poster Ferroni

Acceleration noise from actuation gain fluctuations

- Same electrodes used to apply both force and torque
- Constant stiffness $\rightarrow$ pull on both sides of TM
- (mostly) uncorrelated fluctuations between electrodes

$$
\delta g = 2 \left( g_1 \alpha_1 + g_2 \alpha_2 + g_3 \alpha_3 + g_4 \alpha_4 \right)
$$

Gain fluctuations in 4 electrode actuators
Modeling, measuring and projecting actuation gain noise

Apply large actuation forces (without crashing TM into SC) to accentuate effect of actuation noise in $\Delta g$ measurement

Blue bars: electrode force vectors for force
Red bars: electrode force vectors for torque

Standard «URLA» 50 pN authority
(10 pN net force TM2)

Big (5 nN) force authority

Big (4 nN) common mode force
Actuation measurement campaign: results
(fit of model to both noise in $\Delta g$ and $\Delta \gamma_\phi$)

- Detect actuation gain noise
- Dominated by uncorrelated gain fluctuations
- Agreement with ground testing
  $\rightarrow$ Noisier amps (positive forces) on ground are noisier in space!!!
Actuation gain fluctuations: projection to noise in $\Delta g$

- Actuation gain noise not dominant in lowest actuation authority (URLA)
  - Typically $4 \text{ fm/s}^2/\text{Hz}^{1/2}$ at 0.1 mHz
  - Dominated by uncorrelated fluctuations and applied $\phi$ torques

- Agreement with ground model
  - FEE actuation noise as expected
  - LPF low noise $\rightarrow$ thanks to LPF gravitational balancing

See: talk Ferraioli (FEE performance), poster Mance (FEE for LISA)
White noise 3-8 mHz: Brownian motion from residual gas

Evidence for Brownian noise:

• 1/t-t0 fit with t0 = 3 Feb 2016 (venting day)
• roughly compatible with radiometric pressure
• Increases with temperature
  → «event» around 225 days after launch (NASA thruster turn on)

• Now below LISA spec ... will it continue to decrease?
  → Fit «saturates» at 2 fm/s^2/Hz^{1/2} ...

See talk: Rita Dolesi
Correlation between Brownian noise and «quasi-DC» $\Delta g$

$\Delta g \rightarrow$ average $\Delta p$ across TM

Possible link between quasi-DC force and Brownian pressure

- similar time, temperature response
- Implies 1 per 1000 imbalance in outgassing / conductance across TM
LPF noise budget: stray electrostatic forces

Interaction between TM charge and stray electrostatic fields

- **Measured DC stray fields**
  - $dF/dq$ with UV charge burst
  - Order 10’s mV (as on ground)
  - Compensated with applied voltages

- **Measured stray field fluctuations**
  - Noise with charged TM
  - Similar to ground upper limits

- **Measured TM charge fluctuations**
  - Noise with large DC E field
  - Dedicated long term charge measurement

See talk Peter Wass
LPF noise budget: «explained» noise, 16 May 2016

NB: Charge noise interaction with DC bias off the chart small (compensation)
LPF noise budget: what’s missing here

- Noise budget curve only shows consolidated sources
- Avoid overestimating «known» noise where we have (only) upper limits

- **Magnetic noise**
  Measured susceptibility, B noise
  Working on DC gradients

- **Thermal gradient noise**
  Measured dF/dΔT
  Working on noise in ΔT

- **Crosstalk**
  IFO (debump) + cross-stiffness
  Actuation to be revisited

Talk Miquel Nofrarias
Talk Rita Dolesi, poster Ferran Gibert
Evolution of low frequency noise and slope \(\frac{d\Delta g}{dt}\)

- Ave PSD(Dg), 0.1-0.4 mHz
- Ave PSD(Dg), act gain noise, 0.1-0.4 mHz
- LISA Requirement

Origin: 2015-12-03 04:04:00.000 - Time [D]
Observed correlation between $d\Delta g/dt$ and low frequency noise

NB: noise from actuation gain fluctuations removed
LPF low frequency noise and drifting $\Delta g$: actuation?

- Observed correlation between $d\Delta g/dt$ and $S_{\Delta g}$ at low frequencies

- Need to accurately subtract changing actuation force:

$$\delta (\Delta g) = \frac{1}{M} \left( F^{CMD}_{x2} - F^{ACT}_{x2} \right)$$

- Actuation «LSB» – 153 µV in Volts $\rightarrow$ roughly 10 fm/s$^2$ in $\Delta g$

- Typical (big) drifts: 500 fm/s$^2$ / day $\rightarrow$ 1-2 bits / hour

  - Accurate digitization «smoothed» by high-f $\Sigma–\Delta$ dither (force CMD)
  - Inaccurate digitization – even if stable – gives low frequency force noise

- Peak output changes by 1 LSB
- Analog output amplitude changes by 0.85 – 1.15 LSB
- FPGA «double digitization»

LPF low frequency noise and drifting $\Delta g$: actuation?

- 1 LSB change in peak amplitude produces 0.85-1.15 LSB change in effective amplitude (typical 10 µV errors)
- Observed in lab (prototype FEE) and by simplified analysis

- Could explain our low frequency noise increase with drift

One possible actuation scheme

Need to find true code:

$$V_x^{CMD} \rightarrow V_x^{ACT}$$
Low frequency noise (10-100 µHz)  
Very interesting for LISA (last year of mergers)

Studying this with LPF requires:
Subtraction accuracy
• Centrifugal
• Actuation (digitization)
• Charge and thermal
• Other?

LPF challenges: glitches

- Roughly 1-2 dozen unexplained events in $\Delta g$ in 2 month noise dataset
- Most fast (minutes), a few long (hours)
- Under investigation ... for observatory, want to discriminate!
Physical model of the LISA Pathfinder differential acceleration measurement

- Meeting LISA goal down to 1 mHz
- Most of noise in most of LISA bandwidth understood
- Low frequency tail (and time evolution) under study

Extra
(section 4D)

Actuation contribution to noise in $\Delta_x$ Measurements with charged TM (1-4 May 2016)

- Coupling of stray fields ($\Delta_x$) to TM potential:

$$F_x = -\Delta_x \left. \frac{\partial C_x}{\partial x} \right|_{V_{TM}}$$

- Contribution of actuation voltages to $\Delta_x$:

$$\Delta^{ACT}_{x} = V_{EL\ 1} + V_{EL\ 2} - V_{EL\ 3} - V_{EL\ 4}$$

- Noise in $\Delta g$ as function of TM potential allows measurement of noise in $\Delta_x$
Measured noise in $\Delta_x$ looks compatible with actuation noise from FEE

- NB 4 electrodes contribute, assumed here to sum incoherently
- Important for surface electrostatics fluctuations (compatible with zero)
- Reference is breadboard level testing, S2-HEV-RP-3042, page 43
Evolution of low frequency noise and «quasi-DC» $\Delta g$
Actuation voltage inaccuracies: ELM-light test

Scanning 60 Hz VAC-x1 actuation command
- 50 μV step every 20 s

Green: 152.588 μV staircase

Blue: lock-in measurement of demodulated 60 Hz x VAC

Observation: not all LSB jumps are same size (120-180 μV)
- Repeatable on same electrode
- **Repeatable also on other +X electrode**
  - big and small jumps occur for same voltage commands
  - systematic digitization issue
Is 10 μV RMS error to be expected?

Actuation waveform generation

First digitization steps:
1. 16-bit digitization (153 μV) of actuation peak amplitude (10 Hz)
2. Multiplication by 16-bit sine look-up table and truncation
   • 153 μV, 16-bit + sign
   • 12 kHz sampling (x waveform at 60 Hz, 200 pts / cycle)
3. Summing with ϕ waveform and DC voltages
4. Rescaling of bits to match feedback ADC
5. Interpolation to 96 kHz (adds 3 bits between 16 bit levels)

What happens in first two steps?
Simplified calculation for first two digitization steps

Digitize peak amplitude

\[ n_{\text{LSB}} = \text{ROUND} \left( \frac{V_x}{\text{LSB}} \right) \times \text{ROUND} \left( 2^{16} \sin \omega_x t_j \right) \]

16-bit sine LUT

Truncation to 153 µV LSB

NB: it does not do this:

\[ n_{\text{LSB}} \neq \text{ROUND} \left( \frac{V_x \sin \omega_x t_j}{\text{LSB}} \right) \]

- Simple calculation implemented with matlab
  
  Input peak amplitude → target waveform → effective amplitude

- Some uncertainty (use of «round» or «floor»)
- Further stages (including interpolation) can change but not improve non-uniformity of effective force LSB
«Corrected» data, 19 June 2016
5 day measurement with roughly -0.5 pm/s$^2$/day

Visible error of order fm/s$^2$, changing as expected on 2000-5000 s time scales
Typical LPF $\Delta g$ time series: comparison with SMBH merger
Typical LPF $\Delta g$ time series: comparison with SMBH merger

With LPF level of acceleration noise, can resolve every cycle «by eye» a week before merger

[Waveform thanks to Antoine Petiteau, APC]