

# The Fast and the Fiducial

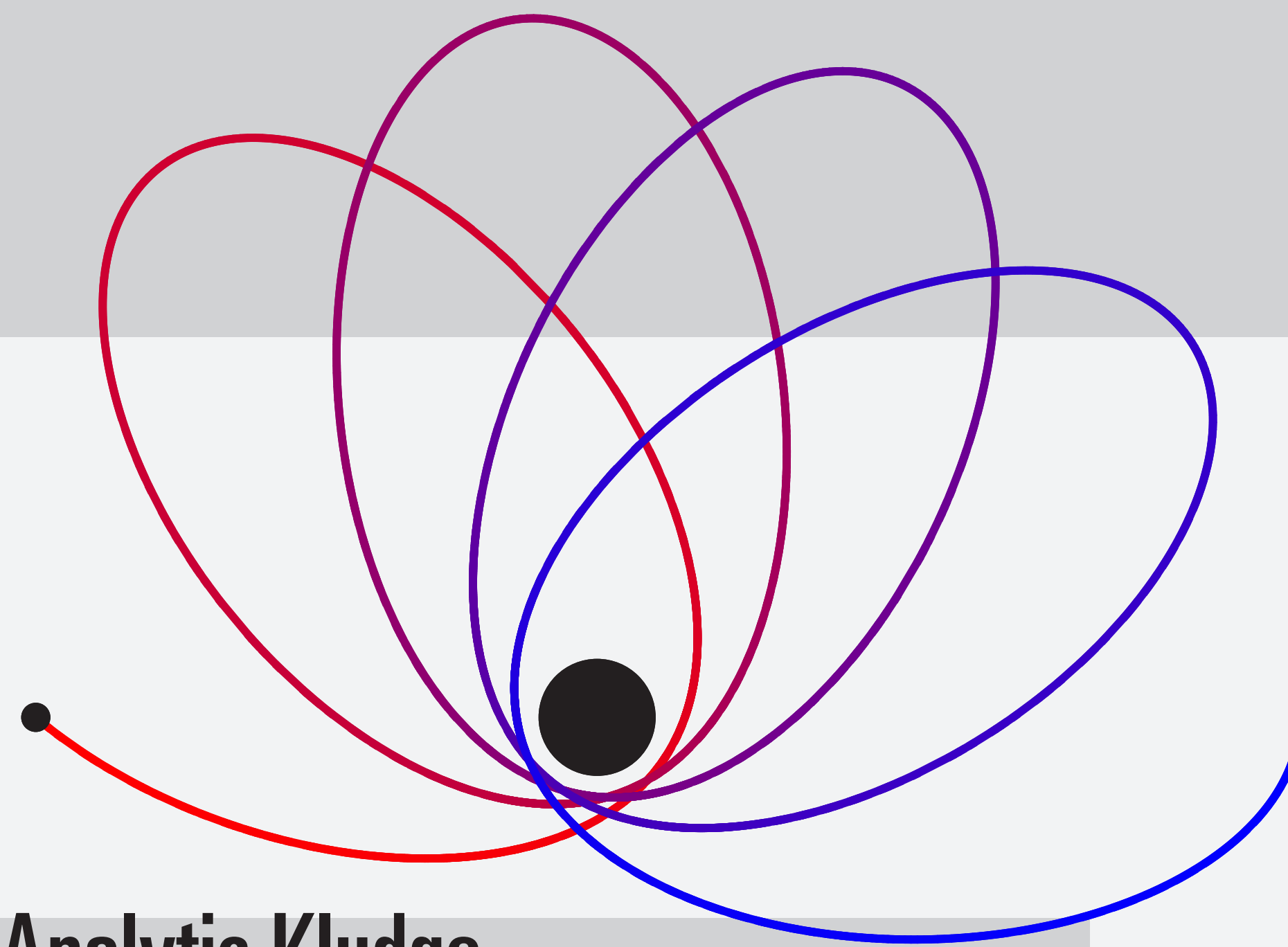
## Improving kludge waveforms for EMRIs

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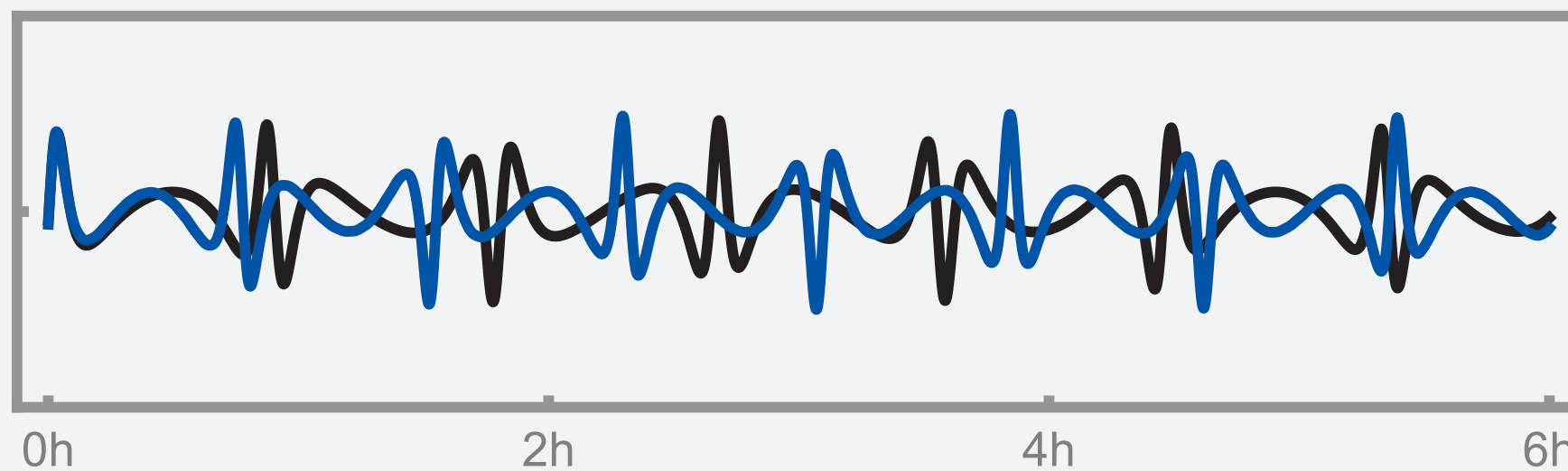
### Overview

EXTREME-MASS-RATIO INSPIRALS will be an important class of source for LISA, but present severe challenges in data analysis due to the length and complexity of their gravitational-wave signals.

We introduce an augmentation of Barack and Cutler's analytic kludge EMRI model with a frequency map and other improvements [1,3,4]. It is hoped that the augmented model will be fast and accurate enough to provide short-duration detection templates in a hierarchical search.

After sources are localised in parameter space, longer self-force waveforms are needed to measure their parameters precisely. We propose a technique that uses Gaussian process regression to include information from fiducial waveforms when searching with faster but less accurate ones [2–4].

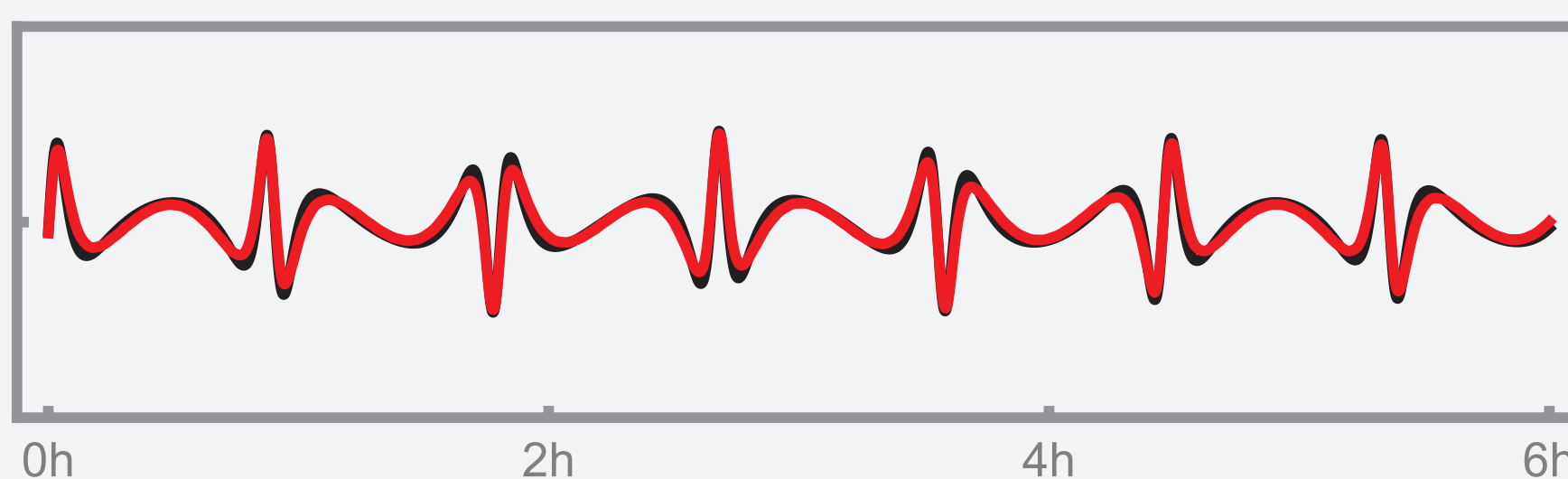
### Detection: Augmented Analytic Kludge



Analytic kludge model builds orbital trajectory out of precessing Keplerian ellipses; its waveforms are fast to generate but dephase relative to more accurate waveforms within hours

Matching Keplerian, periastris precession and Lense–Thirring frequencies of each ellipse to **fundamental frequencies** of corresponding Kerr geodesic induces three-dimensional map over space of orbits; we map black-hole mass and spin, along with orbital semi-latus rectum

$$(M, a, p) \mapsto (\tilde{M}, \tilde{a}, \tilde{p}) \quad (\omega_{\text{Kep}}, \omega_{\text{peri}}, \omega_{\text{LT}}) \Big|_{(\tilde{M}, \tilde{a}, \tilde{p})} = (\omega_r, \omega_\phi - \omega_r, \omega_\phi - \omega_\theta) \Big|_{(M, a, p)}$$



Augmented analytic kludge uses local fit to mapped trajectory of fiducial model and higher-order post-Newtonian equations; its waveforms are still fast to generate but now dephase over months

Source code for the augmented analytic kludge and other kludge waveforms (with shared front end) is available at [github.com/alvincjk/EMRI\\_Kludge\\_Suite](https://github.com/alvincjk/EMRI_Kludge_Suite)

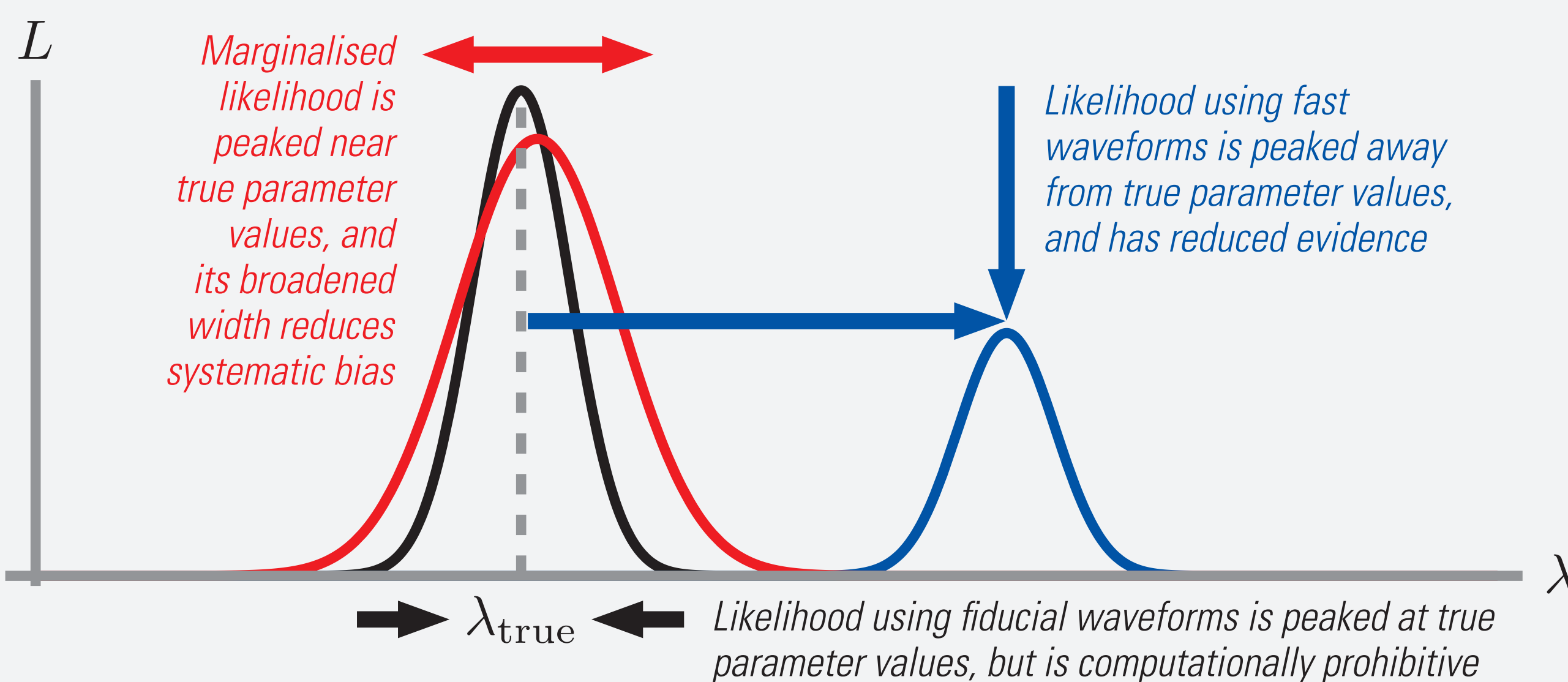
### Parameter Estimation: Gaussian Process Regression

Difference between fast and fiducial waveforms is modelled by zero-mean **Gaussian process** and interpolated from set of pre-computed differences in relevant region of parameter space

Waveform difference is marginalised over analytically in likelihood; **marginalised likelihood** is computationally affordable as it only uses fast waveforms and Gaussian process interpolant

$$h_{\text{fid}} - h_{\text{fast}} = \delta h \sim \mathcal{N}(\mu_{\text{GP}}, \sigma_{\text{GP}}^2)$$

$$L \propto \frac{1}{1 + \sigma_{\text{GP}}^2} \exp \left( -\frac{1}{2} \frac{\langle s - (h_{\text{fast}} + \mu_{\text{GP}}) | s - (h_{\text{fast}} + \mu_{\text{GP}}) \rangle}{1 + \sigma_{\text{GP}}^2} \right)$$



### References

- [1] A. J. K. CHUA & J. R. GAIR. Improved analytic extreme-mass-ratio inspiral model for scoping out eLISA data analysis. *Class. Quantum Grav.* 32:232002, 2015.
- [2] C. J. MOORE, C. P. L. BERRY, A. J. K. CHUA & J. R. GAIR. Improving gravitational-wave parameter estimation using Gaussian process regression. *Phys. Rev. D* 93:064001, 2016.
- [3] A. J. K. CHUA. Augmented kludge waveforms and Gaussian process regression for EMRI data analysis. *J. Phys.: Conf. Ser.* 716:012028, 2016.
- [4] A. J. K. CHUA, C. J. MOORE & J. R. GAIR. The fast and the fiducial: Improving kludge waveforms for extreme-mass-ratio inspirals. In preparation, 2016.

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