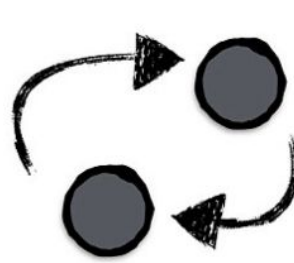


# Kicked Waveforms

Observing Black Hole Merger Kicks  
Directly With Gravitational Waves



**Christopher Moore, Cambridge, UK**  
**11th International LISA Symposium**

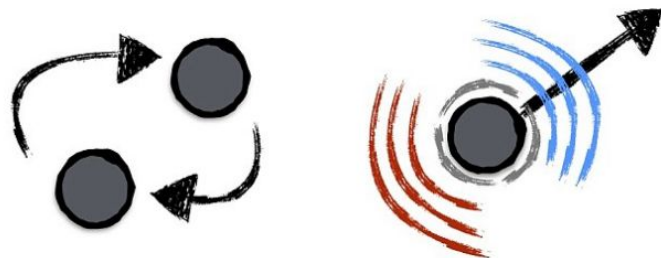
**08/09/2016**

Gerosa and Moore, Physical Review Letters **117** 011101 (2016), arXiv:1606.04226

Work done in collaboration with Davide Gerosa and Riccardo Barbieri

# Outline

- Introduction to black hole kicks
- Modelling the kick in the GW signal
- Detecting the kick: prospects for LISA
- Parameter estimation with kicks
- Concluding remarks



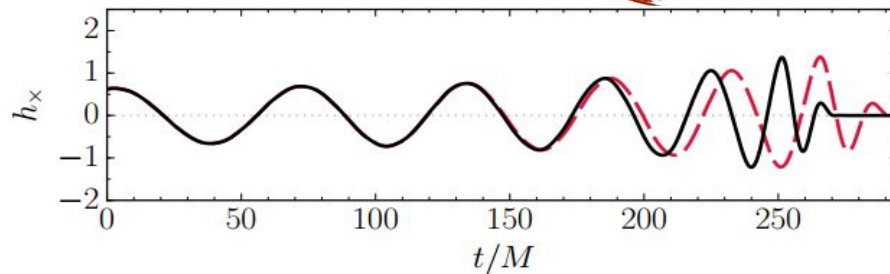
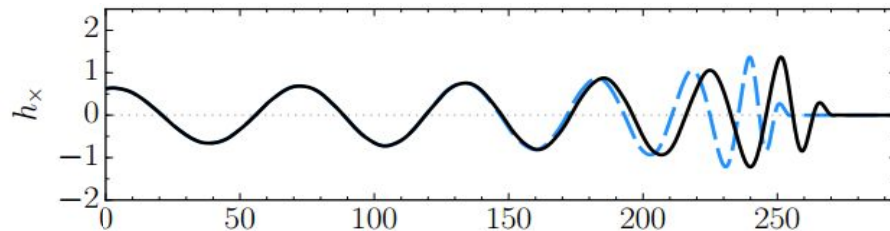
# Black hole kicks

Merging BHs recoil due to asymmetric GW emission in final stages of merger

Recoil, or **kick**, velocities of up to 5000 km/s are possible, which is sufficient to eject the remnant from the galaxy

Smaller kicks can also have astrophysical consequences as the BH displacement transfers energy to the stars in the galactic nucleus

Kicks leave an imprint on the GW signal via a Doppler shift of the final few wave cycles



# Black hole kicks

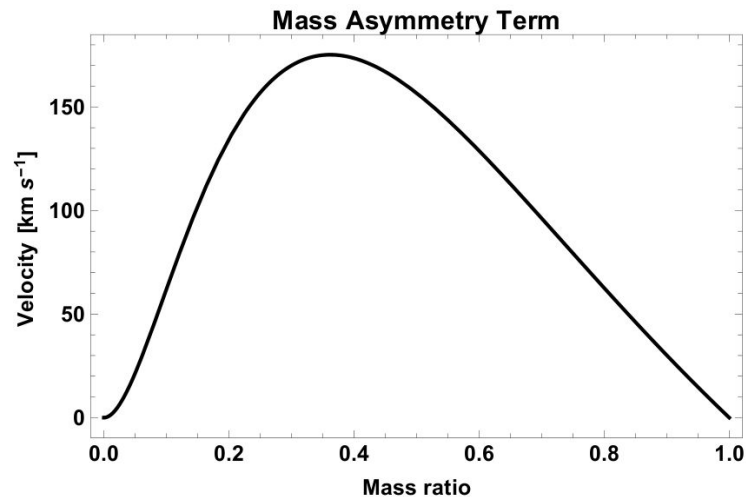
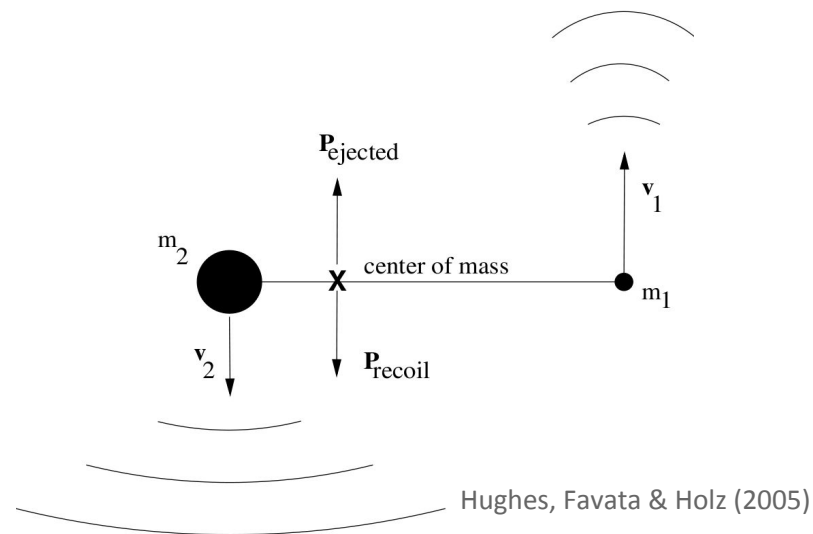
It is the presence of asymmetry in a BH binary which leads to the kick

## Mass asymmetry:

Smaller object moves faster than larger object, so emitted waves can be considered as being “beamed” in the forward direction

Mass ratios of  $0.3 \lesssim q \lesssim 0.4$  produce the largest **mass kicks** around 160 km/s

J. A. González et al. (2007)



# Black hole kicks

It is the presence of asymmetry in a BH binary which leads to the kick

## Spin asymmetry:

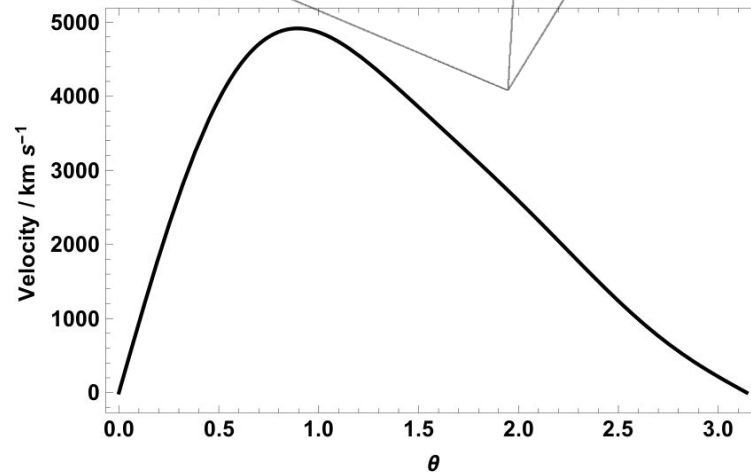
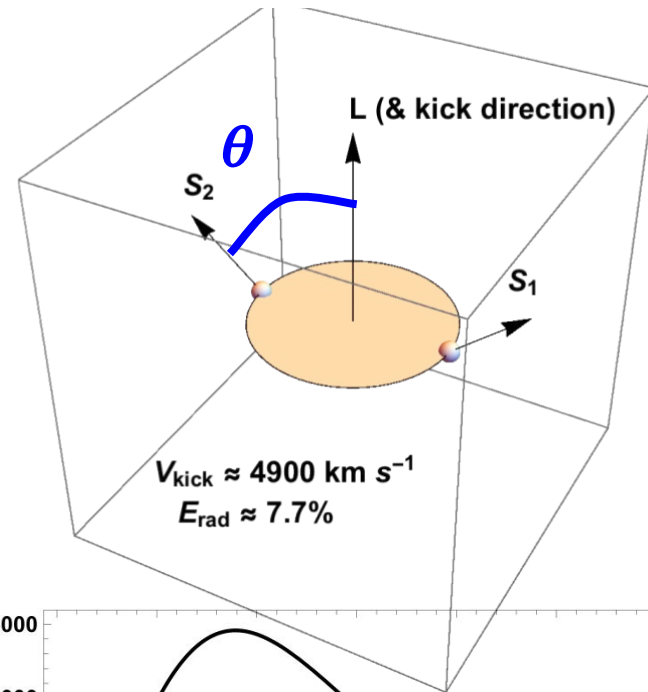
Spin asymmetry can lead to larger kicks, especially if the spin components in the orbital plane are antiparallel

The **superkick** configuration ( $\theta = \pi/2$ ) has the spin vectors confined to the orbital plane and produces kick speed up to  $\approx 2500$  km/s

J. A. González et al. (2007) and M. Campanelli et al. (2007)

The **hangup kick** configuration ( $\theta = 0.28\pi$ ) produces the largest known kicks with speeds up to  $\approx 5000$  km/s

C. O. Lousto & Y. Zlochower (2011)



# Kick or push?

The kick is not instantaneous, velocity accumulates over a period of  $\approx 20M$  around merger

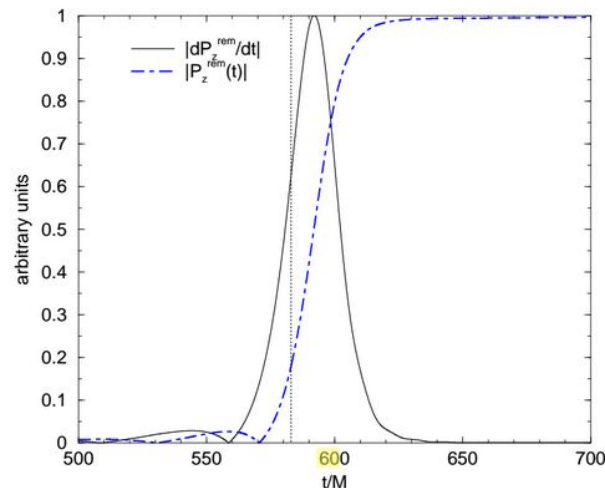
While quite a lot is known about the kick speed, relatively little is known about the kick acceleration profile

The small number of profiles reported in the literature are generally approximately **Gaussian** profiles

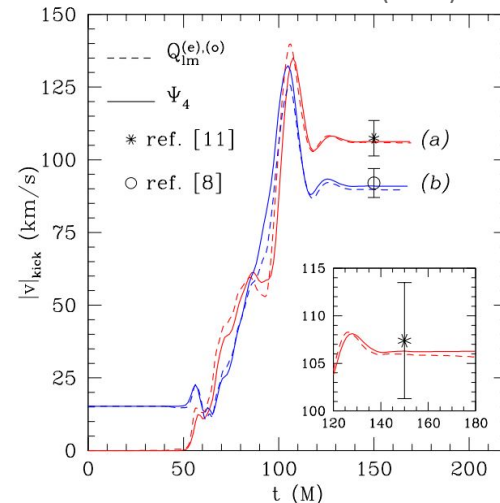
B. Brüggmann et al. (2008) and C. O. Lousto & Y. Zlochower (2008)

The one notable exception is the **antikick**, where there is some deceleration after merger; i.e. the acceleration profile changes sign

M. Koppitz et al. (2007) and L. Rezzolla et al. (2010)



Lousto & Zlochower (2014)



M. Koppitz et al. (2008)

# SXS kicks

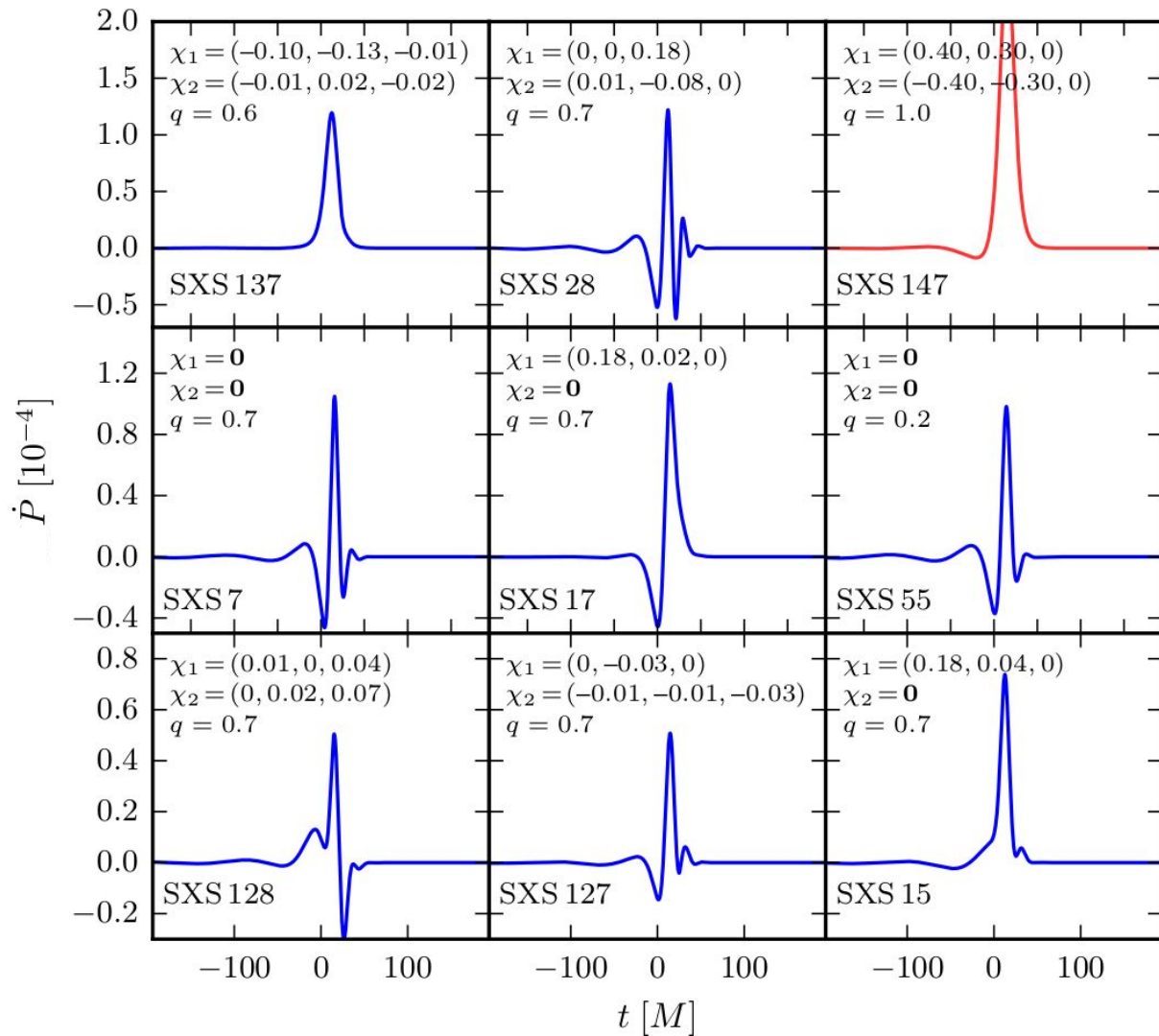
A selection of kick profiles from the public SXS catalog

The sources are observed along the direction of the kick

There is a lot of diversity in the kick profiles

Shown in red is the largest kick in the catalog, 1550 km/s

This binary has equal mass components each with dimensionless spin of 0.5, arranged in the *superkick* configuration



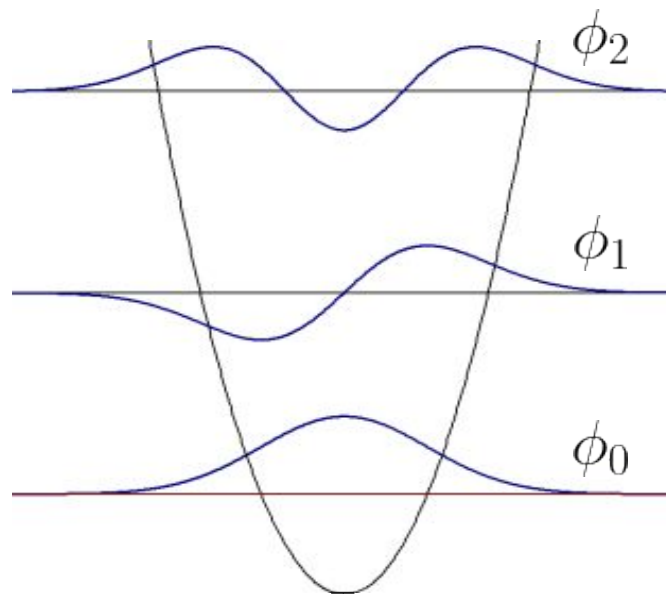
# Kick or push?

We adopt an agnostic approach to modelling the kick, expanding the acceleration profile as a combination of a simple set of basis functions

We choose a basis with the property that the first term models a **Gaussian** kick profile, while the second models an **antikick**

$$a(t) = \sum_{n=0}^N a_n \phi_n \left( \frac{t - t_c}{\sigma} \right)$$
$$\phi_n(x) = \frac{\exp\left(-\frac{x^2}{2}\right) H_n(x)}{\sqrt{2^n n!} \sqrt{\sigma^2 \pi}}$$

We treat  $\{\sigma, t_c, a_0, a_1, \dots, a_n\}$  as free parameters which are to be measured from the data



These are familiar from the solutions to the quantum harmonic oscillator



# Kicked waveforms

Start from a fast, frequency domain inspiral-merger-ringdown model, allowing for precessing systems: **IMRPhenomP**

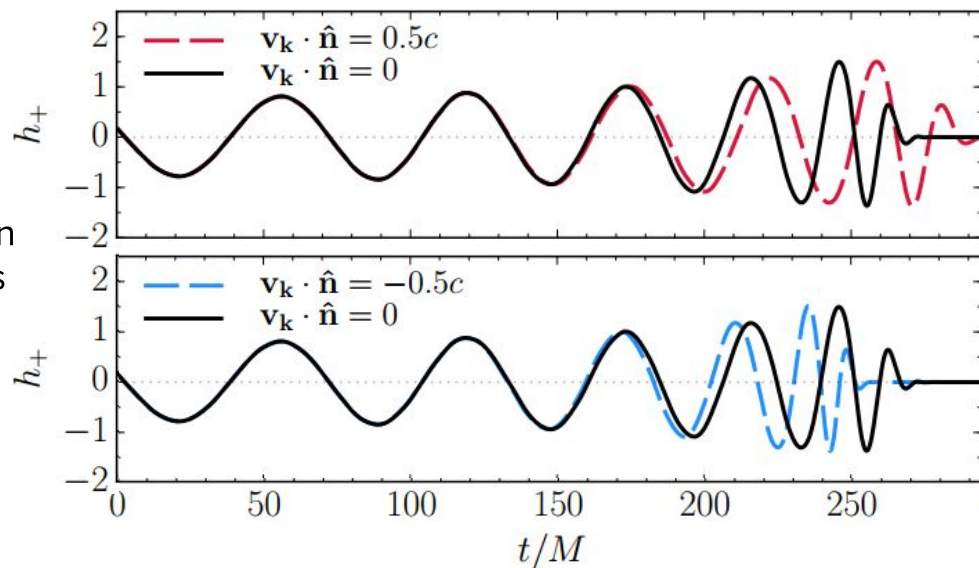
M. Hannam, et al. (2014)

Binary properties are specified at some low  $f_{\text{ref}}$ , spin configuration is evolved to small orbital separations

Fitting formulae, tuned to NR simulations at small separations, are used to find the kick speed

D. Gerosa & M. Kesden (2016) and references therein

The system is evolved again, with the kick included, to obtain the GW signal. The effect of the kick is to stretch or compress the final few wave cycles



Gaussian kick, with total speed  $v_k = 0.5c$ , and duration  $\sigma = 60M$   
(Unphysically large for clarity)

# Detecting the kick

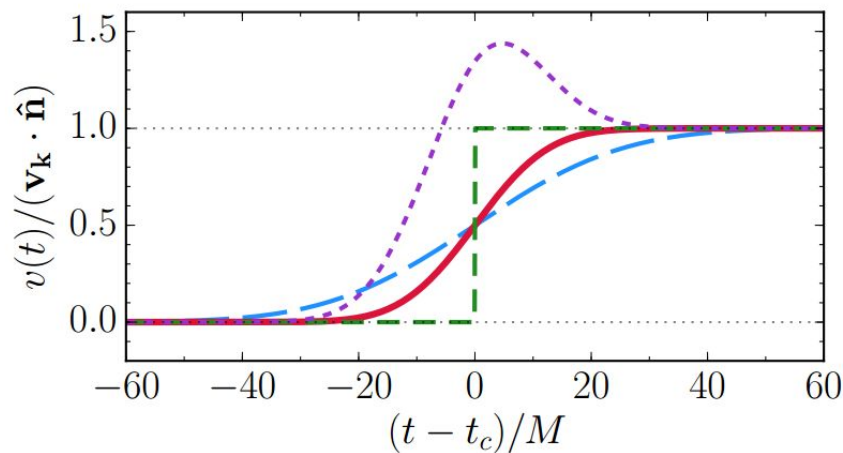
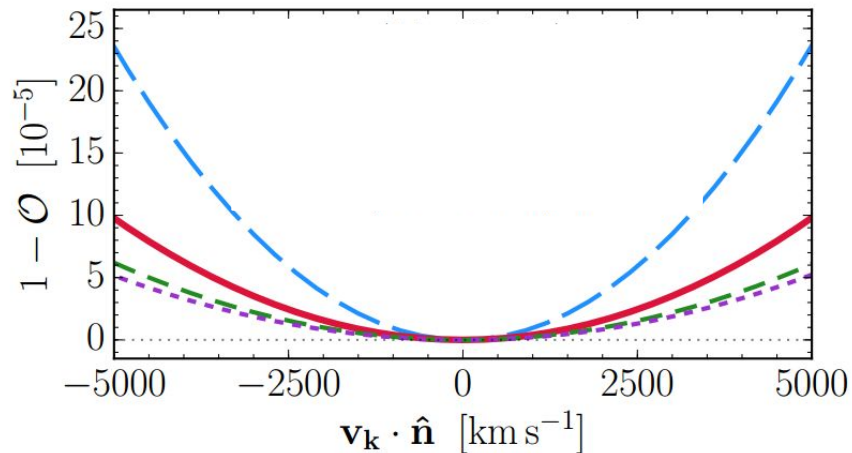
The **un-kicked** ( $h_0$ ) and **kicked** ( $h_k$ ) waveform models have a **mismatch** which depends strongly on the kick speed, but also on the shape of the kick profile

$$\text{Mismatch: } 1 - \mathcal{O} = 1 - \max_{t_c, \phi} \left\{ \frac{\langle h_0 | h_k \rangle}{\sqrt{\langle h_0 | h_0 \rangle \langle h_k | h_k \rangle}} \right\}$$

The signal-to-noise ratio in the **ringdown** ( $\rho_r$ ) portion of the waveform governs the detectability of the kick

A back-of-the-envelope calculation yields the following simple criteria for the kick to be detectable

$$\rho_r \geq \frac{0.322}{v_k/c}$$



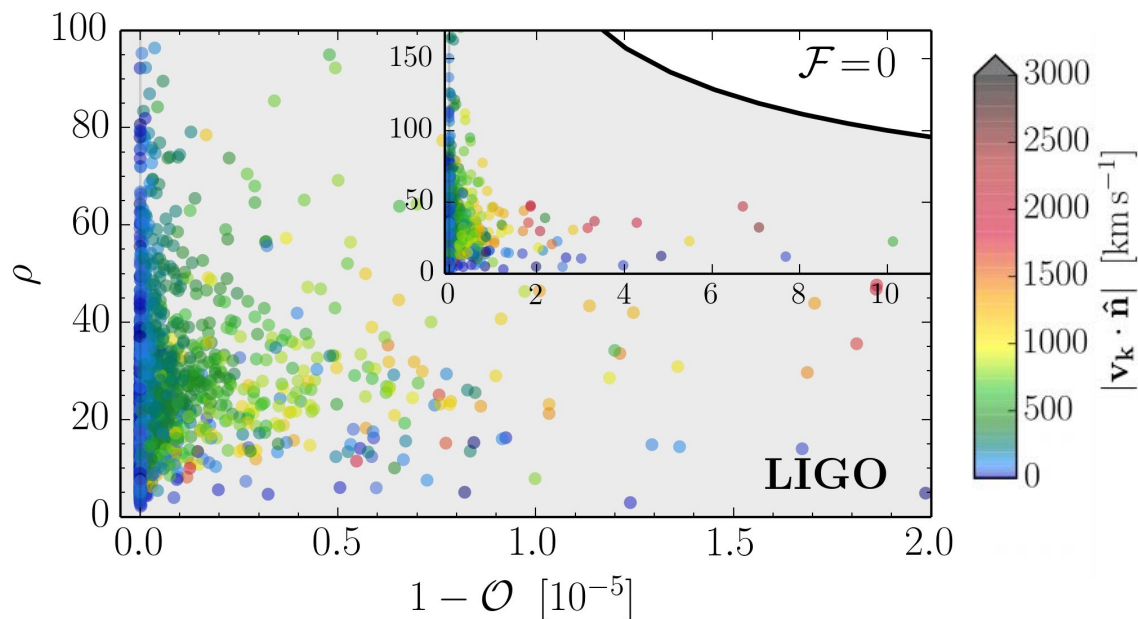
# Detecting the kick

Generate a simple population of binary BHs for advanced LIGO

Reference frequency	$f_{\text{ref}} = 20 \text{ Hz}$
Total mass	$M \in [10, 100] M_{\odot}$
Mass ratio	$q \in [0.05, 1]$
Spin magnitudes	$\chi_1, \chi_2 \in [0, 1]$
Comoving distance (distributed uniformly in comoving volume)	$D_c \in [0.1, 1] \text{ Gpc}$
Spin orientations	Isotropic

The shaded region indicates systems with **undetectable** kicks, assessed using a simple criteria based on the mismatch

$$1 - \mathcal{O} \geq \rho^{-2}$$



Assuming "Zero-Det-High\_P" sensitivity curve  
from <https://dcc.ligo.org/LIGO-T0900288/public>

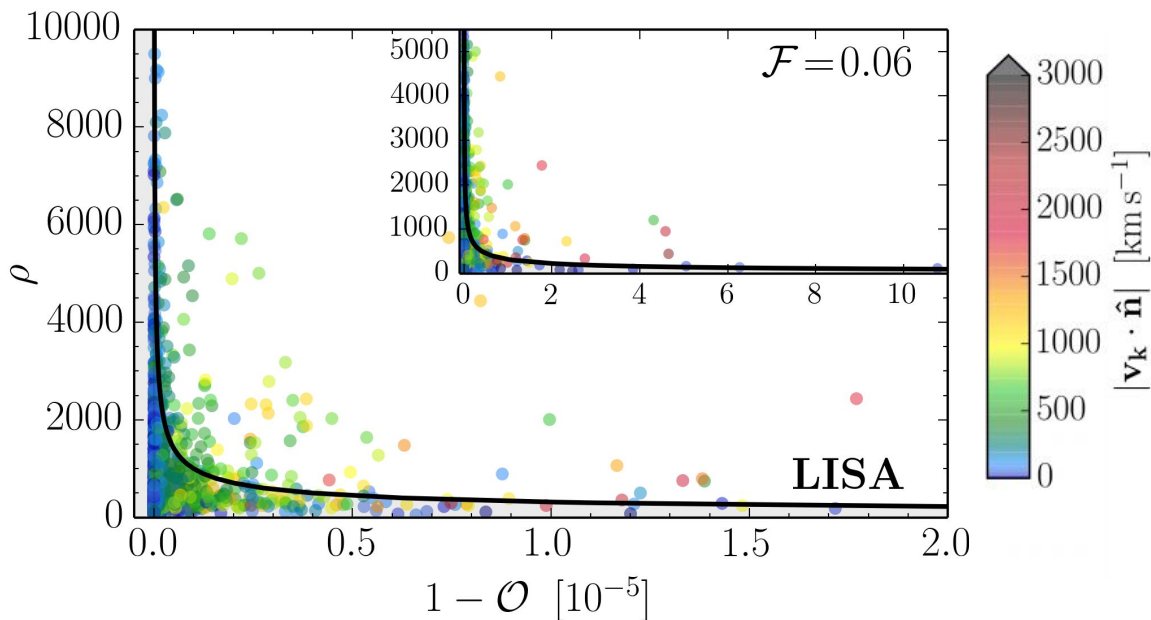
# Detecting the kick

Generate a simple population of binary BHs for advanced LISA

Reference frequency  $f_{\text{ref}} = 2 \text{ mHz}$   
Total mass  $M \in [1, 10] \times 10^5 M_{\odot}$   
Mass ratio  $q \in [0.05, 1]$   
Spin magnitudes  $\chi_1, \chi_2 \in [0, 1]$   
Comoving distance  $D_c \in [1, 10] \text{ Gpc}$   
(distributed uniformly in comoving volume)  
Spin orientations Isotropic

A significant fraction of binaries have detectable kicks  
(6% for this simple population)

LISA may detect kicks in as many as  $\approx 30$  binary BH systems



Assuming "N2A5L6" sensitivity curve from  
A. Klein et al. *Phys. Rev. D* 93, 024003 (2016)

# Parameter estimation with kicks

Using the mismatch as a criteria for detecting the kick assumes the kick is not degenerate with any other parameters in the system

We expect this to be the case because all other system parameters affect inspiral and merger, while the kick only affects the merger

To address this more carefully, we performed a Fisher matrix (**FM**) analysis for a **Golden System**:  
 $m_1=m_2=1.3\times 10^6 M_\odot$ , extremal spins, superkick config, kick directed away from Earth, signal-to-noise ratio  $\rho = 10^4$

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We describe the kick by 3 parameters: **speed**  $v_k$ , **duration**  $\sigma$ , and size of the **antikick** component  $\alpha \equiv a_1/a_0$

FM analysis found no strong degeneracies between the kick parameters and the usual binary parameters

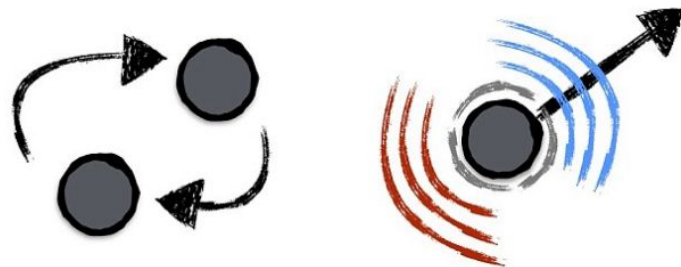
FM analysis also provides estimates for the errors on the kick parameters

$$\Delta v_k \approx 200 \text{ km s}^{-1}$$

$$\Delta \sigma \approx 1 M$$

$$\Delta \alpha \approx 10\%$$

# Conclusions



Shown how BH merger kicks may be included into existing GW waveform models

Used these kicked waveform models to assess the detectability of kicks for several detectors

Demonstrated that a few percent of supermassive binary BHs observed with LISA can be expected to have detectable kicks

For a golden system LISA can detect kicks as small as 200 km/s, which are expected to be common, can detect presence of an anti-kick at the 10% level, and measure kick duration at the level of  $1.0M$

Modelling the kick in greater detail will be necessary for the detailed analysis of the merger and ringdown of the loudest binary BHs observed with LISA

Gerosa and Moore, Physical Review Letters **117** 011101 (2016), arXiv:1606.04226

**Thank you for listening**