Precision Measurement of Planetary Gravitomagnetic field and Laser Interferometry in Space

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1. Motivations and Preliminary Mission Concepts

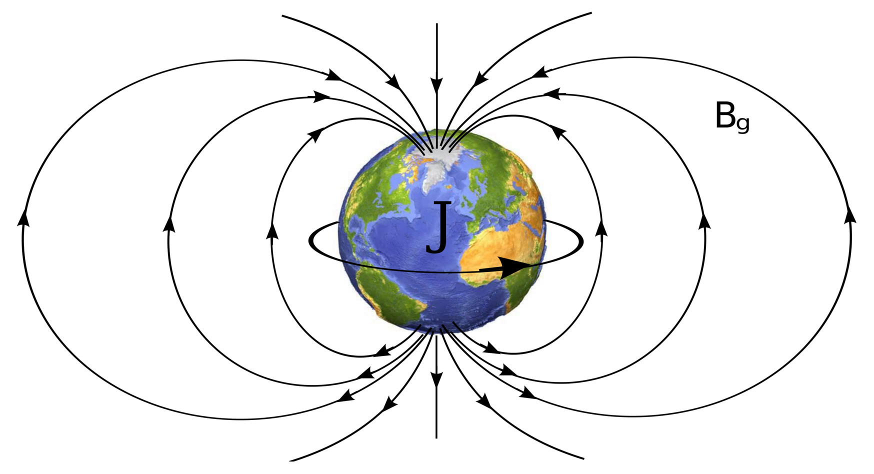
Possible future TechDemoSat for the planned Chinese space-borne gravitational wave antenna.

- ***A differential measurement of Earth's gravitomagnetic field predicted by Einstein's GR to unprecedented accuracy better than 1%.***
- Improve the accuracy in the measurement of some post-Newtonian parameters.
- Track the temporal variation of the Earth gravity field.
- Set constraints on low energy effective theory related to string theory and quantum gravity, such as Chern-Simons gravity and torsion gravity.

Precision measurement of the GravitoMagnetic (Frame-Dragging) effects as one of the outstanding tests of GR in the 21st century

- Poorly tested, remained the major challenge in experimental relativity.
- Related to fundamental issues such as the origins of inertial and etc..
- Applications to future space science such as the determinations of inertial frames, synchronizations of clocks in deep space and etc..

In weak field and slow motion limits $\frac{GM}{c^2 r} \sim \frac{v^2}{c^2} \sim \mathcal{O}(\epsilon^2)$, there exists rich correspondences between electrodynamics and GR.



$$\nabla \cdot \frac{1}{2}\mathbf{B}_g = 0, \quad \nabla \times \frac{1}{2}\mathbf{B}_g = \frac{\partial}{\partial t}\mathbf{E}_g - 4\mathbf{j}.$$

Preliminary Mission Concepts

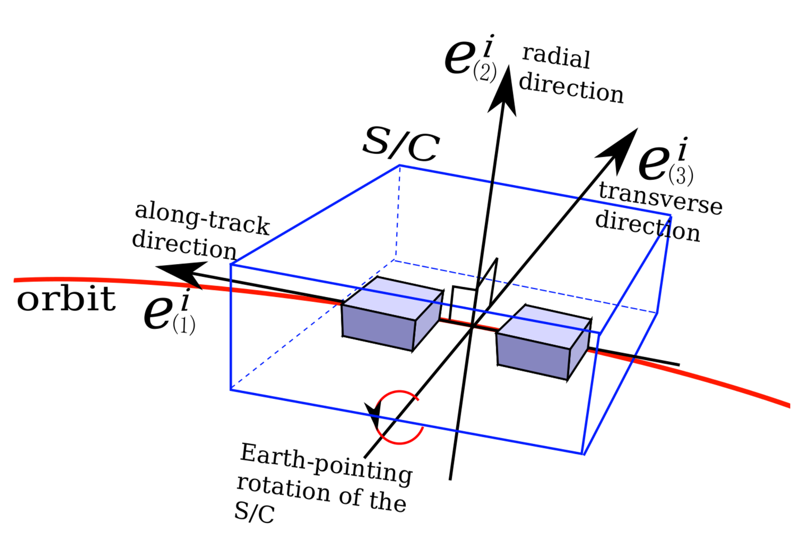
- Near Polar orbit with altitude about **2000*km***.
- Freely-falling spacecraft in the Earth pointing orientation.
- Two drag-free TMs located at the along track direction with distance about **50*cm***.
- On-board laser interferometers as read out system.
- Two TMs located at transverse direction with distance about **50*cm*** to remove errors caused by jitters or random rotations of the SpaceCraft (S/C)about the radial axis.
- Attitude control.
- ***The gravitomagnetic signal s^{GM} in the transverse direction will reach a few nanometers in about two days operations***

2. Physical Picture

For the two drag-free TMs at the along-track direction along a nearly circular orbit.

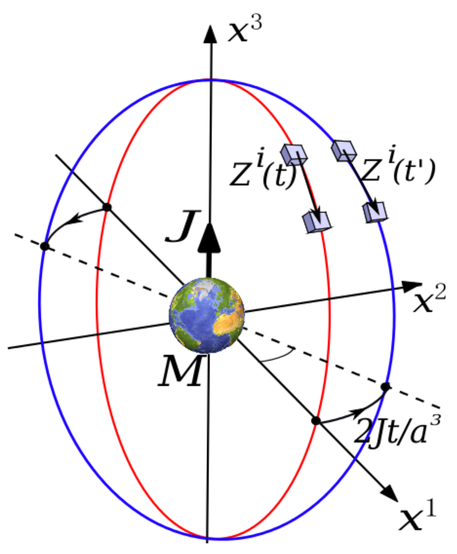
- The freely-falling S/C is given an initial angular velocity to maintain its Earth pointing orientation, which can be viewed as a gyroscope moving along the orbit.
- Due to the frame-dragging effect, the orientation of the S/C (a freely-falling gyroscope) will precess slowly about the Earth rotation axis with rate

$$\Omega^{S/C} = \frac{GJ\sin I}{2c^2a^3} + \mathcal{O}(J^2).$$



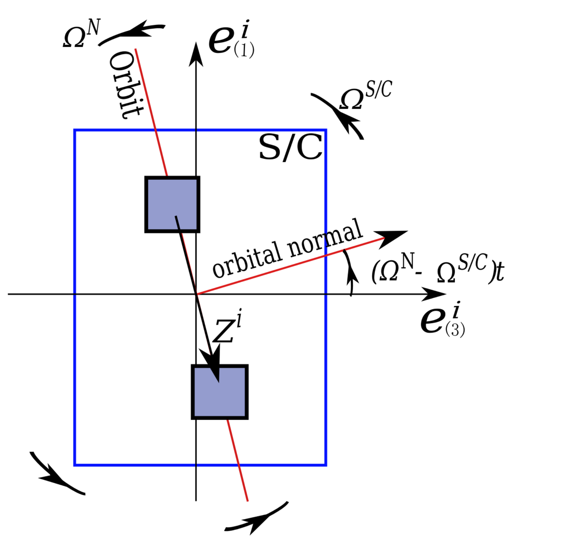
- The two drag-free TMs can be viewed as the two markers on the orbit. When the orbit precess slowly about the Earth rotation axis, the position difference vector Z^i will also precess with rate

$$\Omega^N = \frac{2GJ\sin I}{c^2a^3} + \mathcal{O}(J^2),$$

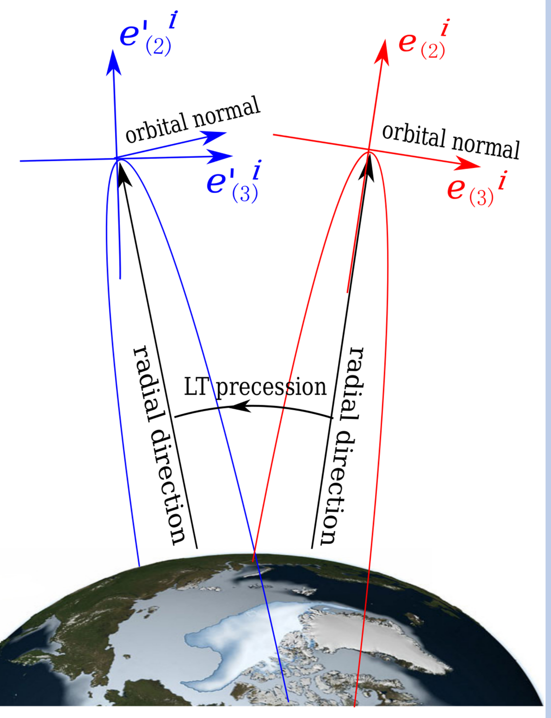


The existence of a constant offset between these two precessing rates will give rise to a relative oscillation between the two TMs along the transverse direction

$$s^{GM} \approx d\sin(\Omega^N t - \Omega^{S/C} t)\sin(\omega t) \approx \frac{3GdJt\sin I\sin(\omega t)}{2c^2a^3},$$



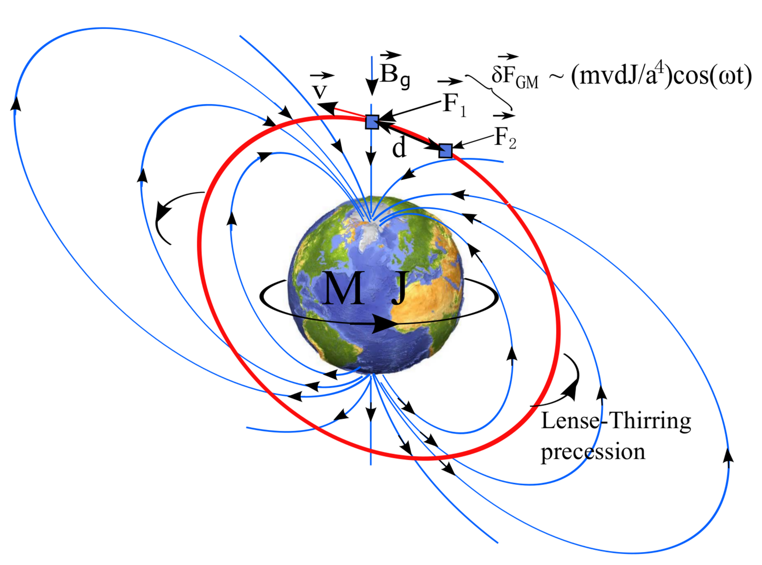
The growing oscillations along the transverse direction as a differential measurements of the GM effect.

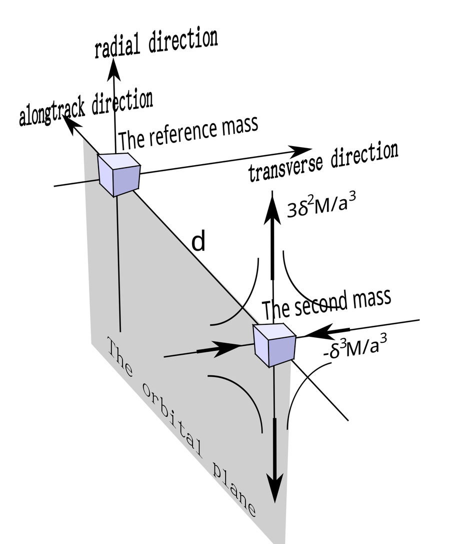


3. Mechanical Principle (I)

An orbiting proof mass m satisfies the PN equations of motion

$$m\frac{d^2\vec{x}}{dt^2} = -\frac{GmM}{r^3}\vec{x} + \frac{GmM}{c^2r^3}[(\frac{4GM}{r} - v^2)\vec{x} + 4(\vec{x}\cdot\vec{v})\vec{v}] + \frac{2Gm\vec{v}}{c^2} \times [\frac{\vec{J}}{r^3} - \frac{3(\vec{J}\cdot\vec{x})\vec{x}}{r^5}]$$





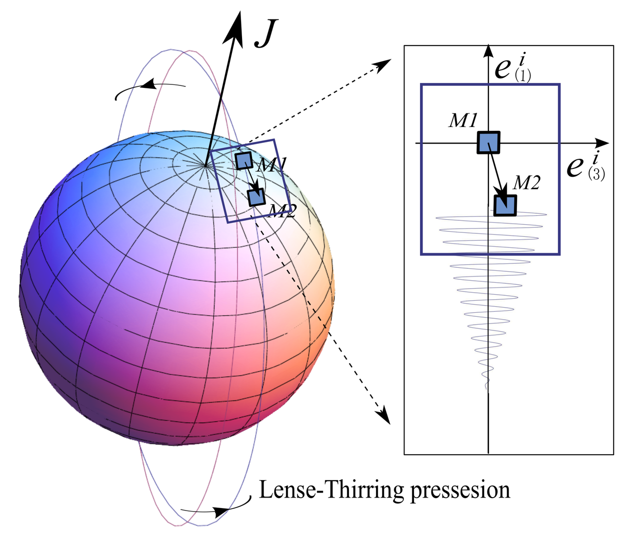
$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{3GM}{a^3} & 0 \\ 0 & 0 & -\frac{GM}{a^3} \end{pmatrix}$$

The GM force $\vec{F}_{GM} = -2m\frac{\vec{v}}{c} \times \vec{B}_g$ contributes the only transverse perturbation along $e_{(3)}{}^i$. ***Their gradient between the two TMs reads***

$$\delta\vec{F}_{GM} \sim \frac{d}{a}\frac{GmJv}{c^2a^3}\sin I\cos(\omega t),$$

whose frequency matches that of the natural frequency of the relative motions along the transverse direction. This gives rise to an resonant oscillation in the transverse direction

$$s^{GM} \sim \frac{GdJt\sin I\sin(\omega t)}{c^2a^3}.$$



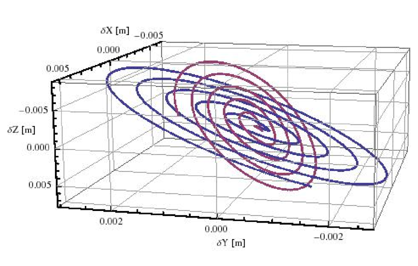
4. Mechanical Principle (II)

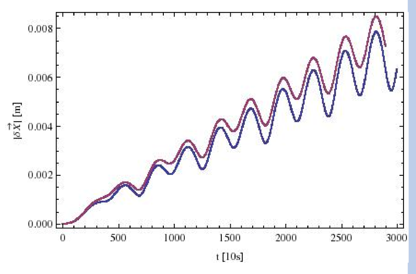
- The PN nearly circular orbit can be solved as

$$x^1 = a\cos\Psi\cos(2GJ\tau/c^2a^3) - a\cos I\sin\Psi\sin(2GJ\tau/c^2a^3),$$

$$x^2 = a\cos I\sin\Psi\cos(2GJ\tau/c^2a^3) + a\cos\Psi\sin(2GJ\tau/c^2a^3),$$

$$x^3 = a\sin I\sin\Psi.$$



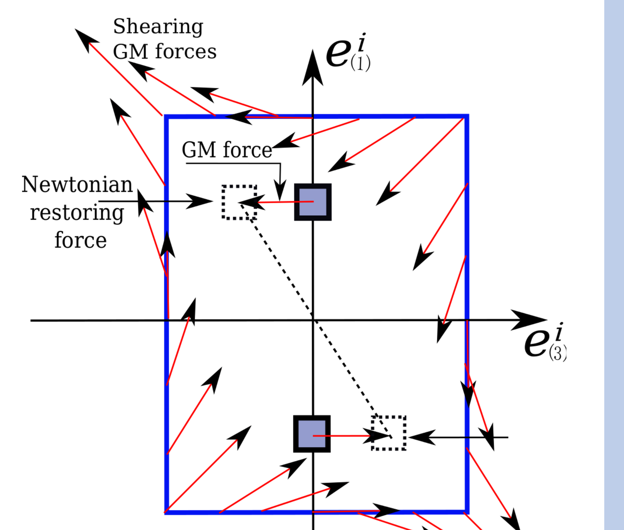


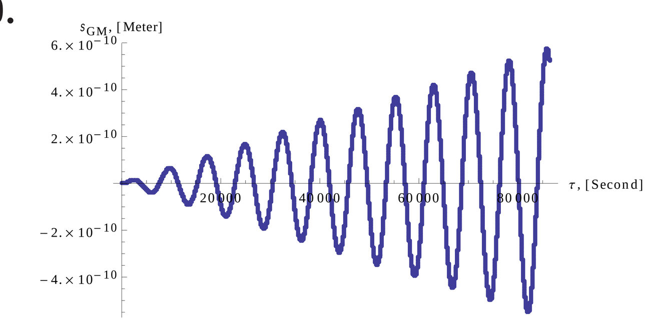
- The PN extension of the Clohessy-Wiltshire Equations that determines the local motions in the freely-falling Earth pointing frame can be written as

$$\begin{pmatrix} \ddot{Z}^{(1)}(\tau) \\ \ddot{Z}^{(2)}(\tau) \\ \ddot{Z}^{(3)}(\tau) \end{pmatrix} + \underbrace{\begin{pmatrix} 0 & 2\omega & 0 \\ -2\omega & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} \dot{Z}^{(1)}(\tau) \\ \dot{Z}^{(2)}(\tau) \\ \dot{Z}^{(3)}(\tau) \end{pmatrix}}_{Coriolis\ forces} + \underbrace{\begin{pmatrix} 0 & 0 & 0 \\ 0 & -3\omega^2 & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}\begin{pmatrix} Z^{(1)}(\tau) \\ Z^{(2)}(\tau) \\ Z^{(3)}(\tau) \end{pmatrix}}_{Newtonian\ tidal\ forces}$$

$$+ \underbrace{\begin{pmatrix} 0 & -\frac{3a^2\omega^3}{c^2} & 0 \\ \frac{3a^2\omega^3}{c^2} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} \dot{Z}^{(1)}(\tau) \\ \dot{Z}^{(2)}(\tau) \\ \dot{Z}^{(3)}(\tau) \end{pmatrix}}_{1PN\ corrections\ to\ Coriolis\ forces}$$

$$+ \underbrace{\begin{pmatrix} \frac{6GJ\omega\cos I}{c^2a^3} & -\frac{9GJ\tau\omega^2\cos I}{c^2a^3} & \frac{3GJ\omega\sin I\cos(\omega\tau)}{c^2a^3} \\ -\frac{9GJ\tau\omega^2\cos I}{c^2a^3} & \frac{6a^2\omega^4}{c^2} - \frac{6GJ\omega\cos I}{c^2a^3} & -\frac{27GJ\omega\sin I\sin(\omega\tau)}{2c^2a^3} - \frac{9GJ\tau\omega^2\sin I\cos(\omega\tau)}{2c^2a^3} \\ \frac{3GJ\omega\sin I\cos(\omega\tau)}{c^2a^3} & -\frac{27GJ\omega\sin I\sin(\omega\tau)}{2c^2a^3} - \frac{9GJ\tau\omega^2\sin I\cos(\omega\tau)}{2c^2a^3} & 0 \end{pmatrix}}_{1PN\ corrections\ to\ tidal\ forces}\begin{pmatrix} Z^{(1)}(\tau) \\ Z^{(2)}(\tau) \\ Z^{(3)}(\tau) \end{pmatrix} = 0.$$





- For the two TMs at the along-track direction, we set the initial values

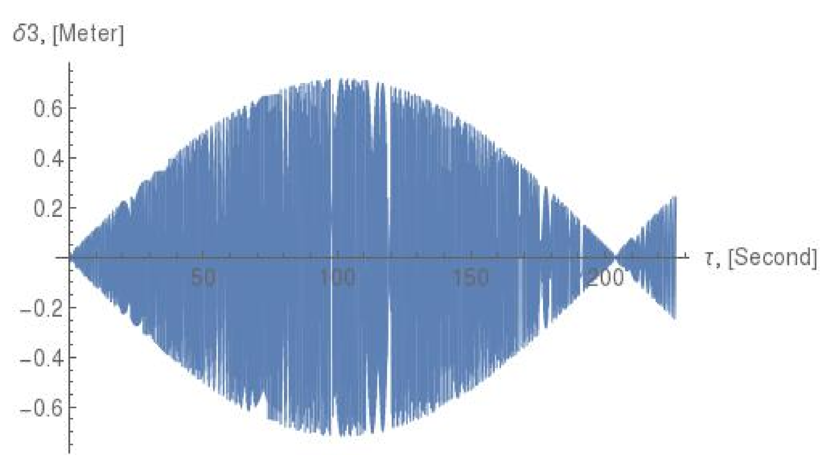
$$\frac{z_0^{(1)}}{d} \sim -1 + \mathcal{O}(\lambda), \quad \frac{z_0^{(2)}}{d} \sim \frac{z_0^{(3)}}{d} \sim \frac{\dot{z}_0^{(n)}}{d\omega} \sim \mathcal{O}(\lambda) \ll 1.$$

The PN corrections $\delta^{(m)}$ to the periodic solutions of the classical Clohessy-Wiltshire Equations read

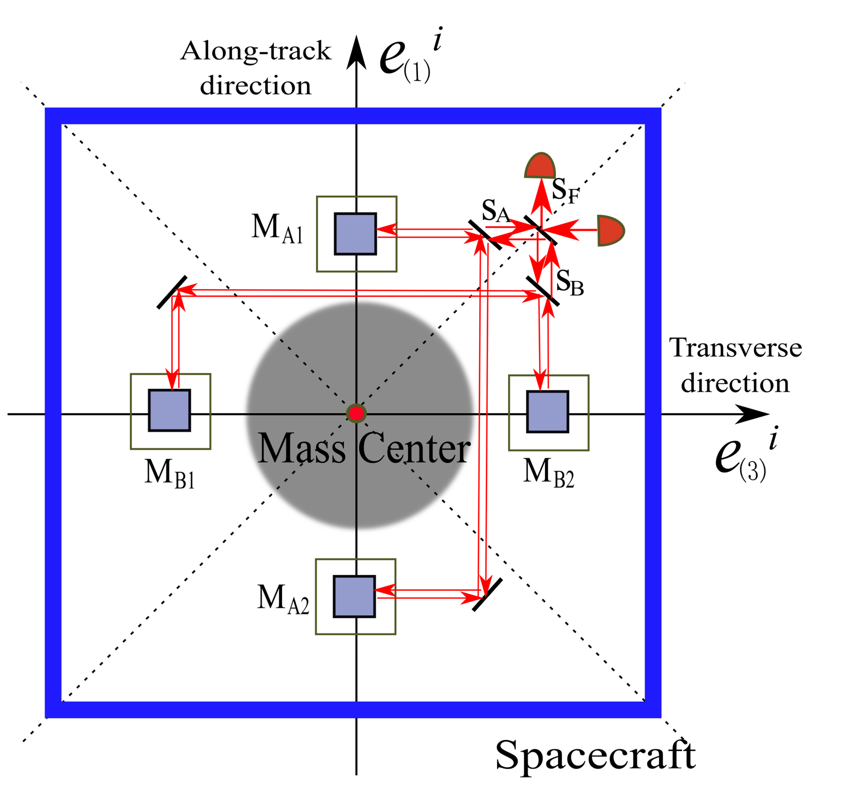
$$\delta^{(1)}(\tau) = \frac{12GdJ\cos I\sin^2(\frac{\omega\tau}{2})}{c^2a^3\omega} + d\mathcal{O}(\epsilon^2\lambda),$$

$$\delta^{(2)}(\tau) = -\frac{3GdJ\cos I(\sin(\omega\tau) - \tau\omega)}{c^2a^3\omega} + d\mathcal{O}(\epsilon^2\lambda),$$

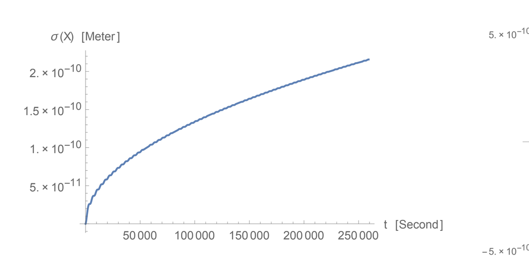
$$\delta^{(3)}(\tau) = \boxed{\frac{3GdJ\sin I\sin(\omega\tau)}{2c^2a^3}\tau} + d\mathcal{O}(\epsilon^2\lambda).$$

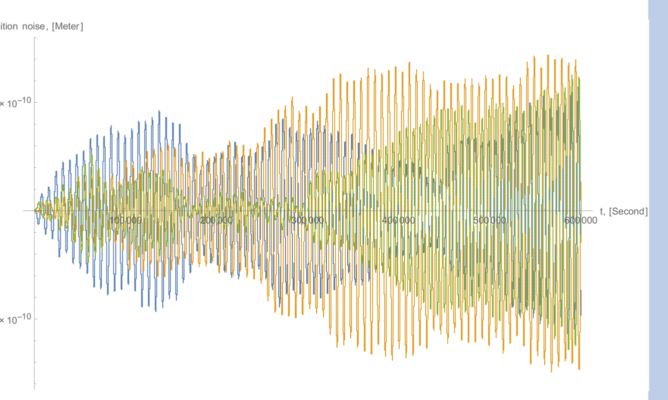


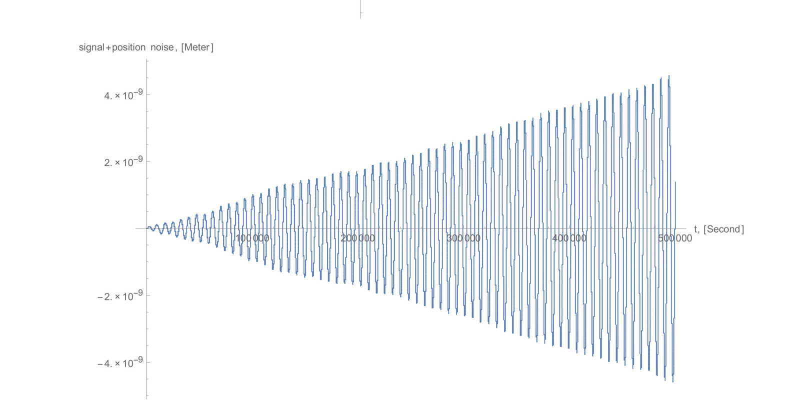
5. Readouts and Error Analysis



- ***In the final readout, the disturbances Z_{CM} of the mass center of the S/C in the $e_{(1)}{}^i - e_{(3)}{}^i$ plan and the errors caused by the jitters or random rotations $\delta\theta$ of the S/C about the radial axis may be removed.***
- Noises n^i caused by the initial deviations of the TM's position from the nominal values can be reduced to nanometer-level.
- Total acceleration noise $\sim 10^{-15}m/s^2Hz^{\frac{1}{2}}$ in the low frequency band. While, along the transverse direction, position disturbances of the signal frequency caused by the residual acceleration noises will be amplified with time as $\sim\sqrt{t}$.







$$Z_{A1/2} = -\frac{3GdJ\sin I\sin(\omega\tau)}{4c^2a^3}\tau + n^i_{A1/2} + n^{acc}_{A1/2} + (\frac{d}{2} + \delta d_{A1/2})\delta\theta \mp \delta Z^{(3)}_{CM} + n^{geo}_{A1/2},$$

$$Z_{B1/2} = -\frac{3GdJ\sin^2(\frac{\omega\tau}{2})}{c^2a^3\omega} + n^i_{B1/2} + n^{acc}_{B2/2} - (\frac{d}{2} + \delta d_{B1/2})\delta\theta \pm \delta Z^{(1)}_{CM} + n^{geo}_{B1/2},$$

$$s_F = -\boxed{\frac{3GdJ\sin I\sin(\omega\tau)}{2c^2a^3}\tau} - \frac{6GdJ\sin I\sin^2(\frac{\omega\tau}{2})}{c^2a^3\omega} + (n^i_{A1} - n^i_{A2} + n^i_{B1} - n^i_{B2}) + (n^{acc}_{A1} - n^{acc}_{A2} + n^{acc}_{B1} - n^{acc}_{B2}) + (\delta d_A - \delta d_B)\delta\theta + n^{geo}_A + n^{geo}_B + n^{\epsilon}.$$

- Noises and errors n^{geo} from geopotential multipoles, especially the J_2 component, may be adjusted and fitted out given the precision measured results from SLR and in EGM08.

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