Modelling EMRIs using Self-Force

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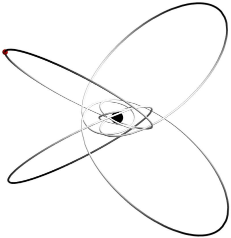
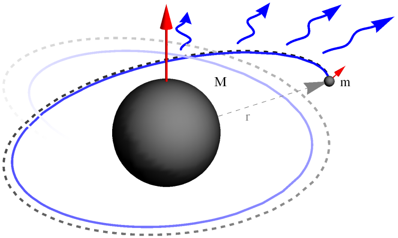
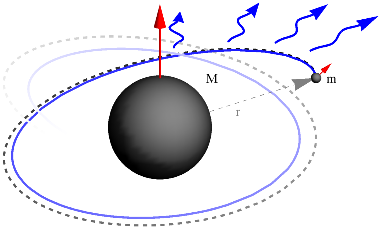


LISA Symposium XI, Zurich, 7 September 2016



Introduction: the challenge of modelling EMRI evolution

The challenges of modelling EMRIs



Challenge 1: relativistic

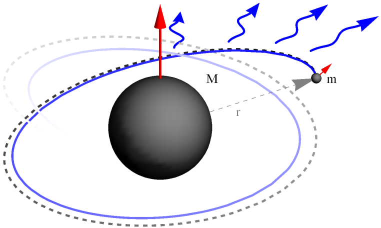
EMRIs are highly relativistic $r \sim \frac{GM}{c^2}$.
Can't use post-Newtonian theory.

Challenge 2: lengthscales

Wildly disparate length scales.
$\frac{Gm}{c^2} : \frac{GM}{c^2} \sim 1 : 10^5$.
Can't use numerical relativity.

Challenge 3: timescales

EMRIs evolve very slowly. A typically EMRI will spend $\sim \frac{M}{m} \sim 10^5$ orbits in the LISA band. Need a model that is accurate to $\sim 10^5$ wave cycles!



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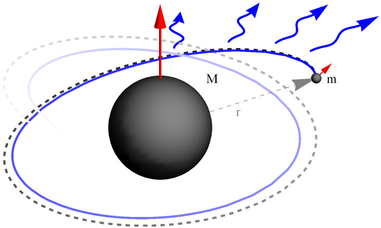

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The “Capra” programme

19th Capra meeting (2016) participants

Goals

- model EMRI evolution accurate to ~ 1 radian over $\sim 10^5$ orbits.
- include general eccentricities
- include general inclination
- include effects of spin on primary object
- include effects of spin on secondary object

"Elk nadeel hep z'n voordeel."
[Johan Cruijff]

Strategy

Use the smallness of the mass-ratio $\mu := \frac{m}{M}$ to our advantage and use it as an expansion parameter using:

- Black hole perturbation theory
- Multi length scale analysis (matched asymptotics)
- Multi time scale analysis

How far do we need to go?

- test-mass limit, geodesics in black hole geometry of primary, no evolution
- Adiabatic approximation "0PA". Only needs orbital average change of energy, angular momentum, etc.
- $\frac{1}{2}$PA: Orbital resonances, amplify local "self-force effects"
- "1PA": Self-forced evolution. Needs:
  1. "first order self-force"
  2. "spin-force"
  3. second order adiabatic quantities

Strategy: perturbation theory

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- $\mu = 0$ test-mass limit, geodesics in black hole geometry of primary, no evolution.
- phase accurate to $\sim \mu^{-1}$: Adiabatic approximation "0PA". Only needs orbital average change of energy, angular momentum, etc.
- $\sim \mu^{-1/2}$ "$\frac{1}{2}$PA: Orbital resonances, amplify local "self-force effects".
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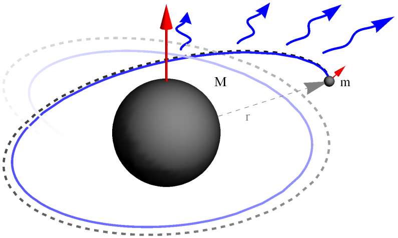
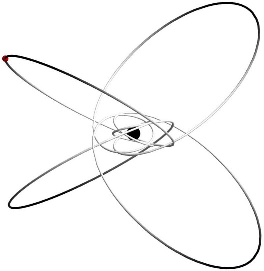


Test mass limit: Geodesic equations

Geodesics in Kerr spacetime



Constants of Motion

Geodesics in Kerr spacetime are characterized by three constants of motion: [Carter, 1968]

1. Energy, E
2. Angular momentum, L_z
3. Carter constant, Q, (related to total angular momentum)

Orbital phases

Position along the orbit is described by three independently evolving phases:

1. q_ϕ: related to azimuthal position
2. q_r: related to radial motion
3. q_z: related to oscillations around equator

Analytic solutions

Analytic solutions are available:

- [Fujita&Hikida, 2009]
- [Hackmann et al., 2008,2010]

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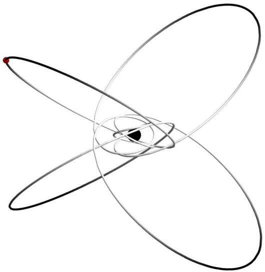

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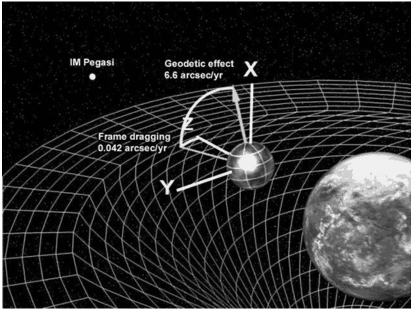
Motion of secondary spin

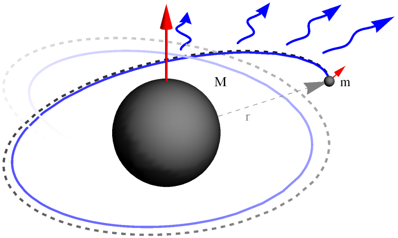
Motion of test spin

- de Sitter precession (geodetic effect)
- Lense-Thirring effect



Parallel transport

- Parallel transport equations can be numerically integrated once geodesic is known.
- No known generic analytic solutions.



Adiabatic approximation

Adiabatic approximation

Averaged fluxes

The (long term) average rate of change of the constants of motion can be gauged by measuring the GW flux towards infinity and into the primary black hole.

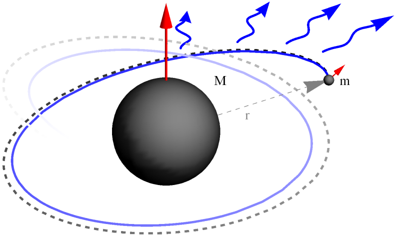
- $\langle \dot{E} \rangle$ from the energy flux.
- $\langle \dot{L}_z \rangle$ from the angular momentum flux.
- $\langle \dot{Q} \rangle$ see [Sago et al., 2006]

Evolution: osculating geodesics

At any instant the inspiral is tangent to some geodesic. Instantaneous values of $\langle \dot{E} \rangle$, $\langle \dot{L}_z \rangle$, and $\langle \dot{Q} \rangle$ are obtained by solving Teukolsky equation sourced by geodesic.

State-of-the-art

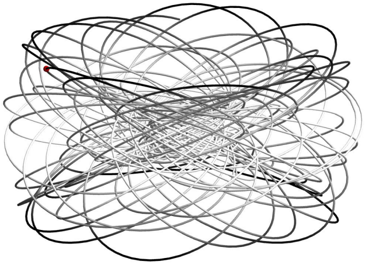
- Flux calculations for sourced by generic orbits in Kerr spacetime. [Drasco & Hughes, 2006][Fujita, Hikida & Tagoshi, 2009].

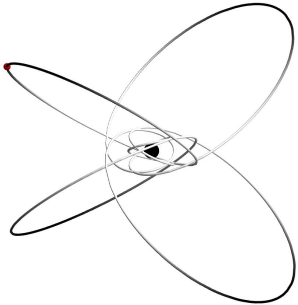


1/2 PA corrections: resonances

Resonant orbits

Generic | vs | Resonant





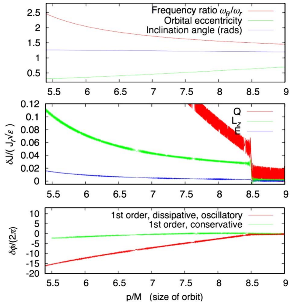
Resonances

Resonances occur when two or more of the independent orbital phase evolutions "sync" up in rational ratio, allowing coherent secular build up of otherwise oscillatory effects. Occur generically in inclined inspirals, and mostly within the eLISA band. [see poster C. Berry]

Effects of Resonances

rz-resonances

- Coherent build of oscillatory self-force effects leads to jumps in constants of motion.[Flanagan& Hinderer, 2012]
- Size of jump is sensitive to orbital phases. Can be obtained from averaged fluxes on resonant geodesics.[MvdM, 2013]
- "Resonant locking" unlikely. [MvdM, 2013]



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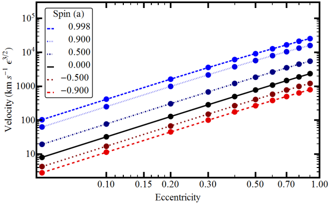

[MvdM, 2014]

$r\phi$- and $z\phi$- resonances[Hirata, 2012][MvdM, 2014]

Resonances involving ϕ motion:

- Cannot affect evolution of "intrinsic" orbital parameters.
- Can affect "extrinsic" parameters of EMRI systems such as CoM velocity ("Kicks")

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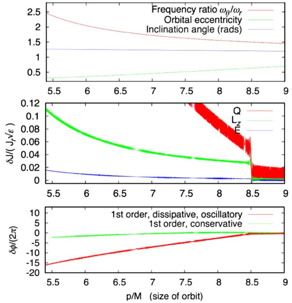

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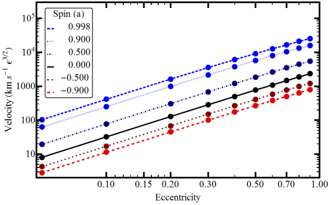

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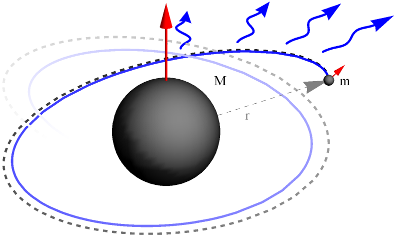
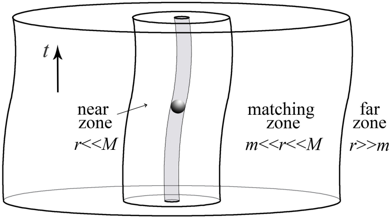


1 PA corrections: Self-force

Matched asymptotic expansions

[Mino, Sasaki & Tanaka, 1997] [Poisson, 2003]



far zone

Kerr geometry of primary plus perturbation generated by secondary.

near zone

Kerr geometry of secondary plus perturbation generated by primary.

Matching identifies

- effective worldline of secondary in far zone.
- source for perturbation in far zone (point particle).
- regular part of (singular) perturbation in far zone responsible for backreaction on effective worldline (gravitational self-force & spin-force).

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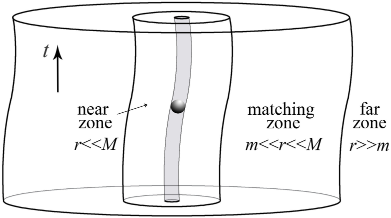

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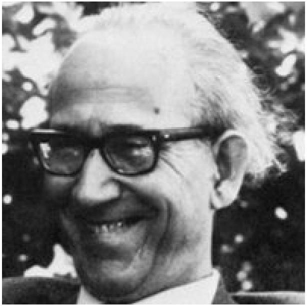
Spin force

Myron Mathisson



Achilles Papapetrou

Mathisson-Papapetrou spin-force

- force induced on geodesic by the presence of spin on test object.
- first order correction in μ
- first derived by Papapetrou [Papapetrou, 1951].
- analytic expression in terms of position, velocity, and spin-vector.

MiSaTaQuWa formula

[Mino,Sasaki&Tanaka,1996][Quinn&Wald,1996]

$$\frac{D}{d\tau}x^{\alpha} = \mu F^{\alpha}[h^{R}]$$

h^R is "regular" part of (retarded) metric perturbation produced by point particle.

Methods for obtaining regular part

1. Mode-sum regularization [Barack&Ori,2001]
2. Effective source methods [Barack&Golbourn,2008]
3. Green's function methods [Mino, Sasaki & Tanaka, 1996]

Time domain

- Decompose field equations in spherical harmonics.
- Numerically solve system of $1+1$D PDEs on a grid.
- [Barack, Lousto, Sago]
- 2+1D and 3+1D methods also explored

Frequency domain

- Further decompose equations in Fourier modes.
- Numerically solve system of ODEs.
- [Barack, Burko, Detweiler, Warburton, Akcay, Kavanagh, Ottewill, Evans, Hopper, ...]

State-of-the-art

- Self-force calculations using a wide variety of methods (Time domain, frequency domain, mode-sum, effective source, etc.)
- eccentricities up to $\lesssim 0.8$. [Osburn, Warburton& Evans, 2016]

Calculating the GSF: Kerr

The problem with Kerr

No spherical symmetry. Field equations do not decouple in "spherical" harmonics.

Time domain

- Decompose field equations in azimuthal m-modes.
- Numerically solve system of $2+1$D PDEs on a grid.
- [Dolan, Wardell, Barack, Thornburg]
- Issues with numerically unstable gauge modes

Frequency domain

- Teukolsky equation for Weyl scalars ψ_0 and ψ_4 does decouple in Fourier modes.[Teukolsky,1972]
- Can be solved using semi-analytical methods.[Mano,Suzuki&Tagasugi,1996]
- Metric perturbation can be reconstructed from ψ_0 and ψ_4 in radiation gauge.[Chrzanowski,Cohen,Kegeles, 1970s]
- [Friedman, Keidl, Shah, MvdM, ...]

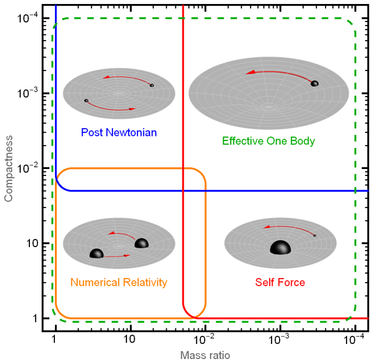
State-of-the-art

- GSF on eccentric equatorial orbits [MvdM, 2016]
- Generic orbits... (coming soon)

Validation

Invariants

- Energy & angular momentum fluxes
- Detweiler-Barack-Sago redshift
- ISCO shift
- Periapsis precession
- Spin precession ("self-torque")
- Tidal invariants



Crosschecks with ...

- Other self-force calculations (different method, gauge, etc.)
- post-Newtonian theory
- Numerical relativity
- Effective-One-Body models

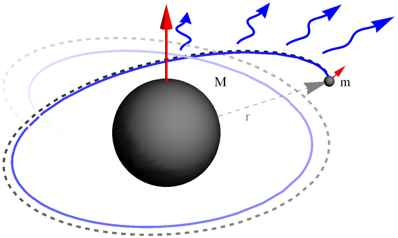
Status of second order calculations

Second order challenge

- Second order GSF essential ingredient for 1PA evolution.
- Technical formalism in place [Pound, Rosenthal, Gralla, Detweiler,...]
- Challenges in “UV”
- Challenges in “IR”

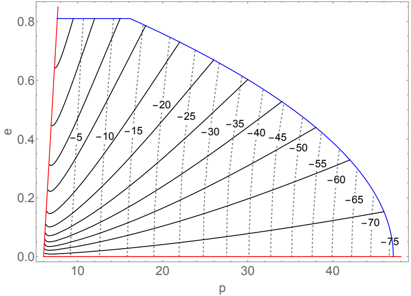
Status

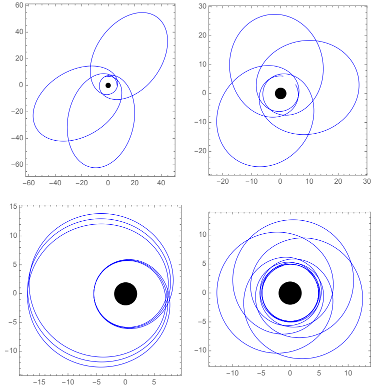
First numerical calculations (Schwarzschild circular orbits) expected “soon”.



Inspiral evolution

[Warburton, Akcay, Barack, Gair & Sago, 2012] [Osburn, Warburton & Evans, 2016]

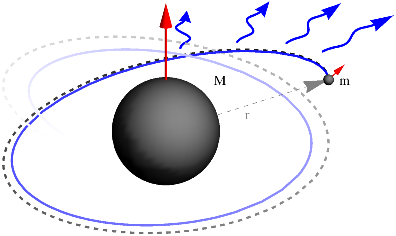




Osculating geodesics

- GSF sourced by instantaneously tangent geodesic.
- No second order GSF included.
- Conservative GSF effects add phase difference of several tens of radians over inspiral.

$\mu = 10^{-5}$
initial data: $p = 12$, $e = 0.81$
2115.5, 500, 100, and 1 day(s) before plunge.



Status overview

Status of calculations

		Schwarschild		Kerr		
		circ	ecc	circ	ecc	incl
Geodesics (analytic)		[Hackmann & Lammerzahl, 2008]		[Fujita & Hikida, 2009] [Hackmann et al, 2010]		
Adiabatic	orbit	[Cutler et al, 1994]		[Shibata et al, 1994]		[Hughes, 1999]
				[Drasco & Hughes, 2006]		
	evolution				[Glamp. & Ken., 2002]	[Hughes, 2001]
				to do		
$\frac{1}{2}$ PA: resonances				[Flanagan & Hinderer, 2012] [Flanagan, Hughes & Ruangsri, 2014] [MvdM, 2014]		
1PA:	1GSF	[Barack & Sago, 2007]	[Barack & Sago, 2010]	[Shah et al, 2012]	[MvdM & Shah,2015] [MvdM, 2016]	in progress
	2GSF	in progress	to do	to do		
	spin force	[Papapetrou, 1951]		[Papapetrou, 1951]		
	evolution	[Warburton et al, 2012] [Osburn et al, 2015]		to do		

Conclusions

Status

- Formalism mostly in place
- 1GSF calculations in Schwarzschild now routine
- 1GSF in Kerr now available for equatorial orbits
- First self-forced inspirals

To do...

- Numerical 2GSF calculations (soon...)
- 1GSF on Kerr generic orbits (soon...)
- self-forced inspirals in Kerr
- include secondary spin effects & 2GSF
- waveforms

The End

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