A frequency-domain implementation of the particle-without-particle approach to EMRIs

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INTRODUCTION

Extreme-Mass-Ratio Inspiral (EMRI) systems are one of the main sources of gravitational waves (GWs) for ground- and space-based detectors like the eLISA mission. EMRIs are binary systems which consist of a stellar compact object (SCO) with mass $m_*$ orbiting a massive black hole (MBH) with a mass $M_\bullet$. The mass ratio $q = m_*/M_\bullet$ is the singular nature of the gauge where the metric perturbations are usually computed, making the self-force calculation computationally challenging.

We present here a frequency-domain implementation of the particle-without-particle (PWP) technique that was previously developed for the computation of the scalar self-force—helpful for tests for the gravitational self-force. We expect that this will yield significant improvements in computational time and hope that it will provide useful hints towards circumventing the gauge singularity issues and ultimately computing the self-force.

SCALAR SELF-FORCE

We consider a simplified EMRI model: the SCO is a charged scalar particle (with charge $q$) orbiting a non-rotating MBH (with a fixed geometry, the Schwarzschild metric $g_{\text{Sch}}$) along a geodesic $\gamma$ (with worldline $z^\mu$ and 4-velocity $u^\mu$). The field equation and EOM are respectively [1] and jump conditions:

$$\begin{align*}
\left(\Box + V_j(r)\right)\psi^{(m)} &= -g^{(m)}\delta(r - r_p(t))
\end{align*}$$

FREQUENCY DOMAIN

Since we have bound orbits, we can expand the fields in discrete Fourier series, making the field equations ODEs:

$$\begin{align*}
\psi_n^{(m)}(t, r) &= e^{-in\omega t} \sum_{m=-n}^{n} e^{-im\phi} R_n^{(m)}(r)
\end{align*}$$

NUMERICAL IMPLEMENTATION

We use a pseudospectral collocation method to find a numerical solution $R_n^{(m)}$ to the homogeneous ODE from the horizon to apocenter with arbitrary BCs at the latter, and $R_n^{+}$ from the pericenter to spatial infinity with arbitrary BCs at the former. The solution is $R_n = C_n R_n^{+} + C_n^{\text{reg}} R_n^{\text{reg}}$, where $C_n^{\text{reg}}$ are scaling coefficients determined by solving a linear system which arises from the jump conditions. Schematically:

RESULTS AND WORK IN PROGRESS

Once the fields are computed numerically, their value can be used to calculate the self-force.

- Thus far, using this method, the known value of the self-force has been recovered for circular orbits (in agreement with the PWP in time-domain [3–5] and the results of other methods in the literature).
- We are working on extending this to generic (eccentric) orbits.
- We aim to also extend this method to rotating black holes (the Kerr spacetime).

REFERENCES