A frequency-domain implementation of the particle-without-particle approach to EMRIs

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ABSTRACT

The gravitational waves emitted by binary systems with extreme- or intermediate- mass ratios carry unique astrophysical information expected to be detected by the next generation of gravitational wave detectors. The detection of these binaries rely on an accurate modelling of the gravitational self-force that drives their orbital evolution. Although the theoretical formalism to compute the self-force has been fully developed, the mathematical tools needed to implement it are under development, and the self-force computation is still an open problem. The main obstacle is the singular nature of the gauge where the metric perturbations are usually computed, making the self-force calculation computationally challenging.

We present here a frequency-domain implementation of the particle-without-particle (PwP) technique that was previously developed for the computation of the scalar self-force – a helpful testbed for the gravitational self-force. We expect that this will yield significant improvements in computational time and hope that it will provide useful hints for circumventing the gauge singularity issues and ultimately computing the metric perturbations in the full gravitational case.

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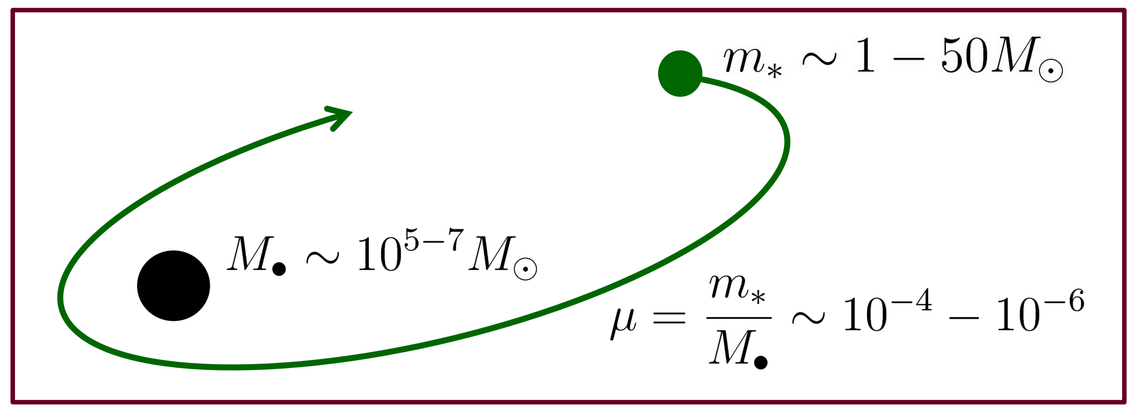
INTRODUCTION

Extreme-Mass-Ratio Inspirals (EMRIs) are one of the main sources of gravitational waves (GWs) for space-based detectors like the eLISA mission. EMRIs are binary systems which consist of a stellar compact object (SCO; with a mass m_*) orbiting a massive black hole (MBH; with a mass $M_\bullet$).



The challenge in modeling EMRIs is to compute the perturbations generated by the SCO in the (background) gravitational field of the MBH, and how these perturbations affect the motion of the SCO itself. The most extended approach [1] consists in modelling the SCO by using a point-like description and then, to describe the radiation reaction effects on the dynamics as the action of a local *self-force* that is responsible for the deviations from geodesic motion.

SCALAR SELF-FORCE

We consider a simplified EMRI model: the SCO is a charged scalar particle (with charge q associated to a scalar field Φ) orbiting a non-rotating MBH (with a fixed geometry, the Schwarzschild metric $g^{\rm BH}_{\mu\nu}$) along a geodesic γ (with worldline z^μ and 4-velocity u^μ). The field equation and EOM are respectively [1]:

$$g^{\alpha\beta}_{\rm BH}\nabla_\alpha\nabla_\beta\Phi(x) = -4\pi q\int_\gamma {\rm d}\tau\, \delta_4(x - z(\tau))$$

$$m_*\frac{{\rm d}u^\mu}{{\rm d}\tau} = F^\mu = q(g^{\mu\nu}_{\rm BH} + u^\mu u^\nu)\,(\nabla_\nu\Phi)|_\gamma$$

The field's harmonic modes $\Phi^{lm}(t,r)$ decouple, leading to a wave-like PDE for $\psi^{lm} = r\Phi^{lm}$:

$$(\Box + V_l(r))\,\psi^{lm} = -S^{lm}\delta(r - r_{\rm p}(t)),\ \Box = \partial_t^2 - \partial_{r_*}^2$$

where $r_* = r + 2M_\bullet \ln(r/2M_\bullet - 1)$, $V_l(r)$ is the "Regge-Wheeler potential," S^{lm} is the source term coefficient, proportional to q and dependent on the particle location, and $r_{\rm p}$ the particle radial location.

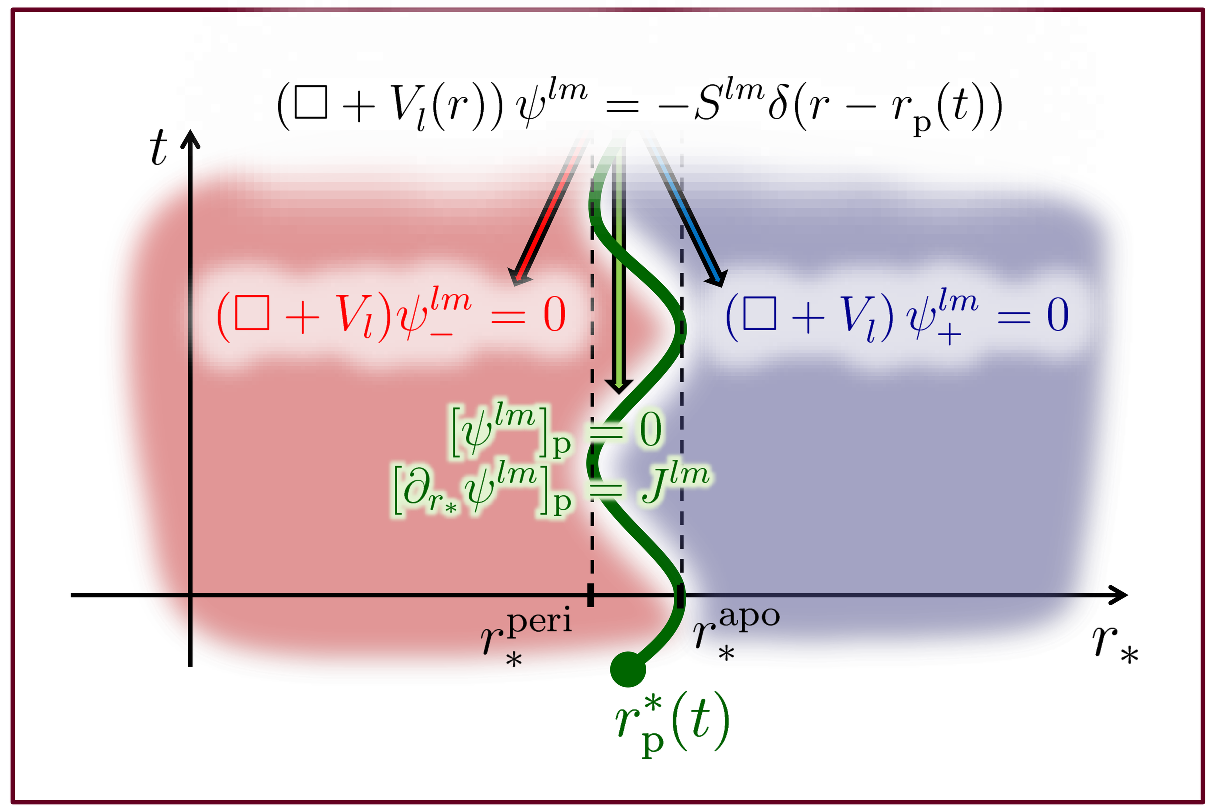
Once the field is solved for, its singular part must be subtracted (via "mode-sum regilarisation" [2]).

THE PWP FORMULATION

The full solution has to be found numerically, but the presence of singularities makes the task difficult. To circumvent this, the PwP [3–5] splits the computational domain into two disjoint regions whereby any non-singular quantity $\mathcal{Q}(t,r)$ is decomposed as

$$\mathcal{Q} = \mathcal{Q}^-\,\Theta^-_{\rm p} + \mathcal{Q}^+\,\Theta^+_{\rm p},\quad \Theta^\pm_{\rm p} = \Theta(r \mp r_{\rm p})$$

and we denote by $[\mathcal{Q}]_{\rm p} = \lim_{r\to r_{\rm p}}(\mathcal{Q}^+ - \mathcal{Q}^-)$ its jump across the particle location. Then we get homogeneous equations together with jump conditions for the fields:



where J^{lm} is proportional (up to a factor depending on the particle location) to S^{lm}.

FREQUENCY DOMAIN

Since we have bound orbits, we can expand the fields in discrete Fourier series, making the field equations ODEs:

$$\psi^{lm}_\pm(t,r) = e^{-{\rm i}m\omega_\varphi t}\sum_{n=-\infty}^{+\infty} e^{-{\rm i}n\omega_r t}\, R^\pm_{lmn}(r)$$

$$\Rightarrow \left(\frac{{\rm d}^2}{{\rm d}r_*^2} - V_l(r) + \omega^2_{nm}\right) R^\pm_{lmn} = 0,\quad \omega_{nm} = n\omega_r + m\omega_\varphi$$

supplemented by appropriate (non-reflecting) BCs:

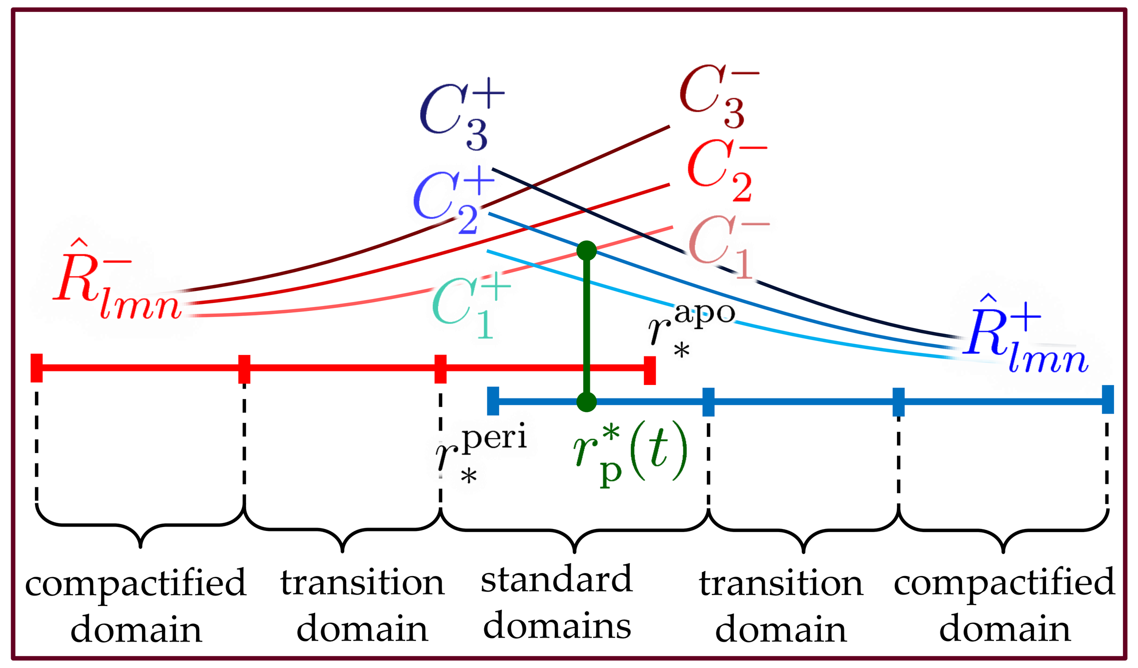
$$\left(\pm(-{\rm i}\omega_{mn}) + \frac{\rm d}{{\rm d}r_*}\right) R^\pm_{lmn}\bigg|_{r_*\to\pm\infty} = 0$$

and jump conditions:

$$\sum_{n=-\infty}^{+\infty} e^{-{\rm i}n\omega_r t}\,[R_{lmn}]_{\rm p} = 0,\ e^{-{\rm i}m\omega_\varphi t}\sum_{n=-\infty}^{+\infty} e^{-{\rm i}n\omega_r t}\left[\frac{{\rm d}R_{lmn}}{{\rm d}r_*}\right]_{\rm p} = J^{lm}$$

NUMERICAL IMPLEMENTATION

We use a pseudospectral collocation method to find a numerical solution $\hat{R}^-_{lmn}$ to the homogeneous ODE from the horizon to apocenter with arbitrary BCs at the latter, and $\hat{R}^+_{lmn}$ from the pericenter to spatial infinity with arbitrary BCs at the former. Then the solution is $R_{lmn} = C^-_{lmn}\hat{R}^-_{lmn}\Theta^-_{\rm p} + C^+_{lmn}\hat{R}^+_{lmn}\Theta^+_{\rm p}$ where $C^\pm_{lmn}$ are scaling coefficients determined by solving a linear system which arises from the jump conditions. Schematically:



RESULTS AND WORK IN PROGRESS

Once the fields are computed numerically, their value can be used to calculate the self-force.

- Thus far, using this method, the known value of the self-force has been recovered for circular orbits (in agreement with the PwP in time-domain [3–5] and the results of other methods in the literature).
- We are working on extending this to generic (eccentric) orbits.
- We aim to also extend this method to rotating black holes (the Kerr spacetime).

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