Constraining inflationary physics with primordial gravitational waves at small scales

Based on the REVIEW: M. C. Guzzetti, N. Bartolo, M. Liguori and S. Matarrese, *Gravitational waves from inflation*, Riv. Nuovo Cim. **39**, 9 (2016), arXiv:1605.01615

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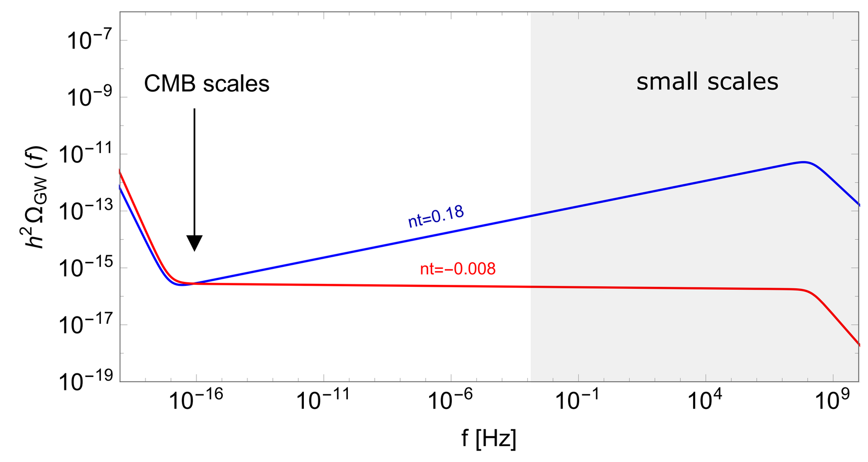
THEORETICAL PICTURE

Gravitational waves from inflation

Any inflationary scenario predicts the production of a stochastic gravitational-wave (GW) background, because of **quantum fluctuations of the gravitational field**. The dynamics of such tensor perturbations h_{ij} is driven by a wave equation. In particular, their amplitude turns out to depend on evolution of the scale factor of the universe $a(t)$. As for perturbations of the field(s) that drives the dynamics, perturbations of the gravitational field are stretched by the accelerated expansion of the universe and pulled outside the horizon where they get frozen.
Their power-spectrum is usually parameterized as a power-law:

$$P_{\rm T}(k) = A_{\rm T}(k_*)\left(\frac{k}{k_*}\right)^{n_{\rm T}}$$

Then, entering the causal region during the subsequent epochs, they provide a GW background spread on a extremely large range of scales, and which it is still filling the universe.



From the inflationary GW power-spectrum, taking into account the subsequent evolution of the universe, it is possible to calculate the **present-time GW spectral energy-density**:

$$\Omega_{\rm GW}(k,\tau_0) \equiv \frac{1}{\rho_c}\frac{{\rm d}\rho_{\rm gw}}{{\rm d}\ln k} = \frac{1}{12}\left(\frac{k}{aH}\right)^2 T(k)\,P_{\rm T}(k)$$

where $T(k)$ is the transfer function from the primordial to the present time. The amplitude and the scale-dependence reflect the primordial values of $A_{\rm T}$ and $n_{\rm T}$.

Single-field slow-roll inflation

- **Amplitude**: $A_{\rm T} \sim H^2$ → measure of the energy scale of inflation;
- **Spectral index**: $n_{\rm T} = -2\epsilon \lesssim 0$ → measure of the deviation from a de-Sitter background;

$$P_{\rm T}(k) = \frac{8}{M_{\rm pl}^2}\left(\frac{H}{2\pi}\right)^2\left(\frac{k}{aH}\right)^{-2\epsilon}$$

✓ Primordial GW constitute a **smoking-gun for the cosmological inflationary model** and carry information about the **energy-scale** of inflation, its dynamics and the **field excursion** of the inflaton.

Mechanisms of gravitational wave production during inflation

The GW signal produced by vacuum fluctuations of the gravitational field characterize any inflationary model. Moreover, a further contribution to inflationary GW can be generated by a *classical* mechanism.

***Classical* production of GW**: the presence of other fields during inflation, besides the gravitational one, can introduce an efficient *source term* (of the second order) in the equation of motion of GW:

$$h''_{ij} + 2\mathcal{H}h'_{ij} - \nabla^2 h_{ij} = \frac{2}{M_{\rm pl}^2}\hat{\Pi}_{ij}^{lm}T_{lm},$$

where T_{lm} is a generic stress-energy tensor and $\hat{\Pi}_{ij}^{lm}$ projects on the transverse and trace-less part.

GW generated by the presence of a source term, often present **non-gaussianity** and **chirality** levels different from those due to vacuum oscillation.

GW PRODUCTION	Discriminant	Specific discriminant	Examples of specific models
Vacuum oscillations quantum fluctuations of the gravitational field stretched by the accelerated expansion	**theory of gravity**	General Relativity	*single-field slow-roll*; all other models in GR
		MG/EFT approach	G-Inflation; Potential-driven G-Inflation; EFT approach
Classical production second-order GW generated by the presence of a source term in GW equation of motion	**source term**	vacuum inflaton fluctuations	*all models*
		fluctuations of extra scalar fields	inflaton+spectator fields; curvaton
		gauge particle production	pseudoscalar inflaton+gauge field; scalar infl.+pseudoscalar+gauge
		scalar particle production	scalar inflaton+ scalar field
		particle production during preheating	chaotic inflation; hybrid inflation

Table 1: Taken from [1].

Total GW spectral energy-density:

$$\Omega_{\rm gw} = \Omega_{\rm gw}^{vacuum} + \Omega_{\rm gw}^{sourced} \longrightarrow$$ possible **enhancement of GW amplitue at small scales** $n_{\rm T} > 0$ ⟶ interesting for detectors related to small scales

✓ Inflationary GW represent a crucial **discriminant among the variety of inflationary models** and represent the possibility of **testing the theory of gravity** underlying the inflationary scenario. In particular, GW power-spectra with enhanced amplitude at small scales represent interesting signals for laser interferometer detectors.

Inflationary consistency relation and its violations

For single-field slow-roll inflation, at the lowest order in slow-roll parameters, the following consistency relation between the tensor-to-scalar ratio $r \equiv A_{\rm T}/A_{\rm S}$ and the GW spectral index holds:

$$\boldsymbol{r = -8n_{\rm T}} \;\rightarrow\; n_{\rm T} < 0$$

A gravity theory different from General Relativity or an extra GW production due to a source term, can lead to a violation of such an equality.

GW SPECTRAL INDEX:

RED: $n_{\rm T} < 0$

BLUE: $n_{\rm T} > 0$ → clear violation of the consistency relation

See fig.1.

	Model	Tensor power-spectrum	Tensor spectral index		Consistency relation
GW signal due to vacuum fluctuations	Standard infl.	$P_{\rm T} = \frac{8}{M_{\rm pl}^2}\left(\frac{H}{2\pi}\right)^2$	$n_{\rm T} = -2\epsilon$	red	$r = -8n_{\rm T}$
	EFT inflation[(a)]	$P_{\rm T} = \frac{8}{c_{\rm T}M_{\rm pl}^2}\left(\frac{H}{2\pi}\right)^2$	$n_{\rm T} = -2\epsilon + \frac{2}{3}\frac{m_{\rm T}^2}{\alpha H^2}\left(1+\frac{4}{3}\epsilon\right)$	r/b	-
	EFT inflation[(b)]	$P_{\rm T} = \frac{8}{c_{\rm T}M_{\rm pl}^2}\frac{2^{\frac{-p}{1+p}}}{\pi}\Gamma^2\left(\frac{1}{2(1+p)}\right)\left(\frac{H}{2\pi}\right)^2$	$n_{\rm T} = \frac{p}{1+p}$	blue	violation
	Gen. G-Infl.	$P_{\rm T} = \frac{8}{M_{\rm pl}^2}\gamma_{\rm T}\frac{\mathcal{G}_{\rm T}^{1/2}}{\mathcal{F}_{\rm T}^{3/2}}\left(\frac{H}{2\pi}\right)^2$	$n_{\rm T} = 3 - 2\nu_{\rm T}$	r/b	-
	Pot.-driv. G-Infl.	$P_{\rm T} = \frac{8}{M_{\rm pl}^2}\left(\frac{H}{2\pi}\right)^2$	$n_{\rm T} = -2\epsilon$	r/b	$r \simeq -\frac{32\sqrt{6}}{9}n_{\rm T}$
Extra GW signal due to a source term	Particle prod.	$P_{\rm T}^{+} = 8.6\times10^{-7}\frac{4H^2}{M_{\rm pl}^2}\left(\frac{H}{2\pi}\right)^2\frac{e^{4\pi\xi}}{\xi^6}$	-	blue	violation
	Spectator field	$P_{\rm T} \simeq 3\frac{H^4}{c_{\rm S}^{18/5}M_{\rm pl}^4}$	$n_{\rm T} \simeq 2\left(\frac{2m^2}{3H^2} - 2\epsilon\right) - \frac{18}{5}\frac{\dot{c}_{\rm S}}{Hc_{\rm S}}$	r/b	violation

Table 2: Taken from [1].

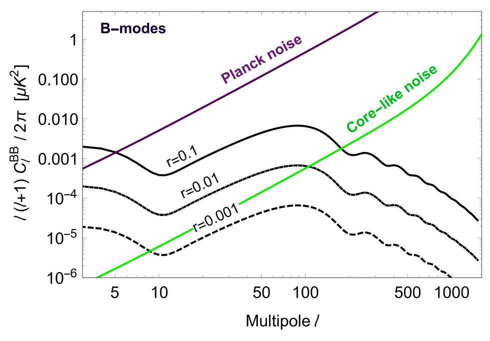
Single-field slow-roll inflation

- Consistency relation: $r = -8n_{\rm T}$
- GW spectral index $n_{\rm T}$: slightly negative, that is **red tilted** (see fig.1).

✓ Constraining the inflationary amplitude and spectral index is required in order to test the consistency relation and then the inflationary physics. GW experiments **at small scales** play a crucial role in order to constrain the **GW spectral index**, and then in testing the consistency relation.

GW SIGNATURES

- **CMB temperature and polarization** power-spectra
- **CMB spectral distorsions**
- **Gravitational lensing**
- **Pulsar timing**
- **Mass distribution of the universe**



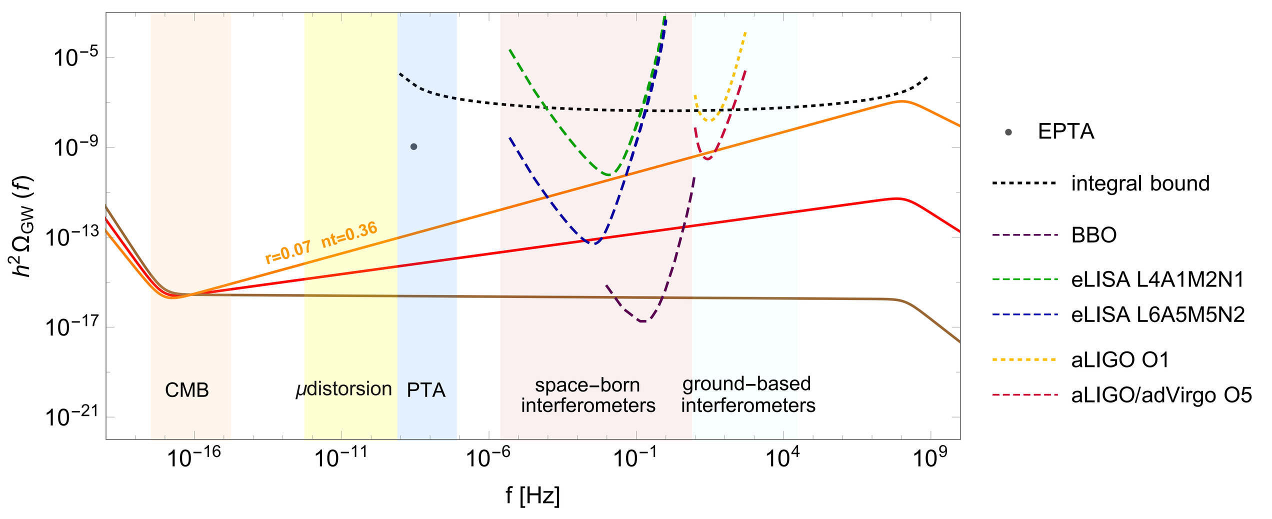
- **Evolution of the scale factor**: reflected, for example, on the Big Bang Nucleosynthesis

Preliminary bounds from primordial black holes

Constraints on primordial black holes provide interesting limits on scalar perturbations at small scales, which can be translated into significant bounds on primordial GW.

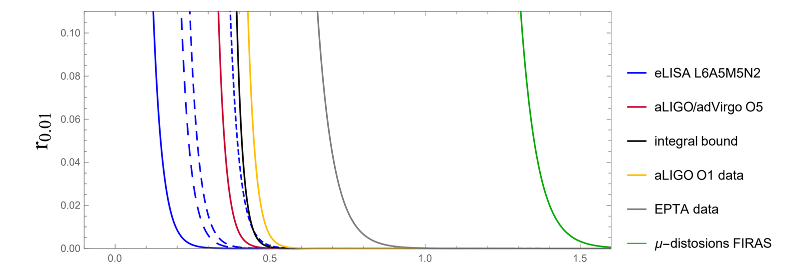
PROSPECTS EXPLOITING GW AT SMALL SCALES

Current bounds and observational prospects



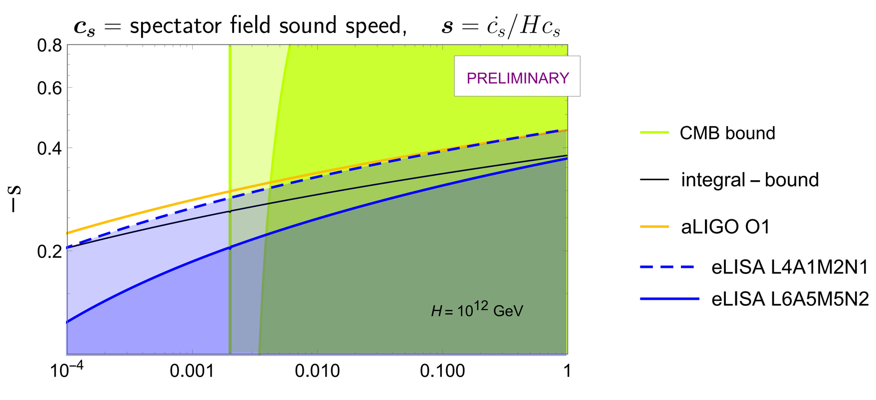
Taken from [1]. Current direct/indirect bounds and expected sensitivity curves for future experiments.

Constraints on GW power-spectrum parameters



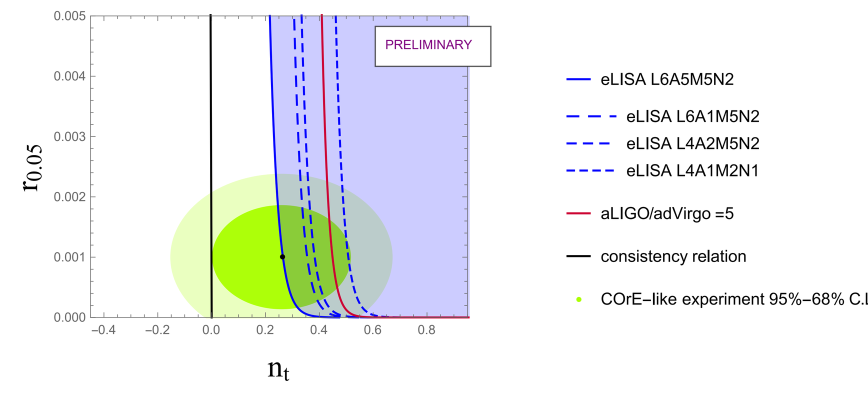
Constraints provided by current GW bounds and limits expected by future experiments at 95% C.L. (on the left, region admitted for a non-detection by each of them). Detectors working at small scales might significantly improve current bounds.

Constraints on parameter-space of inflationary models, an example



Constraints on the parameter-space of inflation **with a spectator field**, provided by current experiments and in case of a non-detection by eLISA at 95% C.L.. Due to the presence of the extra field, besides vacuum fluctuations, a certain amount of GW is expected to be produced *classically*. Being the sourced GW power-spectrum admitted to be blue (quantified by the parameter s), GW experiments at small scales might assume an interesting role in constraining this model parameters.

Test of the consistency relation



Constraints expected for a COrE-like CMB experiment for fiducial values of $r = 0.001$, $n_{\rm T} = 0.26$: the validity of the consistency relation cannot be excluded by this experiment. For the same amount of GW, a detection by eLISA is expected at 95% C.L., which would clearly exclude the validity of the consistency relation. It follows that, **in some cases, detectors at small scales are expected to be more powerful than CMB experiments** in testing the consistency relation.

References: The work is based on the following REVIEW:
[1] Guzzetti et al., *Gravitational waves form inflation* (2016), Riv. Nuovo Cim. **39**, 9 (2016), arXiv:1605.01615.

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