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Understanding the importance of transient resonances in extreme mass ratio inspirals

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Extreme-mass-ratio inspirals (EMRIs) occur when a compact object orbits a much larger one—like a solar-mass black hole around a supermassive black hole. The orbit has 3 orbital frequencies (Ω_r, Ω_θ, Ω_ϕ) which evolve through the inspiral. If the radial frequency Ω_r and polar frequency Ω_θ become commensurate, the system passes through a **transient resonance** [1]. Evolving through resonance can cause a jump in the evolution of the orbital parameters. In [2] we study these jumps and their impact on EMRI detection. Jumps are smaller for lower eccentricity orbits. Since most EMRIs have small eccentricities when passing through resonances, we expect that the impact of detection will be small: there is a loss of $\sim 4\%$ of signals if jumps are not included in the EMRI templates.

Resonances

When Ω_r and Ω_θ are a rational ratio of each other there is a resonance. The orbit does not cover the whole allowed r–θ plane, but cycles over a single loop [3]. The orbital phase can be expressed as

$$\varphi \sim (n_r\Omega_r - n_\theta\Omega_\theta)\, t + \left(n_r\dot{\Omega}_r - n_\theta\dot{\Omega}_\theta\right) t^2 + \ldots = \Omega t + 2\pi\frac{t^2}{\tau_{\text{res}}^2} + \ldots$$

On resonance $\Omega = 0$ so the first term, which normally dominates, goes to zero. During resonance the second term governs dynamics for a duration set by the resonance time τ_{res} [4]. The evolution of the inspiral is determined by the gravitational self-force, and across a resonance, terms that usually average to zero can combine coherently, significantly impacting the orbital motion [1].

Resonances are not included in **adiabatic** models. These average the self-force over the r–θ plane (or equivalently the r–θ phase). Adiabatic waveforms quickly dephase from true EMRI waveforms if passing through resonance causes a jump in the orbital parameters.

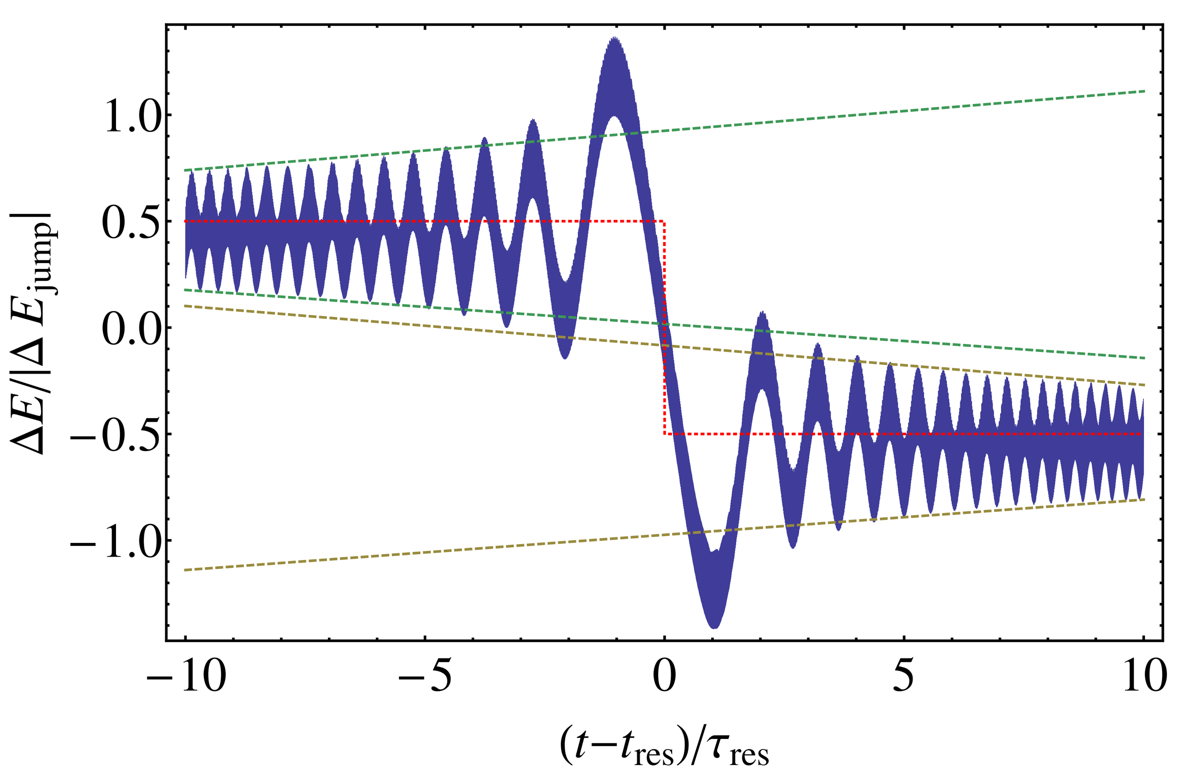

Figure 1. The difference in orbital energy ΔE between an instantaneous evolution and an adiabatic model that matches at resonance ($t = t_{\text{res}}$). This is for a 2:3 resonance ($n_\theta = 2$, $n_r = 3$). The size of the jump ΔE_{jump} here is calculated from the bounding lines.

Jumps

The enhancement (or decrement) of change in the orbital parameters across resonance leads to a **jump**. Fig. 1 shows an example of a jump for a 2:3 resonance.

The size of the jump in quantity $\mathcal{I}$ is [1,5]

$$\Delta\mathcal{I}_{\text{jump}} = \eta \sum_{s\neq 0} F_s^{(1)} \tau_{\text{res},\,s} \exp\left[i\left(s\varphi_0 + \frac{\pi}{4}\operatorname{sgn} s\dot{\Omega}\right)\right],$$

where $\eta \ll 1$ is the mass ratio, $F_s^{(1)}$ is the Fourier component of the force evolving $\mathcal{I}$, $\tau_{\text{res},\,s}$ is the resonance time for the s-th harmonic, φ_0 is the phase on resonance, and the exponential is a function which averages to zero over all phases.

The size of jumps decreases for lower eccentricity or lower inclination orbits [6]: there can be no resonances for circular orbits. Fig. 2 shows how the size of the jump for the orbital semilatus rectum changes with eccentricity.

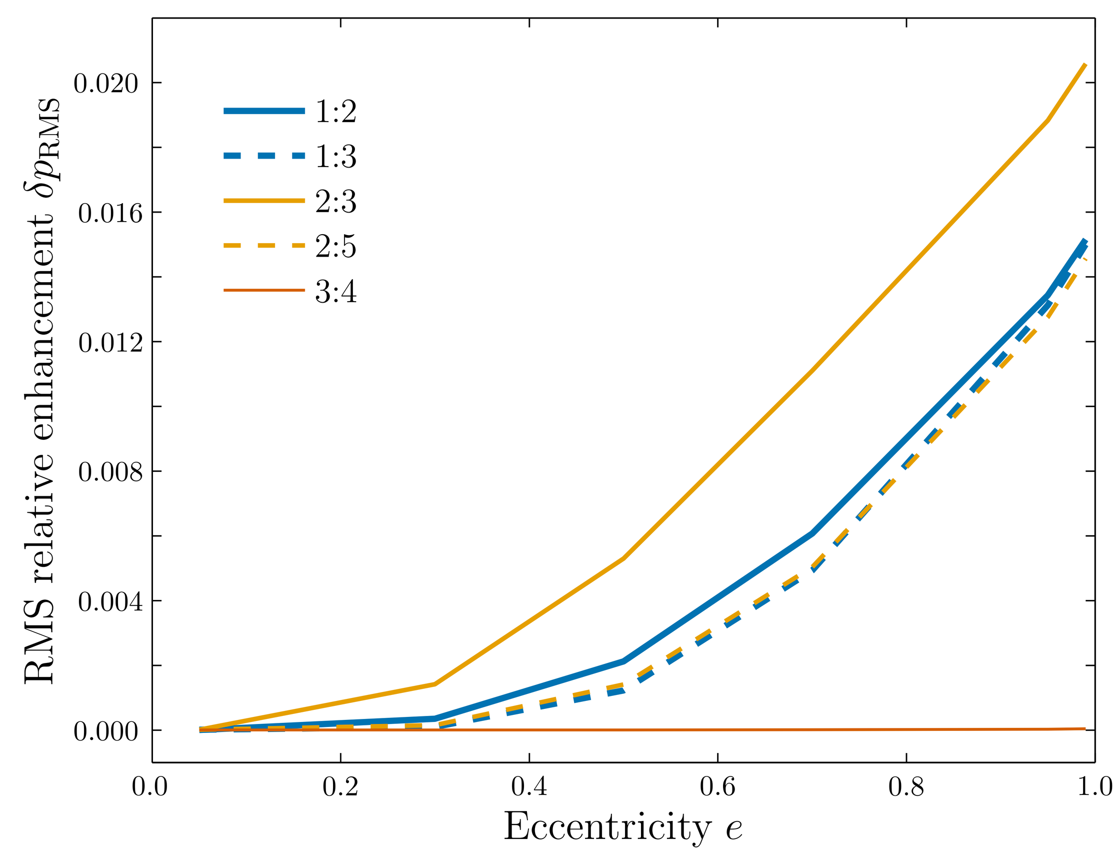

Figure 2. Jump in the semilatus rectum p as a function of eccentricity e. The relative enhancement is the resonant jump divided by the adiabatic change across resonance $\delta p = \Delta p_{\text{jump}}/(\dot{p}_{\text{ad}}\tau_{\text{res}})$. The plotted value is the root-mean-square across a grid of spins and inclinations.

eLISA EMRI Population

We studied an **astrophysical population** of EMRIs that could be detectable with eLISA. The eccentricity distribution follows [7]. As gravitational-wave emission circularises orbits, when important low-order resonances are encountered the eccentricity is small and so are the resonant jumps. Overlaps with adiabatic waveforms are good enough to still detect EMRIs in most cases. Fig. 3 shows the distribution of signal-to-noise ratios (SNRs) ρ for the EMRIs before accounting for resonances and after including the reduction from imperfect waveforms: $\sim 4\%$ of signals drop below the assumed detection threshold $\rho_{\text{thres}} = 15$.

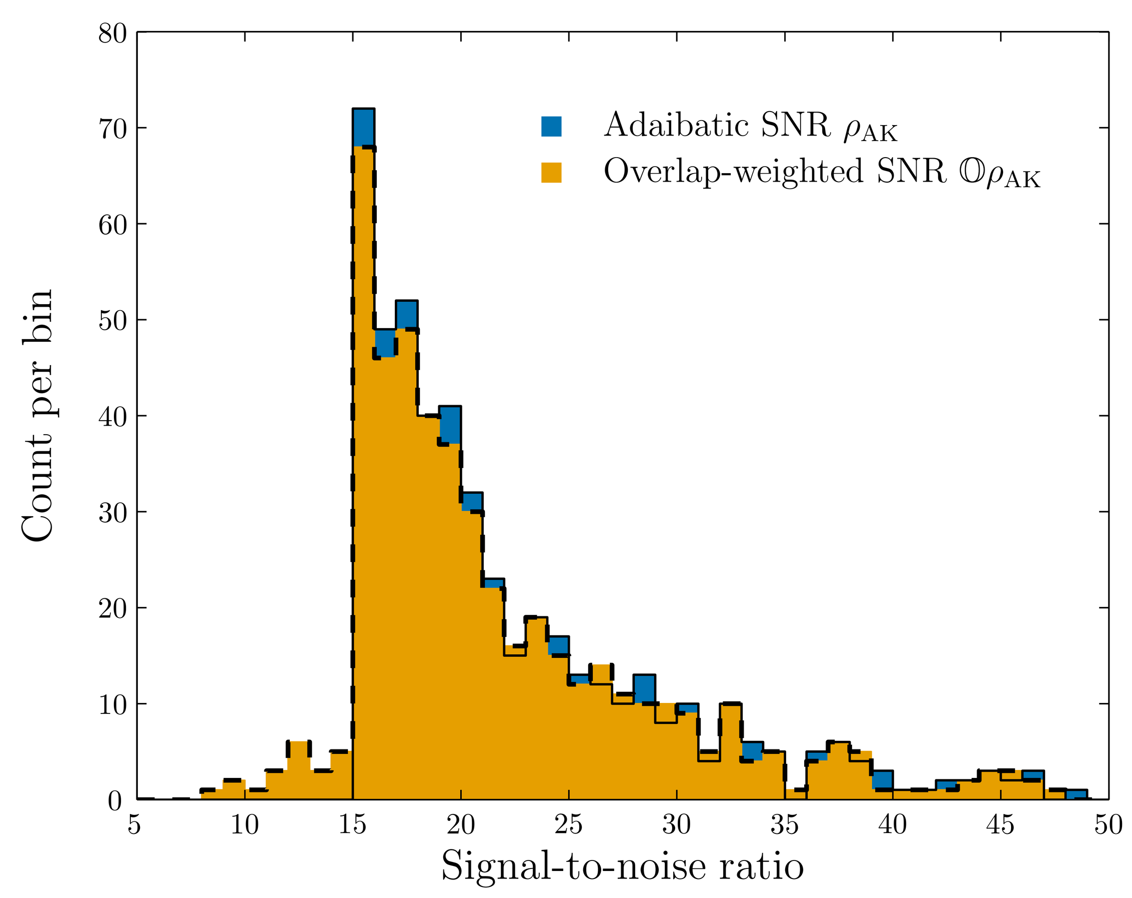

Figure 3. SNRs for detectable EMRIs. The solid outlined (blue) histogram shows the intrinsic SNRs and the dash outlined (orange) factors in the overlap $\mathbb{O}$ between adiabatic and instantaneous waveforms.

Conclusions

- Transient resonances are a generic feature of EMRIs.
- Passing through resonance causes a jump in the evolution.
- Ignoring resonances leads to waveform mismatch.
- The jump depends upon the phase on resonance. Its size depends upon the orbit and the resonance.
- Jumps are small for most EMRIs because of their eccentricity: **detectability** is not significantly reduced.
- Effect of resonances on **parameter estimation** is to be investigated.

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