The self-force for non-geodesic motion

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Abstract

The self-force uses a perturbation of Einstein's field equations in the mass ratio to describe the motion of a point particle in curved spacetime. EMRIs can be modelled as a point mass in Kerr spacetime making them an ideal application. We calculate the self-force for non-geodesic motion (a requisite for self-consistent orbital evolution) in two parts: regularization and computation. For the former, by the mode-sum approach we give the required, previously unknown, *regularization parameters*, as well as higher-order terms that increase the mode-sum convergence rate. For computing the latter, we consider accelerated circular and bound eccentric orbits calculated in the frequency-domain. We discuss how the 'memory' of the self-force can be probed by considering certain accelerated trajectories.

The Detweiler-Whiting singular field

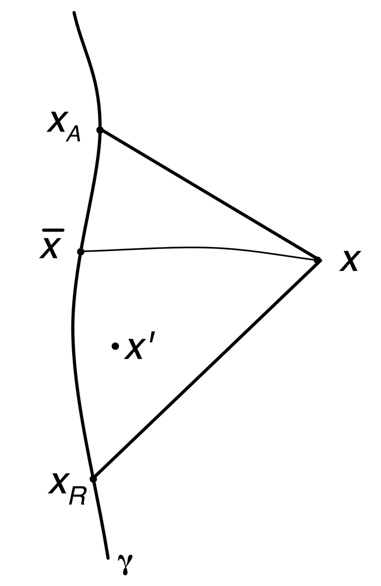
In self-force calculations, a point mass in curved space-time gives rise to a field that diverges at the particle [1]. By isolating and subtracting the singular component from the retarded field, we are left with the regular part, which is (by construction) wholly responsible for the self-force:

$$\Phi_R = \Phi_{ret} - \Phi_S \quad \text{where} \quad (\Box - \xi R - m^2)\Phi_{ret} = q \int \sqrt{-g}\,\delta_4(x, x(\tau))\,d\tau.$$

We use the Detweiler-Whiting singular field [2] which for a scalar field gives,

$$\Phi_S(x) = q \int_\gamma G_S(x, z(\tau))\,d\tau = \frac{q}{2}\left[\frac{U(x,x')}{\sigma_{c'}u^{c'}}\right]_{x'=x_R}^{x'=x_A} + \frac{q}{2}\int_{\tau_R}^{\tau_A} V(x, z(\tau))\,d\tau.$$

The resulting regular field also satisfies the *homogeneous* wave equation.



We expand all bitensors around an arbitrary point $\bar{x}$ on the worldline, γ.

Mode-sum rotated coordinates

Previous mode-sum calculations [3, 4, 5] profited from working in a rotated coordinate frame after calculating the singular field:

$$w_1 = 2\sin\left(\frac{\alpha}{2}\right)\cos\beta, \qquad w_2 = 2\sin\left(\frac{\alpha}{2}\right)\sin\beta,$$

where α and β are the rotated angular coordinates:

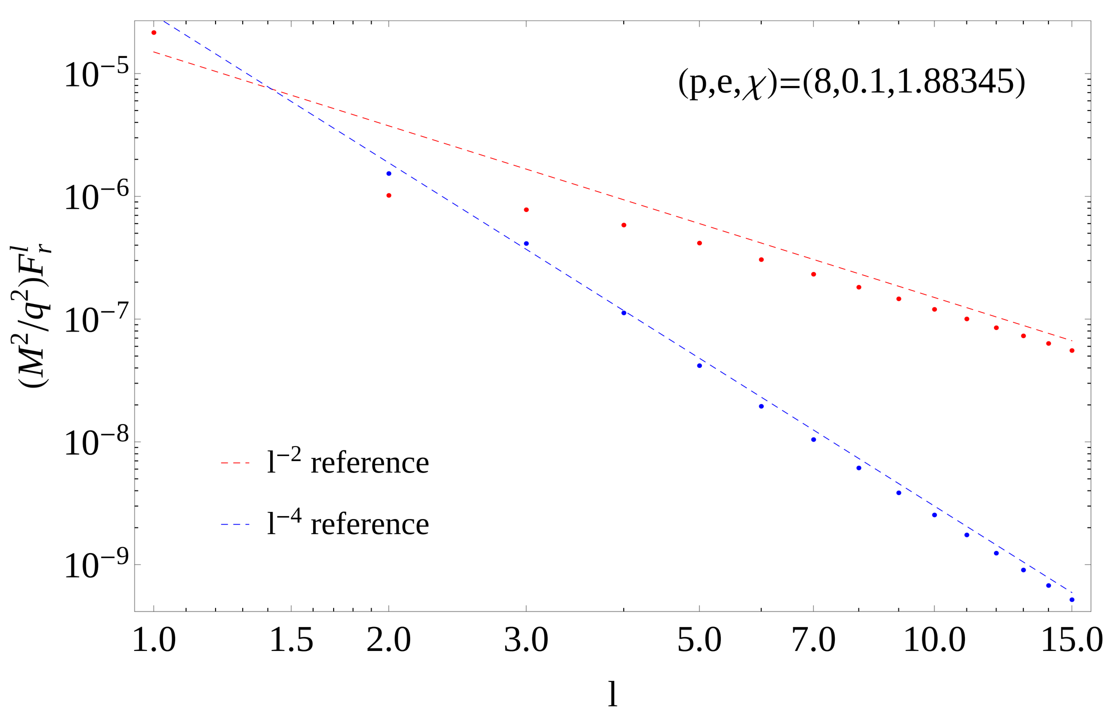
$$\sin\theta\cos\phi = \cos\alpha,$$
$$\sin\theta\sin\phi = \sin\alpha\cos\beta,$$
$$\cos\theta = \sin\alpha\sin\beta.$$

By carrying out this rotation prior to any calculations, we obtain a much higher accuracy.

Mode-sum regularisation

We obtain a spherical harmonic decomposition of the singular field.This allows us to analytically derive the required reqularization parameters [6, 7], which when combined with numerical calculation of the physical field, give us the self-force. The more regularisation parameters, the faster the convergence.

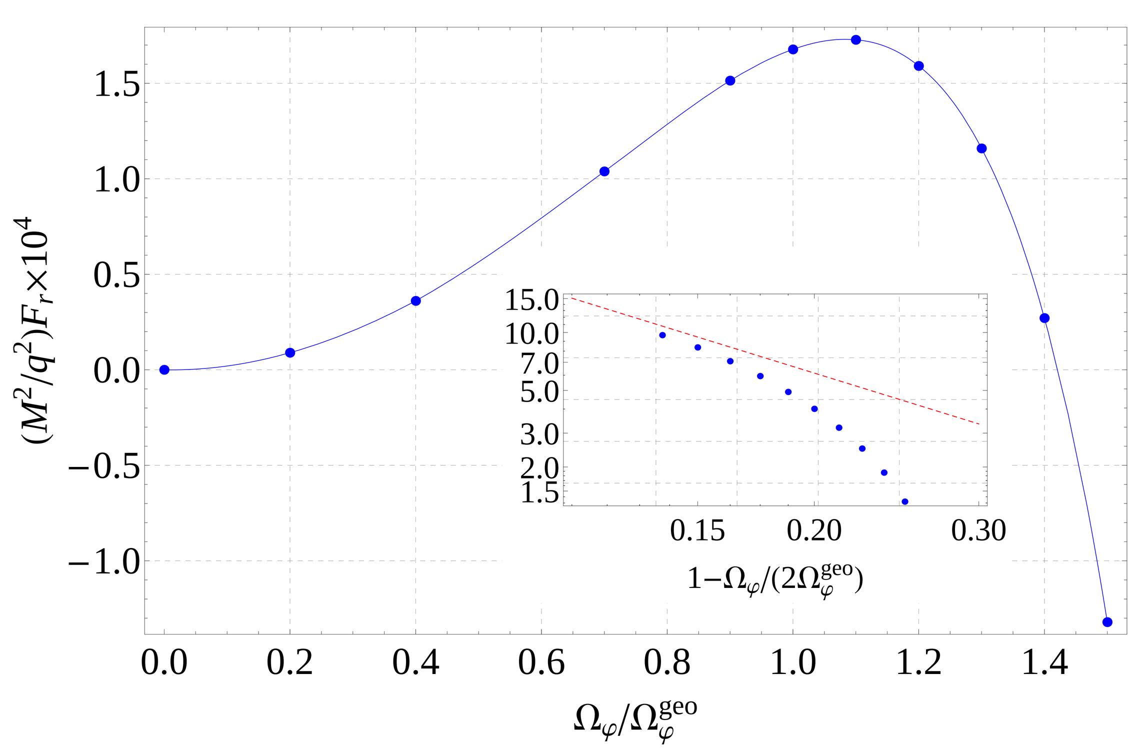
Convergence of the mode-sum



Frequency domain

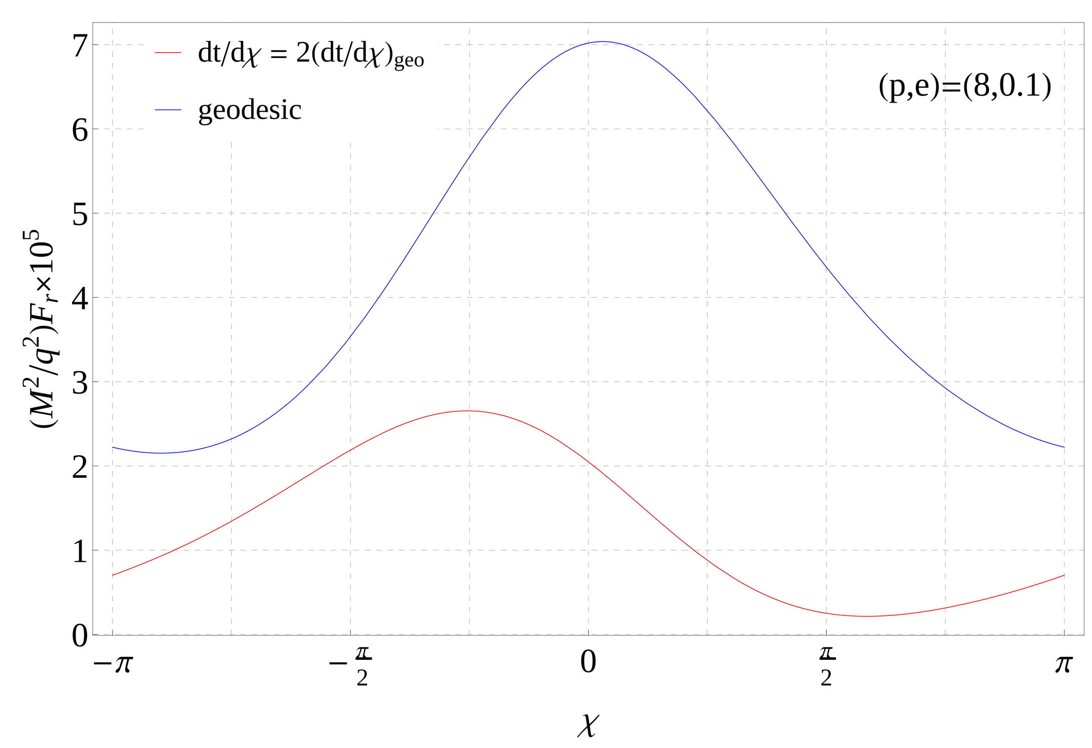
We modify the frequency-domain code in [8], which numerically calculates the physical field within the mode-sum method, by including generic accelerated motion via the forced geodesic-equation $u^\beta\nabla_\beta(\mu u^\alpha) = \mu a^\alpha$.

Circular orbits



Contravariant radial self-force: Starting at the self-force of a static particle = 0, we apply a uniform acceleration and observe the self-force.

Bound elliptic orbits

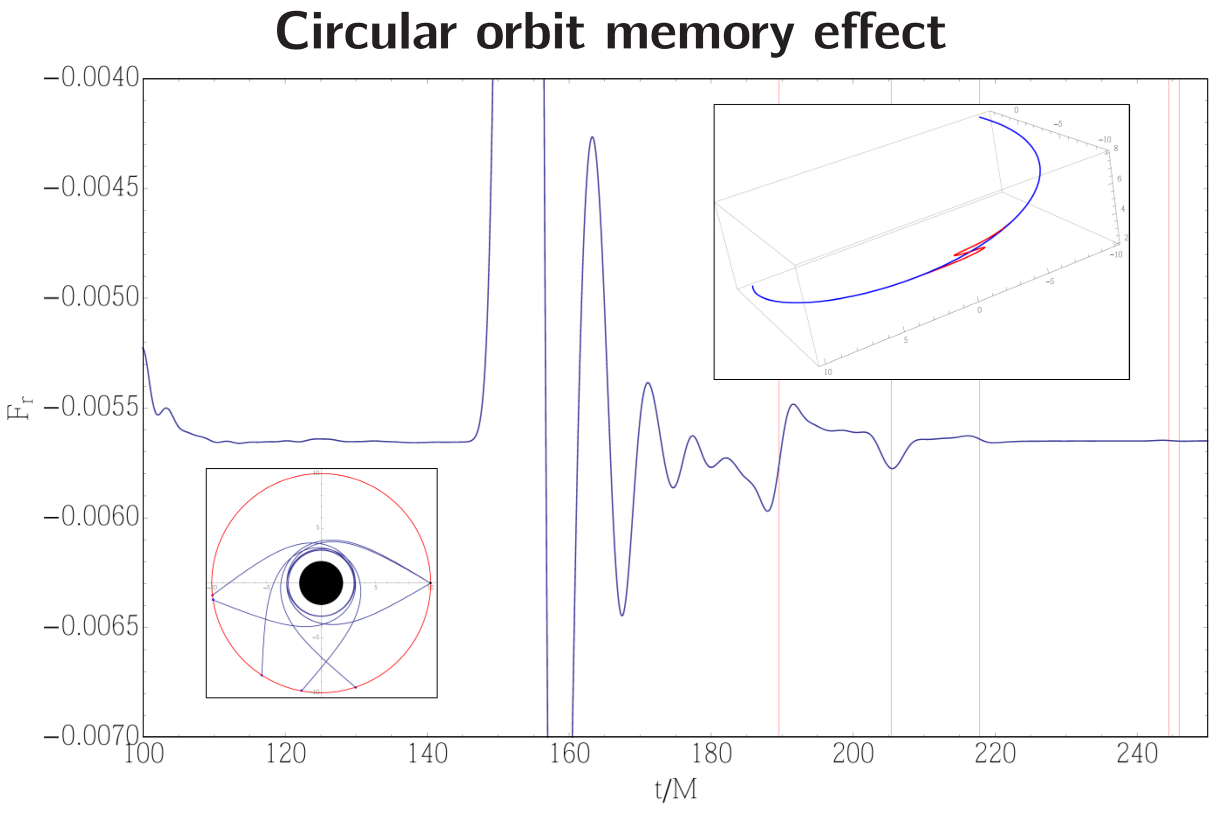


Covariant radial self-force: We maintain the same spatial trajectory of a bound elliptic geodesic but apply a force to half the particle's velocity.

The memory effect in time domain

Numerically calculating the self-force in the time domain, within the mode-sum method, we examine the case of a circular orbit, where we force the particle of it's geodesic and then return it. Ignoring the 'junk' below $t/M = 180$, we see small kinks in the self-force. These coincide with the particle intersecting its own light cone, illustrating the self-force's dependence on its past worldline. We see these effects become negligible as we go further back along the particle's worldline.

Circular orbit memory effect



Conclusions

By considering non-geodesic motion, we were able to probe the nature of the self-force, in particular, the interesting memory effect. The self-force of a particle is formally a functional of its entire past worldline; we were able to illustrate the diminishing contribution of a particle's history in time. We looked at uniform acceleration to observe how the self-force changes when forced off its geodesic, confirming the previously observed change in sign [9] and the divergence of the self-force as it approaches the speed of light. We note how when kept on the same spatial trajectory, the self-force still resembles that of a geodesic. We also produce previously unknown regularisation parameters which increase convergence, and hence accuracy/efficiency, for any future non-geodesic mode-sum self-force calculations. As a first step, this work was carried out in the scalar case; extending to gravity is not so clear as acceleration will affect the source. Extending to Kerr spacetime would also allow us to probe further 'kinks' in the self-force seen by other researchers in the Kerr scalar case [10].

References

[1] Poisson, E., Pound, A., Vega, I.: Living Rev. Relativity 14 (2011), 7
[2] Detweiler, S., Whiting, B.F.: Phys. Rev. D 67, 024025 (2003)
[3] Barack, L., Ori, A.: Phys. Rev. D 61, 061502(R) (2000)
[4] Detweiler, S., Messaritaki, E., Whiting, B.F.: Phys. Rev. D 67, 104016 (2003)
[5] Haas, R., Poisson, E.: Phys. Rev. D 74, 044009 (2006)
[6] Heffernan, A., Ottewill, A., Wardell, B.: Phys. Rev. D 82, 104023 (2012)
[7] Heffernan, A., Ottewill, A., Wardell, B.: Phys. Rev. D 89, 024030 (2014)
[8] Warburton, N., Barack, L.: Phys.Rev.D83:124038 (2011)
[9] Burko, L.M.: Phys. Rev. Lett. 84, 4529 (2000)
[10] Thornburg, J., Wardell, B. in preparation (2016)

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