Gravitational Wave Backgrounds

By Ciarán Conneely, Supervisor Prof. Andrew Jaffe.

The direct detection of gravitational waves in 2015 by Advanced LIGO [1] has started a new era of astronomy and we assume that there are sufficient sources to incoherently sum to an all-pervasive background which can be studied statistically. These waves have many properties that are analogous to photons – especially in terms of their polarisation. Both types of wave have two polarisations: “plus” and “cross” for gravitational waves and “vertical” and “horizontal” for photons. However, these are coordinate system dependent and so analysis of photon polarisation backgrounds (such as measurements of the Cosmic Microwave Background (CMB)) is done in terms of coordinate independent “E” and “B” modes. In an almost mathematically identical way, a gravitational wave background can be considered in terms of similar modes.

This project considers the relative strength of these modes and the relative sensitivity of detectors to them.

CMB Style Decompositions

The gravitational wave background can either be written in terms of + and x polarisations or decomposed using spherical tensor harmonics into E and B modes as below [2]

\[ h_{\mu\nu}(f, k) = H_+(f, k)h_{\mu\nu}^+(k) + H_\times(f, k)h_{\mu\nu}^\times(k) = \sum_{lm} \left( a_{lm}^E(f)Y_{lm}^E(\hat{k}) + a_{lm}^B(f)Y_{lm}^B(\hat{k}) \right) \]

where the strengths of each mode are not necessarily equal. If they are not equal then this indicates something special about the background.

Comparisons of CMB and Gravitational Wave E and B Modes

Both sets of E and B modes can be calculated in terms of spin ± 2 weighted spherical harmonics.

\[ a_{lm}^E(f) = -\frac{1}{2}(a_{2,lm}(f) + a_{-2,lm}(f)) \]

\[ a_{lm}^B(f) = i\frac{1}{2}(a_{2,lm}(f) - a_{-2,lm}(f)) \]

For the single + component, the E and B power spectra are statistically equal.

Power Spectra for Galactic White Dwarf Binaries

Galactic white dwarf binaries are one known stochastic background. Consider a toy model for the galactic distribution where the probability of a white dwarf binary being present decreases with distance from the galactic centre (decreasing more rapidly with galactic latitude than galactic longitude) and simulate it – Fig. 1. Each binary is given randomised parameters [3] and the power spectra computed – Fig. 2.

Examples

Power Spectra for a Cosmic String Loop

A single cosmic string loop viewed from a particular direction can produce pure + polarised gravitational waves [4]. The E and B power spectra are computed in Fig. 3 for a single + polarised source in the z-direction.

Conclusions

- For the white dwarf background, the E and B power spectra are statistically equal.
- For the single + polarised source (i.e. representing a single cosmic string loop) the E and B spectra only diverge at small scales close to the pixel scale. This makes sense because the source is small and so on large scales there should be no significant difference.

This last point is for a single loop with a particular orientation. An unaligned, stochastically distributed ensemble of loops would have equal strength E and B modes.

References: